CHAPTERWISE SOLUTION

Based on **NCERT** Textbook

MATHEMATICS

Class

9

Student Advisor
(Editorial Team)

INDEX

 Published and a product of Student Advisor Publication Pvt. Ltd. E-38, Industrial Area, Mathura (U.P.)

1 Number Systems

Textbook Exercises 1.1

Q.1. Is zero a rational number ? Can you write it in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$?

Sol. Yes, zero is a rational number.

Yes, it can be written in the form $\frac{p}{q}$ as follows :

$\frac{0}{1}=\frac{0}{2}=\frac{0}{3}$ etc., denominator q can also be taken as negative integer. **Ans.**

Q.2. Find six rational numbers between 3 and 4.

Sol. To find six rational numbers between 3 and 4, we write rational numbers 3 and 4 with denominator 7(6 + 1) as follows :

$$3 = \frac{3\times7}{7}=\frac{21}{7};$$

$$4 = \frac{4\times7}{7}=\frac{28}{7}$$

Six rational numbers between 3 and 4 are

$$\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7} \text{ and } \frac{27}{7}$$ **Ans.**

Q.3. Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

Sol. To find 5 rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$, we multiply the numerator and denominator of $\frac{3}{5}$ and $\frac{4}{5}$ by 6(5 + 1).

$$\frac{3}{5} = \frac{3\times6}{5\times6}=\frac{18}{30}$$

$$\frac{4}{5} = \frac{4\times6}{5\times6}=\frac{24}{30}$$

Five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$ are:

$$\frac{19}{30}, \frac{20}{30}, \frac{21}{30}, \frac{22}{30} \text{ and } \frac{23}{30}$$ **Ans.**

Q.4. State whether the following statements are true or false. Give reasons for your answers:

(i) Every natural number is a whole number.

(ii) Every integer is a whole number.

(iii) Every rational number is a whole number.

Sol. (i) True, since the collection of the whole numbers contains all the natural numbers.

(ii) False, since negative integers are not whole numbers.

(iii) False, since $\frac{2}{5}$ is a rational number but not a whole number.

Textbook Exercises 1.2

Q.1. State whether the following statements are true or false. Justify your answers.

(i) Every irrational number is a real number.

(ii) Every point on the number line is of the form $\sqrt{m}$, where m is a natural number.

(iii) Every real number is an irrational number.

Sol. (i) True, since the collection of rational and irrational numbers is said to be real numbers.

(ii) False, since negative number cannot be square root of any natural number.

(iii) False, since real numbers are the collection of rational and irrational numbers.

Q.2. Are the square roots of all positive integers irrational ? If not, give an example of the square root of a number that is a rational number.

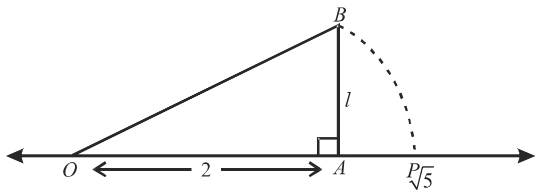
Sol. Square root of all positive integers are not irrational number.

Since $\sqrt{16}$ = 4, which is a rational number.

Hence, the given statement is wrong.

Q.3. Show how $\sqrt{5}$ can be represented on the number line.

Sol. Draw a number line. Take O as origin. It represents 0. Let OA = 2 units and draw $AB \perp OA$ such that AB = 1 unit. Join OB.



By Pythagoras theorem,

$$OB^2 = OA^2 + AB^2$$

$$\Rightarrow \quad OB^2 = 2^2 + 1^2 \Rightarrow OB^2 = 4 + 1$$

$$\Rightarrow \quad OB^2 = 5 \quad \Rightarrow OB = \sqrt{5} \text{ units}$$

with O as centre and OB as radius, draw an arc which cuts the number line at P, then

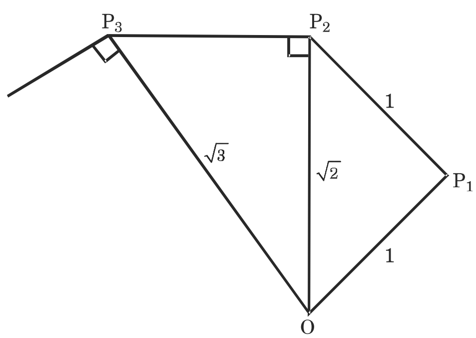
$$OP = OB = \sqrt{5} \text{ units.}$$

Thus, P represents $\sqrt{5}$ on the number line.

Ans.

Q.4. Perform a Class room activity using the 'square root spiral'.

Sol. Take a large sheet of paper and construct the 'square root spiral' in the following fashion.



Start with a point O and draw a line segment OP_1 of unit length. Draw a line segment P_1P_2 perpendicular to OP_1 of unit length (see figure). Now draw a line segment P_2P_3 perpendicular to OP_2. Then draw a line segment P_3P_4 perpendicular to OP_3. Continuing in this manner, you can get the line segment $P_{n-1}P_n$ by drawing a line segment of unit length perpendicular to OP_{n-1}. In this manner, you will have created the points $P_2, P_3, \ldots, P_n, \ldots$ and joined them to create a beautiful spiral depicting $\sqrt{2}, \sqrt{3}, \sqrt{4}, \ldots$

Textbook Exercises 1.3

Q.1. Write the following in decimal form and say what kind of decimal expansion each has :

(i) $\frac{36}{100}$ (ii) $\frac{1}{11}$ (iii) $4\frac{1}{8}$

(iv) $\frac{3}{13}$ (v) $\frac{2}{11}$ (vi) $\frac{329}{400}$.

Sol. (i) By long division, we have

$$\begin{array}{r|l} 100 & 36 \quad 0{\cdot}36 \\ & \underline{0} \\ & 360 \\ & \underline{300} \\ & 600 \\ & \underline{600} \\ & \times \end{array}$$

Hence, $\frac{36}{100} = 0{\cdot}36.$

It has terminating decimal expansion.

(ii)

$$\begin{array}{r|l} 11 & 1 \quad 0{\cdot}090909\ldots \\ & \underline{0} \\ & 10 \\ & \underline{0} \\ & 100 \\ & \underline{99} \\ & 10 \\ & \underline{0} \\ & 100 \\ & \underline{99} \\ & 10 \\ & \underline{0} \\ & 100 \\ & \underline{99} \\ & 1 \end{array}$$

Hence, $\frac{1}{11} = 0{\cdot}090909 = 0{\cdot}\overline{09}$.

It has non-terminating and repeating decimal expansion. **Ans.**

(iii) $4\frac{1}{8} = \frac{33}{8}$

$$\begin{array}{r|l} 8 & 33 \quad 4{\cdot}125 \\ & \underline{32} \\ & 10 \\ & \underline{8} \\ & 20 \\ & \underline{16} \\ & 40 \\ & \underline{40} \\ & \times \end{array}$$

Hence, $4\frac{1}{8} = 4{\cdot}125.$

It has terminating decimal expansion.

Ans.

(iv)

$$\begin{array}{r|l} 13 & 30.0000000 \quad 0.2307692307..... \\ & -26 \\ & \quad 40 \\ & \quad -39 \\ & \quad\quad 100 \\ & \quad\quad -91 \\ & \quad\quad\quad 90 \\ & \quad\quad\quad -78 \\ & \quad\quad\quad 120 \\ & \quad\quad\quad -117 \\ & \quad\quad\quad\quad 30 \\ & \quad\quad\quad\quad -26 \\ & \quad\quad\quad\quad\quad 40 \\ & \quad\quad\quad\quad\quad -39 \\ & \quad\quad\quad\quad\quad 100 \\ & \quad\quad\quad\quad\quad -91 \\ & \quad\quad\quad\quad\quad\quad 9 \end{array}$$

Hence, $\mathbf{\frac{3}{13}} = 0{\cdot}2307692307692...$

$= 0{\cdot}\overline{230769}$.

It has non-terminating repeating decimal expansion. **Ans.**

(v)

$$\begin{array}{r|l} 11 & 2 \quad 0{\cdot}1818 \\ & 0 \\ & 20 \\ & 11 \\ & \quad 90 \\ & \quad 88 \\ & \quad 20 \\ & \quad 11 \\ & \quad\quad 90 \\ & \quad\quad 88 \\ & \quad\quad\quad 2 \end{array}$$

Hence, $\frac{2}{11} = 0{\cdot}1818...$

$= 0{\cdot}\overline{18}$.

It has non-terminating repeating decimal expansion.

(vi)

$$\begin{array}{r|l} 400 & 329 \quad 0{\cdot}8225 \\ & 0 \\ & 3290 \\ & 3200 \\ & \quad 900 \\ & \quad 800 \\ & \quad 1000 \\ & \quad 800 \\ & \quad 2000 \\ & \quad 2000 \\ & \quad\quad \times \end{array}$$

Hence, $\frac{329}{400} = 0{\cdot}8225$.

It has terminating decimal expansion.

Ans.

Q.2. You know that $\frac{1}{7} = 0{\cdot}\overline{142857}$. Can you predict what the decimal expansions of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ are, without actually doing the long division ? If so, how ?

Sol. Yes, we can predict what the decimal expansions of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ are without actually doing the long division.

Now 1 divide by 7

$$\begin{array}{r|l} 7 & 1 \quad 0{\cdot}142857 \\ & 0 \\ & 10 \\ & \;\;7 \\ & \;\;30 \\ & \;\;28 \\ & \quad 20 \\ & \quad 14 \\ & \quad\; 60 \\ & \quad\; 56 \\ & \quad\quad 40 \\ & \quad\quad 35 \\ & \quad\quad\; 50 \\ & \quad\quad\; 49 \\ & \quad\quad\quad 1 \end{array}$$

We observe that remainder repeat after six division. So, it has repeating block of six digits. The given rational numbers $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}$ and $\frac{6}{7}$ will also have repeating block of six digits in the decimal expansions. So, to obtain the decimal expansions of given rational numbers, multiplying $0{\cdot}\overline{142857}$ successively by 2, 3, 4, 5 and 6 as follows:

$$\frac{2}{7} = 2 \times \frac{1}{7} = 2 \times 0{\cdot}\overline{142857} = 0{\cdot}\overline{285714}$$

$$\frac{3}{7} = 3 \times \frac{1}{7} = 3 \times 0{\cdot}\overline{142857} = 0{\cdot}\overline{428571}$$

$$\frac{4}{7} = 4 \times \frac{1}{7} = 4 \times 0{\cdot}\overline{142857} = 0{\cdot}\overline{571428}$$

$$\frac{5}{7} = 5 \times \frac{1}{7} = 5 \times 0{\cdot}\overline{142857} = 0{\cdot}\overline{714285}$$

and $\frac{6}{7} = 6 \times \frac{1}{7} = 6 \times 0{\cdot}\overline{142857} = 0{\cdot}\overline{857142}$.

Ans.

Q.3. Express the following in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$:

(i) $0{\cdot}\overline{6}$ (ii) $0{\cdot}4\overline{7}$ (iii) $0{\cdot}\overline{001}$.

Sol. (i) Let $x = 0{\cdot}\overline{6} = 0{\cdot}6666...$ ...(i)

Multiplying equation (i) by 10, we get

$$10x = 6{\cdot}\overline{6} \quad \text{...(ii)}$$

Subtracting equation (i) from (ii), we get

$$\begin{array}{rl} 10x = 6{\cdot}\overline{6} & \text{...(ii)} \\ x = 0{\cdot}\overline{6} & \text{...(i)} \\ - \quad - & \\ \hline 9x = 6{\cdot}0 \Rightarrow x = \frac{6}{9} = \frac{2}{3} & \end{array}$$

Hence, $0{\cdot}\overline{6} = \frac{2}{3}$. **Ans.**

(ii) Let $x = 0{\cdot}4\overline{7} = 0{\cdot}47777...$ ...(i)

Since, there is one non-repeating digit after the decimal.

Therefore, multiplying equation (i) by 10. So that this digit shifted before the decimal, we get

$$10x = 4{\cdot}\overline{7} = 4{\cdot}7777... \quad \text{...(ii)}$$

Multiplying the equation (ii) by 10, we get

$$100x = 47{\cdot}\overline{7} \quad \text{...(iii)}$$

Subtracting equation (ii) from (iii), we get

$$\begin{array}{rl} 100x = 47{\cdot}\overline{7} & \text{...(iii)} \\ 10x = 4{\cdot}\overline{7} & \text{...(ii)} \\ - \quad - & \\ \hline 90x = 43{\cdot}0 \Rightarrow x = \frac{43}{90} & \end{array}$$

Hence, $0{\cdot}4\overline{7} = \frac{43}{90}$. **Ans.**

(iii) Let $x = 0{\cdot}\overline{001} = 0{\cdot}001001001...$ ...(i)

Multiplying equation (i) by 1000, we get

$$1000x = 1{\cdot}\overline{001} \quad \text{...(ii)}$$

Subtracting equation (i) from (ii), we get

$$\begin{array}{rl} 1000x = 1{\cdot}\overline{001} & \text{...(ii)} \\ x = 0{\cdot}\overline{001} & \text{...(i)} \\ - \quad - & \\ \hline 999x = 1 \Rightarrow x = \frac{1}{999} & \end{array}$$

Hence, $0{\cdot}\overline{001} = \frac{1}{999}$. **Ans.**

Q.4. Express $0{\cdot}99999...$ in the form $\frac{p}{q}$. Are you surprised by your answer ? With your teacher and classmates discuss why the answer makes sense.

Sol. Let $x = 0{\cdot}99999... = 0{\cdot}\overline{9}$ ...(i)

Multiplying equation (i) by 10, we get

$$10x = 9{\cdot}99999... = 9{\cdot}\overline{9} \quad \text{...(ii)}$$

Subtracting equation (i) from (ii), we get

$$\begin{array}{rl} 10x = 9{\cdot}\overline{9} & \text{...(ii)} \\ x = 0{\cdot}\overline{9} & \text{...(i)} \\ - \quad - & \\ \hline 9x = 9 & \\ x = \frac{9}{9} = 1 & \end{array}$$

Hence, $0{\cdot}99999... = 1$

We observe that there is no gap between 1 and $0{\cdot}99999...$ and hence they are equal.

Ans.

Q.5. What can the maximum number of digits be in the repeating block of digits in the decimal expansion of $\frac{1}{17}$? Perform the division to check your answer.

Sol. By long division method, we have

$$\begin{array}{r|l} & 0.0588235294117647.... \\ 17 & 1.0000000000000000 \\ & -85 \\ & \overline{150} \\ & -136 \\ & \overline{140} \\ & -136 \\ & \overline{40} \\ & -34 \\ & \overline{60} \\ & -51 \\ & \overline{90} \\ & -85 \\ & \overline{50} \\ & -34 \\ & \overline{160} \\ & -153 \\ & \overline{70} \\ & -68 \\ & \overline{20} \\ & -17 \end{array}$$

$$\begin{array}{r} 30 \\ -17 \\ \hline 130 \\ -119 \\ \hline 110 \\ -102 \\ \hline 80 \\ -68 \\ \hline 120 \\ -119 \\ \hline 1 \end{array}$$

We observe that the remainder start repeating after 16 divisions.

Hence, $\frac{1}{17} = 0{\cdot}\overline{0588235294117647}$. **Ans.**

Q.6. Look at several examples of rational numbers in the form $\frac{p}{q}$ $(q \neq 0)$, where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy ?

Sol. Let us consider the various rational numbers be $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{7}{20}, \frac{23}{25}, \frac{12}{125}$, etc.

In all these rational numbers multiplying the denominator by such numbers so that it becomes 10 or power of 10, we get

$$\frac{1}{2} = \frac{1\times5}{2\times5} = \frac{5}{10} = 0{\cdot}5$$

$$\frac{1}{4} = \frac{1\times25}{4\times25} = \frac{25}{100} = 0{\cdot}25$$

$$\frac{1}{8} = \frac{1\times125}{8\times125} = \frac{125}{1000} = 0{\cdot}125$$

$$\frac{7}{20} = \frac{7\times5}{20\times5} = \frac{35}{100} = 0{\cdot}35$$

$$\frac{23}{25} = \frac{23\times4}{25\times4} = \frac{92}{100} = 0{\cdot}92$$

$$\frac{12}{125} = \frac{12\times8}{125\times8} = \frac{96}{1000} = 0{\cdot}096$$

In these cases we observe that to obtain a terminating decimal form of given rational numbers we have to multiplied the denominator of these rational numbers by a suitable integer. But it is possible when denominator of given rational numbers be 2^n5^m as the prime factor, where m and n are non-negative integers. Thus we have the following property :

If the prime factorisation of denominator of given rational numbers is of the form 2^n5^m, where m, n are non-negative integers, then it can be represented as a terminating decimal. **Ans.**

Q.7. Write three numbers whose decimal expansions are non-terminating non recurring.

Sol. Three numbers whose decimal expansions are non-terminating non-recurring are

$\cdot01001000100001...$,
$0{\cdot}202002000200002...$ and
$0{\cdot}003000300003...$ **Ans.**

Q.8. Find three different irrational numbers between rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.

Sol. We have,

$$\frac{5}{7} = 0{\cdot}714285714...$$

$$\frac{9}{11} = 0{\cdot}81818181...$$

Three irrational numbers between

$0{\cdot}714285714...$ and $0{\cdot}81818181...$ are :
$0{\cdot}75075007500075000075...$,
$0{\cdot}767076700767000767...$ and
$0{\cdot}808008000800008...$

Hence, three irrational numbers between $\frac{5}{7}$ and $\frac{9}{11}$ are :

$0{\cdot}75075007500075000075...$,
$0{\cdot}767076700767000767...$ and
$0{\cdot}808008000800008...$ **Ans.**

Q.9. Classify the following numbers as rational or irrational :

(i) $\sqrt{23}$ (ii) $\sqrt{225}$
(iii) $0{\cdot}3796$ (iv) $7{\cdot}478478...$
(v) $1{\cdot}101001000100001...$

Sol. (i) Since 23 is not a perfect square. So $\sqrt{23}$ is an irrational number.

(ii) $\sqrt{225} = \sqrt{3\times3\times5\times5} = 3 \times 5 = 15$ which is a rational number.

Hence, $\sqrt{225}$ is a rational number.

(iii) 0·3796 is a terminating decimal expansion, so it is a rational number.

(iv) 7·478478... is non-terminating repeating decimal expansion, so it is a rational number.

(v) 1·101001000100001... is non-terminating non-repeating decimal expansion, so it is an irrational number.

Textbook Exercises 1.4

Q.1. Classify the following numbers as rational or irrational :

(i) $2-\sqrt{5}$ (ii) $(3+\sqrt{23})-\sqrt{23}$

(iii) $\dfrac{2\sqrt{7}}{7\sqrt{7}}$ (iv) $\dfrac{1}{\sqrt{2}}$ (v) 2π.

Sol. We have (i) $2-\sqrt{5}$

Since, difference of a rational and an irrational number is an irrational number. Therefore, $2-\sqrt{5}$ is an irrational number.

(ii) $(3+\sqrt{23})-\sqrt{23}$

$3+\sqrt{23}-\sqrt{23} = 3$

It is a rational number.

(iii) $\dfrac{2\sqrt{7}}{7\sqrt{7}} = \dfrac{2}{7}$

It is a rational number.

(iv) $\dfrac{1}{\sqrt{2}}$

Since, quotient of a rational number and an irrational number is an irrational number.

(v) 2π

Since, π is an irrational number and product of a rational number and an irrational number is an irrational number.

Q.2. Simplify each of the following expressions:

(i) $(3+\sqrt{3})(2+\sqrt{2})$ (ii) $(3+\sqrt{3})(3-\sqrt{3})$

(iii) $(\sqrt{5}+\sqrt{2})^2$ (iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$

Sol. (i) We have

$$(3+\sqrt{3})(2+\sqrt{2}) = 3\times2+3\sqrt{2}+2\sqrt{3}+\sqrt{2}\times\sqrt{3}$$
$$= 6+3\sqrt{2}+2\sqrt{3}+\sqrt{6}.$$

(ii) $$(3+\sqrt{3})(3-\sqrt{3}) = (3)^2-(\sqrt{3})^2$$
$$= 9-3 = 6.$$

(iii) $$(\sqrt{5}+\sqrt{2})^2 = (\sqrt{5})^2+2\times\sqrt{5}\times\sqrt{2}+(\sqrt{2})^2$$
$$= 5+2\sqrt{10}+2 = 7+2\sqrt{10}.$$

(iv) $$(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2}) = (\sqrt{5})^2-(\sqrt{2})^2$$
$$= 5-2 = 3.$$

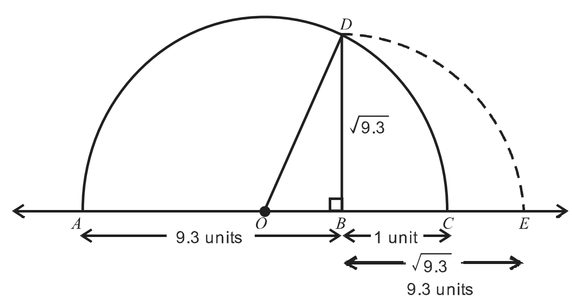
Q.3. Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter (say d). That is, $\pi = \dfrac{c}{d}\cdot$ This seems to contradict the fact that π is irrational. How will you resolve this contradiction ?

Sol. There is no contradiction. Remember that when we measure a length with a scale or any other device, we only get an approximate rational value. So, we may not realise that either c or d is irrational.

Hence, π is an irrational.

Q.4. Represent $\sqrt{9{\cdot}3}$ on the number line.

Sol. Draw a line AB of length 9·3 units. Produce AB to C such that BC = 1 unit. Find the mid-point of AC and marked that point O. Draw a semicircle with centre O and radius OA. Draw a line perpendicular to AC passing through B and intersecting the semicircle at D, then $BD = \sqrt{9{\cdot}3}$ units. Now B as centre and BD as radius, draw an arc meeting AC produced at E, then $BE = BD = \sqrt{9{\cdot}3}$ units.



Q.5. Rationalise the denominators of the following :

(i) $\dfrac{1}{\sqrt{7}}$ (ii) $\dfrac{1}{\sqrt{7}-\sqrt{6}}$

(iii) $\dfrac{1}{\sqrt{5}+\sqrt{2}}$ (iv) $\dfrac{1}{\sqrt{7}-2}$.

Sol. (i) $\dfrac{1}{\sqrt{7}} = \dfrac{1\times\sqrt{7}}{\sqrt{7}\times\sqrt{7}} = \dfrac{\sqrt{7}}{7}$.

(ii) $\dfrac{1}{\sqrt{7}-\sqrt{6}} = \dfrac{1}{(\sqrt{7}-\sqrt{6})}\times\dfrac{(\sqrt{7}+\sqrt{6})}{(\sqrt{7}+\sqrt{6})}$

$= \dfrac{\sqrt{7}+\sqrt{6}}{(\sqrt{7})^2-(\sqrt{6})^2} = \dfrac{\sqrt{7}+\sqrt{6}}{7-6} = \sqrt{7}+\sqrt{6}.$

(iii) $\dfrac{1}{\sqrt{5}+\sqrt{2}} = \dfrac{1}{(\sqrt{5}+\sqrt{2})}\times\dfrac{(\sqrt{5}-\sqrt{2})}{(\sqrt{5}-\sqrt{2})}$

$= \dfrac{\sqrt{5}-\sqrt{2}}{(\sqrt{5})^2-(\sqrt{2})^2} = \dfrac{\sqrt{5}-\sqrt{2}}{5-2} = \dfrac{\sqrt{5}-\sqrt{2}}{3}.$

(iv) $\dfrac{1}{\sqrt{7}-2} = \dfrac{1}{(\sqrt{7}-2)}\times\dfrac{(\sqrt{7}+2)}{(\sqrt{7}+2)}$

$= \dfrac{\sqrt{7}+2}{(\sqrt{7})^2-(2)^2} = \dfrac{\sqrt{7}+2}{7-4} = \dfrac{\sqrt{7}+2}{3}.$

Textbook Exercises 1.5

Q.1. Find : (i) $64^{1/2}$ (ii) $32^{1/5}$ (iii) $125^{1/3}$.

Sol. We have,

(i) $64^{1/2} = (8^2)^{1/2} = 8^{2\times 1/2} = 8^1 = 8.$

(ii) $32^{1/5} = (2^5)^{1/5} = 2^{5\times 1/5} = 2^1 = 2.$

(iii) $125^{1/3} = (5^3)^{1/3} = 5^{3\times 1/3} = 5^1 = 5.$

Q.2. Find : (i) $9^{3/2}$ (ii) $32^{2/5}$ (iii) $16^{3/4}$ (iv) $125^{-1/3}$.

Sol. (i) $9^{3/2} = (3^2)^{3/2} = 3^{2\times 3/2} = 3^3 = 27.$

(ii) $32^{2/5} = (2^5)^{2/5} = 2^{5\times 2/5} = 2^2 = 4.$

(iii) $16^{3/4} = (2^4)^{3/4} = 2^{4\times 3/4} = 2^3 = 8.$

(iv) $125^{-1/3} = (5^3)^{-1/3} = 5^{3\times(-1/3)}$

$= 5^{-1} = \dfrac{1}{5}.$

Q.3. Simplify : (i) $2^{2/3}.2^{1/5}$ (ii) $\left(\dfrac{1}{3^3}\right)^7$ (iii) $\dfrac{11^{1/2}}{11^{1/4}}$ (iv) $7^{1/2}.8^{1/2}$.

Sol. (i) We have,

$2^{2/3}.2^{1/5} = 2^{2/3+1/5}$

$= 2^{(10+3)/15} = 2^{13/15}.$

(ii) $\left(\dfrac{1}{3^3}\right)^7 = \dfrac{1^7}{(3^3)^7} = \dfrac{1}{3^{21}} = 3^{-21}$

(iii) $\dfrac{11^{1/2}}{11^{1/4}} = 11^{1/2-1/4} = 11^{(2-1)/4}$

$= 11^{1/4}.$

(iv) $7^{1/2}.8^{1/2} = (7\times 8)^{1/2} = 56^{1/2}.$

2 Polynomials

Textbook Exercises 2.1

Q.1. Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer :

(i) $4x^2 - 3x + 7$ (ii) $y^2 + \sqrt{2}$

(iii) $3\sqrt{t} + t\sqrt{2}$ (iv) $y + \frac{2}{y}$

(v) $x^{10} + y^3 + t^{50}$.

Sol. (i) We have, $4x^2 - 3x + 7$

Since, the exponent of x in each term is a whole number.

Therefore, the given expression is a polynomial in one variable x.

(ii) We have, $y^2 + \sqrt{2}$

Since, exponent of y is a whole number.

Therefore, the given expression is a polynomial in one variable y.

(iii) We have, $3\sqrt{t} + t\sqrt{2}$

Since, the exponent of t in the Ist term is $\frac{1}{2}$, which is not a whole number.

Therefore, the given expression is not a polynomial.

(iv) We have, $y + \frac{2}{y}$

Since, the exponent of y in the IInd term is -1, which is not a whole number.

Therefore, the given expression is not a polynomial.

(v) We have, $x^{10} + y^3 + t^{50}$

Since, there are three variables x, y, t in the given expression.

Therefore, it is a polynomial in three variables not in one variable.

Q.2. Write the coefficients of x^2 in each of the following:

(i) $2 + x^2 + x$ (ii) $2 - x^2 + x^3$

(iii) $\frac{\pi}{2}x^2 + x$ (iv) $\sqrt{2}x - 1$.

Sol. (i) The coefficient of x^2 in $2 + x^2 + x$ is 1.

(ii) The coefficient of x^2 in $2 - x^2 + x^3$ is -1.

(iii) The coefficient of x^2 in $\frac{\pi}{2}x^2 + x$ is $\frac{\pi}{2}$.

(iv) The coefficient of x^2 in $\sqrt{2}x - 1$ is 0 because there is no term of x^2 in the given expression.

Q.3. Give one example each of a binomial of degree 35, and of a monomial of degree 100.

Sol. Example of a binomial of degree 35 is $3x^{35} - 4$ and example of monomial of degree 100 is $\sqrt{2}y^{100}$ (you can write some more polynomials with different coefficients.)

Q.4. Write the degree of each of the following polynomials :

(i) $5x^3 + 4x^2 + 7x$ (ii) $4 - y^2$

(iii) $5t - \sqrt{7}$ (iv) 3.

Sol. (i) We have, $5x^3 + 4x^2 + 7x$

The highest power term is $5x^3$, and it's exponent is 3. So, the degree of given polynomial is 3.

(ii) We have, $4 - y^2$

The highest power term is $-y^2$, and it's exponent is 2. So, the degree of given polynomial is 2.

(iii) We have, $5t - \sqrt{7}$

The highest power term is $5t$, and it's exponent is 1. So, the degree of given polynomial is 1.

(iv) We have, 3

It is constant polynomial which degree is 0.

Q.5. Classify the following as linear, quadratic and cubic polynomials :

(i) $x^2 + x$ (ii) $x - x^3$ (iii) $y + y^2 + 4$

(iv) $1 + x$ (v) $3t$ (vi) r^2

(vii) $7x^3$.

Sol. (i) The degree of the given polynomial is 2. So, it is the quadratic polynomial.

(ii) The degree of the given polynomial is 3. So, it is the cubic polynomial.

(iii) The degree of the given polynomial is 2. So, it is the quadratic polynomial.

(iv) The degree of the given polynomial is 1. So, it is a linear polynomial.

(v) The degree of the given polynomial is 1. So, it is a linear polynomial.

(vi) The degree of the given polynomial is 2. So, it is a quadratic polynomial.

(vii) The degree of the given polynomial is 3. So, it is a cubic polynomial.

Textbook Exercises 2.2

Q.1. Find the value of the polynomial $5x - 4x^2 + 3$ at :

(i) $x = 0$ (ii) $x = -1$ (iii) $x = 2$.

Sol. Let

$$p(x) = 5x - 4x^2 + 3$$

(i) At $x = 0$, the value of the polynomial $p(x)$ is given by

$$p(0) = 5 \times 0 - 4 \times (0)^2 + 3$$
$$= 0 - 4 \times 0 + 3$$
$$= 0 - 0 + 3 = 3$$

(ii) At $x = -1$, the value of the polynomial $p(x)$ is given by

$$p(-1) = 5 \times (-1) - 4 \times (-1)^2 + 3$$
$$= -5 - 4 + 3$$
$$= -6$$

(iii) At $x = 2$, the value of the polynomial $p(x)$ is given by

$$p(2) = 5 \times 2 - 4 \times (2)^2 + 3$$
$$= 10 - 16 + 3 = -3$$

Q.2. Find $p(0)$, $p(1)$ and $p(2)$ for each of the following polynomials :

(i) $p(y) = y^2 - y + 1$

(ii) $p(t) = 2 + t + 2t^2 - t^3$

(iii) $p(x) = x^3$

(iv) $p(x) = (x - 1)(x + 1)$.

Sol. (i) We have,

$$p(y) = y^2 - y + 1$$
$$p(0) = (0)^2 - 0 + 1$$
$$= 0 - 0 + 1 = 1.$$
$$p(1) = (1)^2 - 1 + 1$$
$$= 1 - 1 + 1 = 1.$$

and $p(2) = (2)^2 - 2 + 1$
$$= 4 - 2 + 1 = 3.$$

(ii) We have, $p(t) = 2 + t + 2t^2 - t^3$

$$p(0) = 2 + 0 + 2 \times (0)^2 - (0)^3$$
$$= 2 + 0 + 0 - 0 = 2.$$
$$p(1) = 2 + 1 + 2 \times (1)^2 - (1)^3$$
$$= 2 + 1 + 2 - 1 = 4.$$

and $p(2) = 2 + 2 + 2 \times (2)^2 - (2)^3$
$$= 2 + 2 + 8 - 8 = 4.$$

(iii) We have,

$$p(x) = x^3$$
$$p(0) = (0)^3 = 0.$$
$$p(1) = (1)^3 = 1.$$

and $p(2) = (2)^3 = 8.$

(iv) We have,

$$p(x) = (x - 1)(x + 1)$$
$$p(0) = (0 - 1)(0 + 1)$$
$$= (-1) \times 1 = -1.$$
$$p(1) = (1 - 1)(1 + 1)$$
$$= 0 \times 2 = 0.$$

and $p(2) = (2 - 1)(2 + 1)$
$$= 1 \times 3 = 3.$$

Hence, $p(0) = -1$, $p(1) = 0$ and $p(2) = 3$.

Q.3. Verify whether the following are zeroes of the polynomial, indicated against them:

(i) $p(x) = 3x + 1$, $x = \frac{-1}{3}$

(ii) $p(x) = 5x - \text{p}$, $x = \frac{4}{5}$

(iii) $p(x) = x^2 - 1$, $x = 1, -1$

(iv) $p(x) = (x + 1)(x - 2)$, $x = -1, 2$

(v) $p(x) = x^2$, $x = 0$

(vi) $p(x) = lx + m$, $x = \frac{-m}{l}$

(vii) $p(x) = 3x^2 - 1$, $x = \frac{-1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

(viii) $p(x) = 2x + 1$, $x = \frac{1}{2}$.

Sol. (i) We have,

$$p(x) = 3x + 1$$

The value of the polynomial $p(x)$ at $x = -\frac{1}{3}$ is given by

$$p\left(\frac{-1}{3}\right) = 3 \times \left(\frac{-1}{3}\right) + 1$$
$$= -1 + 1 = 0$$

Hence, $\frac{-1}{3}$ is a zero of the polynomial $p(x)$.

(ii) We have, $p(x) = 5x - \pi$

The value of the polynomial $p(x)$ at $x = \frac{4}{5}$ is given by

$$p\left(\frac{4}{5}\right) = 5 \times \frac{4}{5} - = 4 - \pi$$

Hence, $\frac{4}{5}$ is not zero of the polynomial $p(x)$.

(iii) We have, $p(x) = x^2 - 1$

The value of the polynomial $p(x)$ at $x = 1$ is given by

$$p(1) = (1)^2 - 1 = 1 - 1 = 0$$

The value of the polynomial $p(x)$ at $x = -1$ is given by

$$p(-1) = (-1)^2 - 1 = 1 - 1 = 0$$

Hence, 1, – 1 both are the zeroes of the polynomial $p(x)$.

(iv) We have, $p(x) = (x + 1)(x - 2)$

The value of the polynomial $p(x)$ at $x = -1$ is given by

$$p(-1) = (-1 + 1)(-1 - 2)$$
$$= 0 \times (-3) = 0$$

The value of the polynomial $p(x)$ at $x = 2$ is given by

$$p(2) = (2 + 1)(2 - 2) = 3 \times 0 = 0$$

Hence, – 1 and 2 both are zeroes of the polynomial $p(x)$.

(v) We have, $p(x) = x^2$

The value of the polynomial $p(x)$ at $x = 0$ is given by

$$p(0) = (0)^2 = 0$$

Hence, 0 is zero of polynomial $p(x)$.

(vi) We have,

$$p(x) = lx + m$$

The value of the polynomial $p(x)$ at $x = \frac{-m}{l}$ is given by

$$p\left(\frac{-m}{l}\right) = l \times \left(\frac{-m}{l}\right) + m$$
$$= -m + m = 0$$

Hence, $\frac{-m}{l}$ is zero of the polynomial $p(x)$.

(vii) We have, $p(x) = 3x^2 - 1$

The value of the polynomial $p(x)$ at $x = \frac{-1}{\sqrt{3}}$ is given by

$$p\left(\frac{-1}{\sqrt{3}}\right) = 3 \times \left(\frac{-1}{\sqrt{3}}\right)^2 - 1$$
$$= 3 \times \frac{1}{3} - 1 = 1 - 1 = 0$$

The value of the polynomial $p(x)$ at $x = \frac{2}{\sqrt{3}}$ is given by

$$p\left(\frac{2}{\sqrt{3}}\right) = 3 \times \left(\frac{2}{\sqrt{3}}\right)^2 - 1$$
$$= 3 \times \frac{4}{3} - 1 = 4 - 1 = 3$$

Hence, $\frac{-1}{\sqrt{3}}$ is a zero of the polynomial $p(x)$ but $\frac{2}{\sqrt{3}}$ is not.

(viii) We have, $p(x) = 2x + 1$

The value of the polynomial $p(x)$ at $x = \frac{1}{2}$ is given by

$$p\left(\frac{1}{2}\right) = 2 \times \frac{1}{2} + 1 = 1 + 1 = 2$$

Hence, $\frac{1}{2}$ is not a zero of the polynomial $p(x)$.

Q.4. Find the zero of the polynomial in each of the following cases :

(i) $p(x) = x + 5$
(ii) $p(x) = x - 5$
(iii) $p(x) = 2x + 5$
(iv) $p(x) = 3x - 2$
(v) $p(x) = 3x$
(vi) $p(x) = ax,\ a \neq 0$
(vii) $p(x) = cx + d,\ c \neq 0$;
where c, d are real numbers.

Sol. (i) We have,

$$p(x) = x + 5$$

For zero of the polynomial, $p(x) = 0$

$\Rightarrow \quad 0 = x + 5 \Rightarrow x = -5$

Hence, -5 is the zero of the polynomial $p(x)$.

(ii) We have, $p(x) = x - 5$

For the zero of the polynomial, $p(x) = 0$

$\Rightarrow \quad 0 = x - 5 \Rightarrow x = 5$

Hence, 5 is the zero of the polynomial $p(x)$.

(iii) $p(x) = 2x + 5$

For the zero of the polynomial, $p(x) = 0$

$\Rightarrow\ 0 = 2x + 5 \Rightarrow 2x = -5 \Rightarrow x = \frac{-5}{2}$

Hence, $\frac{-5}{2}$ is the zero of the polynomial $p(x)$.

(iv) We have, $p(x) = 3x - 2$

For the zero of the polynomial, $p(x) = 0$

$\Rightarrow\ 0 = 3x - 2 \Rightarrow 3x = 2 \Rightarrow x = \frac{2}{3}$

Hence, $\frac{2}{3}$ is the zero of the polynomial $p(x)$.

(v) We have, $p(x) = 3x$

For the zero of the polynomial, $p(x) = 0$

$\Rightarrow \quad 0 = 3x \Rightarrow x = 0$

Hence, 0 is the zero of the polynomial $p(x)$.

(vi) We have,

$$p(x) = ax,\ a \neq 0$$

For the zero of the polynomial, $p(x) = 0$

$\Rightarrow \quad 0 = ax \Rightarrow x = 0$

Hence, 0 is the zero of the polynomial $p(x)$.

(vii) We have, $p(x) = cx + d,\ c \neq 0$; where c, d are real numbers

For the zero of the polynomial, $p(x) = 0$

$\Rightarrow \quad 0 = cx + d$

$\Rightarrow \quad cx = -d \Rightarrow x = \frac{-d}{c}$

Hence, $\frac{-d}{c}$ is the zero of the polynomial $p(x)$.

Textbook Exercises 2.3

Q.1. Determine which of the following polynomials has $(x + 1)$ as a factor :

(i) $x^3 + x^2 + x + 1$
(ii) $x^4 + x^3 + x^2 + x + 1$,
(iii) $x^4 + 3x^3 + 3x^2 + x + 1$
(iv) $x^3 - x^2 - (2+\sqrt{2})x + \sqrt{2}$.

Sol. (i) Let

$$p(x) = x^3 + x^2 + x + 1$$

If $(x + 1)$ is a factor of $p(x)$, then by factor theorem, $p(-1)$ should be zero.

Now, $\quad p(-1) = (-1)^3 + (-1)^2 - 1 + 1$

$\Rightarrow \quad p(-1) = -1 + 1 - 1 + 1$

$\Rightarrow \quad p(-1) = 2 - 2$

$\Rightarrow \quad p(-1) = 0.$

Hence, $(x + 1)$ is a factor of $p(x)$.

(ii) Let $p(x) = x^4 + x^3 + x^2 + x + 1$

If $(x + 1)$ is a factor of $p(x)$, then by factor theorem, $p(-1)$ should be zero.

Now, $\quad p(-1) = (-1)^4 + (-1)^3 + (-1)^2 - 1 + 1$

$\Rightarrow \quad p(-1) = 1 - 1 + 1 - 1 + 1$

$\Rightarrow \quad p(-1) = 1 \neq 0.$

Hence, $(x + 1)$ is not a factor of $p(x)$.

(iii) Let $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$

If $(x + 1)$ is a factor of $p(x)$, then by factor theorem, $p(-1)$ should be zero.

Now, $p(-1) = (-1)^4 + 3 \times (-1)^3 + 3$

$\times (-1)^2 - 1 + 1$

$\Rightarrow \quad p(-1) = 1 - 3 + 3 - 1 + 1$

$\Rightarrow \quad p(-1) = 1 \neq 0.$

Hence, $(x + 1)$ is not a factor of $p(x)$.

(iv) Let $p(x) = x^3 - x^2 - (2+\sqrt{2})x + \sqrt{2}$

If $(x + 1)$ is a factor of $p(x)$, then by factor theorem $p(-1)$ should be zero.

Now,

$p(-1) = (-1)^3 - (-1)^2 - (2+\sqrt{2})(-1) + \sqrt{2}$

$\Rightarrow \quad p(-1) = -1 - 1 + 2 + \sqrt{2} + \sqrt{2}$

$\Rightarrow \quad p(-1) = -2 + 2 + 2\sqrt{2}$

$\Rightarrow \quad p(-1) = 2\sqrt{2} \neq 0.$

Hence, $(x + 1)$ is not a factor of $p(x)$.

Q.2. Use the factor theorem to determine, whether $g(x)$ is a factor of $p(x)$ in each of the following cases :

(i) $p(x) = 2x^3 + x^2 - 2x - 1$; $g(x) = x + 1$

(ii) $p(x) = x^3 + 3x^2 + 3x + 1$; $g(x) = x + 2$

(iii) $p(x) = x^3 - 4x^2 + x + 6$; $g(x) = x - 3$.

Sol. (i) We have,

$$p(x) = 2x^3 + x^2 - 2x - 1$$

If $g(x) = x + 1$ is a factor of $p(x)$, then by factor theorem, $p(-1)$ should be zero.

Now, $\quad p(-1) = 2 \times (-1)^3 + (-1)^2 - 2 \times (-1) - 1$

$\Rightarrow \quad p(-1) = -2 + 1 + 2 - 1$

$\Rightarrow \quad p(-1) = 0.$

Hence, $g(x) = x + 1$ is a factor of $p(x)$.

(ii) We have, $p(x) = x^3 + 3x^2 + 3x + 1$

If $g(x) = x + 2$ is a factor of $p(x)$, then by factor theorem, $p(-2)$ should be zero.

Now, $\quad p(-2) = (-2)^3 + 3 \times (-2)^2 + 3 \times (-2) + 1$

$\Rightarrow \quad p(-2) = -8 + 12 - 6 + 1$

$\Rightarrow \quad p(-2) = -14 + 13$

$\Rightarrow \quad p(-2) = -1 \neq 0.$

Hence, $g(x) = x + 2$ is not a factor of $p(x)$.

(iii) We have, $p(x) = x^3 - 4x^2 + x + 6$

If $g(x) = x - 3$ is a factor of $p(x)$, then by factor theorem, $p(3)$ should be zero.

Now, $\quad p(3) = (3)^3 - 4 \times (3)^2 + 3 + 6$

$\Rightarrow \quad p(3) = 27 - 36 + 3 + 6$

$\Rightarrow \quad p(3) = 36 - 36$

$\Rightarrow \quad p(3) = 0.$

Hence, $g(x) = x - 3$ is a factor of $p(x)$.

Q.3. Find the value of k, if $x - 1$ is a factor of $p(x)$ in each of the following cases :

(i) $p(x) = x^2 + x + k$

(ii) $p(x) = 2x^2 + kx + \sqrt{2}$

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$

(iv) $p(x) = kx^2 - 3x + k$.

Sol. (i) We have,

$$p(x) = x^2 + x + k$$

If $(x - 1)$ is a factor of $p(x)$, then by factor theorem, $p(1)$ should be zero.

$\Rightarrow \quad p(1) = 0$

$\Rightarrow \quad (1)^2 + 1 + k = 0 \Rightarrow 1 + 1 + k = 0$

$\Rightarrow \quad 2 + k = 0 \Rightarrow k = -2$

Hence, $\quad k = -2.$

(ii) We have, $p(x) = 2x^2 + kx + \sqrt{2}$

If $(x - 1)$ is a factor of $p(x)$, then by factor theorem, $p(1)$ should be zero.

$\Rightarrow \quad p(1) = 0$

$\Rightarrow \quad 2 \times (1)^2 + k \times 1 + \sqrt{2} = 0$

$\Rightarrow \quad 2 + k + \sqrt{2} = 0 \Rightarrow k = -2 - \sqrt{2}$

$\Rightarrow \quad k = -(2 + \sqrt{2})$

Hence, $\quad k = -(2 + \sqrt{2}).$

(iii) We have, $p(x) = kx^2 - \sqrt{2}x + 1$

If $(x - 1)$ is a factor of $p(x)$, then by factor theorem, $p(1)$ should be zero.

$\Rightarrow \quad p(1) = 0$

$\Rightarrow \quad k \times (1)^2 - \sqrt{2} \times 1 + 1 = 0$

$\Rightarrow \quad k - \sqrt{2} + 1 = 0 \Rightarrow k = \sqrt{2} - 1$

Hence, $\quad k = \sqrt{2} - 1.$

(iv) We have, $p(x) = kx^2 - 3x + k$

If $(x - 1)$ is a factor of $p(x)$, then by factor theorem, $p(1)$ should be zero.

$\Rightarrow \quad p(1) = 0$

$\Rightarrow k \times (1)^2 - 3 \times 1 + k = 0$

$\Rightarrow \quad k - 3 + k = 0 \Rightarrow 2k - 3 = 0$

$\Rightarrow \quad 2k = 3 \Rightarrow k = \frac{3}{2}$

Hence, $\quad k = \frac{3}{2}.$

Q.4. Factorise :

(i) $12x^2 - 7x + 1$ (ii) $2x^2 + 7x + 3$

(iii) $6x^2 + 5x - 6$ (iv) $3x^2 - x - 4$.

Sol. (i) $12x^2 - 7x + 1 = 12x^2 - (4 + 3)x + 1$

$= 12x^2 - 4x - 3x + 1$

$= (12x^2 - 4x) - (3x - 1)$

$= 4x(3x - 1) - 1(3x - 1)$

$= (3x - 1)(4x - 1).$

Hence, $12x^2 - 7x + 1 = (3x - 1)(4x - 1).$

(ii) $2x^2 + 7x + 3 = 2x^2 + (6 + 1)x + 3$

$= 2x^2 + 6x + x + 3$

$= (2x^2 + 6x) + (x + 3)$

$= 2x(x + 3) + 1(x + 3)$

$= (x + 3)(2x + 1).$

Hence, $2x^2 + 7x + 3 = (x + 3)(2x + 1).$

(iii) $6x^2 + 5x - 6 = 6x^2 + (9 - 4)x - 6,$

$= 6x^2 + 9x - 4x - 6$

$= (6x^2 + 9x) - (4x + 6)$

$= 3x(2x + 3) - 2(2x + 3)$

$= (2x + 3)(3x - 2).$

Hence, $6x^2 + 5x - 6 = (2x + 3)(3x - 2).$

(iv) $3x^2 - x - 4 = 3x^2 - (4 - 3)x - 4,$

$= 3x^2 - 4x + 3x - 4$

$= (3x^2 - 4x) + (3x - 4)$

$= x(3x - 4) + 1(3x - 4)$

$= (3x - 4)(x + 1).$

Hence, $3x^2 - x - 4 = (3x - 4)(x + 1).$

Q.5. Factorise :

(i) $x^3 - 2x^2 - x + 2$ (ii) $x^3 - 3x^2 - 9x - 5$

(iii) $x^3 + 13x^2 + 32x + 20$

(iv) $2y^3 + y^2 - 2y - 1.$

Sol. (i) Let $\quad p(x) = x^3 - 2x^2 - x + 2$

Factors of + 2 are ± 1, ± 2.

Now, put $\quad x = 1$

$\therefore \quad p(1) = (1)^3 - 2 \times (1)^2 - 1 + 2$

$= 1 - 2 - 1 + 2 = 0.$

$\therefore$ By factor theorem, $(x - 1)$ is a factor of $p(x)$. Now, divide $x^3 - 2x^2 - x + 2$ by $(x - 1)$, we get

$$\begin{array}{l} x - 1 \overline{)\, x^3 - 2x^2 - x + 2 \,}(\, x^2 - x - 2 \\ \quad\quad x^3 - x^2 \\ \quad\quad - \quad + \\ \hline \quad\quad\quad - x^2 - x + 2 \\ \quad\quad\quad - x^2 + x \\ \quad\quad\quad + \quad - \\ \hline \quad\quad\quad\quad - 2x + 2 \\ \quad\quad\quad\quad - 2x + 2 \\ \quad\quad\quad\quad + \quad - \\ \hline \quad\quad\quad\quad\quad 0 \\ \hline \end{array}$$

So, $x^3 - 2x^2 - x + 2 = (x - 1)(x^2 - x - 2)$

$= (x - 1)[x^2 - (2 - 1)x - 2]$

$= (x - 1)[x^2 - 2x + x - 2]$

$= (x - 1)[(x^2 - 2x) + (x - 2)]$

$= (x - 1)[x(x - 2) + 1(x - 2)]$

$= (x - 1)(x - 2)(x + 1).$

(ii) Let $p(x) = x^3 - 3x^2 - 9x - 5$

Factors of – 5 are ± 1, ± 5.

Now, put $x = -1$

$\therefore p(-1) = (-1)^3 - 3 \times (-1)^2 - 9 \times (-1) - 5$

$= -1 - 3 + 9 - 5 = -9 + 9 = 0.$

$\therefore$ By factor theorem, $(x + 1)$ is a factor of $p(x)$. Now, divide $x^3 - 3x^2 - 9x - 5$ by $(x + 1)$, we get

$$\begin{array}{l} x + 1 \overline{)\, x^3 - 3x^2 - 9x - 5 \,}(\, x^2 - 4x - 5 \\ \quad\quad x^3 + x^2 \\ \quad\quad - \quad - \\ \hline \quad\quad\quad - 4x^2 - 9x - 5 \\ \quad\quad\quad - 4x^2 - 4x \\ \quad\quad\quad + \quad + \\ \hline \quad\quad\quad\quad - 5x - 5 \\ \quad\quad\quad\quad - 5x - 5 \\ \quad\quad\quad\quad + \quad + \\ \hline \quad\quad\quad\quad\quad 0 \\ \hline \end{array}$$

So, $x^3 - 3x^2 - 9x - 5$

$= (x + 1)(x^2 - 4x - 5)$

$= (x + 1)[x^2 - (5 - 1)x - 5]$

$= (x + 1)[x^2 - 5x + x - 5]$

$= (x + 1)\ [(x^2 - 5x) + (x - 5)]$

$= (x + 1)\ [x(x - 5) + 1(x - 5)]$

$= (x + 1)\ (x - 5)\ (x + 1).$

(iii) Let $p(x) = x^3 + 13x^2 + 32x + 20$

Factors of 20 are $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10$ and ± 20.

Now, put $x = -1$

$\therefore \quad p(-1) = (-1)^3 + 13 \times (-1)^2 + 32 \times (-1) + 20$

$= -1 + 13 - 32 + 20$

$= 33 - 33 = 0.$

$\therefore$ By factor theorem, $(x + 1)$ is a factor of $p(x)$. Now divide $x^3 + 13x^2 + 32x + 20$ by $(x + 1)$, we get

$$
\begin{array}{r|l}
x+1 & x^3 + 13x^2 + 32x + 20 \ (x^2 + 12x + 20 \\
 & x^3 + x^2 \\
 & - \quad - \\ \hline
 & \quad 12x^2 + 32x + 20 \\
 & \quad 12x^2 + 12x \\
 & \quad - \quad\ \ - \\ \hline
 & \qquad\quad 20x + 20 \\
 & \qquad\quad 20x + 20 \\
 & \qquad\quad - \quad\ - \\ \hline
 & \qquad\qquad 0
\end{array}
$$

So, $x^3 + 13x^2 + 32x + 20$

$= (x + 1)\ (x^2 + 12x + 20)$

$= (x + 1)\ [x^2 + (10 + 2)x + 20]$

$= (x + 1)\ [x^2 + 10x + 2x + 20]$

$= (x + 1\ [(x^2 + 10x) + (2x + 20)]$

$= (x + 1)\ [x(x + 10) + 2(x + 10)]$

$= (x + 1)\ (x + 10)\ (x + 2).$

(iv) Let $p(y) = 2y^3 + y^2 - 2y - 1$

Factors of $-1 = \pm 1$

Now, put $y = 1$

$\therefore \quad p(1) = 2 \times (1)^3 + (1)^2 - 2 \times 1 - 1$

$= 2 + 1 - 2 - 1 = 3 - 3 = 0$

$\therefore$ By factor theorem, $(y - 1)$ is a factor of $p(y)$.

Now, divide $2y^3 + y^2 - 2y - 1$ by $(y - 1)$, we get

$$
\begin{array}{r|l}
y-1 & 2y^3 + y^2 - 2y - 1 \ (2y^2 + 3y + 1 \\
 & 2y^3 - 2y^2 \\
 & - \quad\ + \\ \hline
 & \qquad 3y^2 - 2y - 1 \\
 & \qquad 3y^2 - 3y \\
 & \qquad - \quad\ + \\ \hline
 & \qquad\qquad y - 1 \\
 & \qquad\qquad y - 1 \\
 & \qquad\qquad - \quad + \\ \hline
 & \qquad\qquad\quad 0
\end{array}
$$

So, $2y^3 + y^2 - 2y - 1$

$= (y - 1)\ (2y^2 + 3y + 1)$

$= (y - 1)\ [2y^2 + (2 + 1)y + 1]$

$= (y - 1)\ [2y^2 + 2y + y + 1]$

$= (y - 1)\ [(2y^2 + 2y) + (y + 1)]$

$= (y - 1)\ [2y\ (y + 1) + 1\ (y + 1)]$

$= (y - 1)\ (y + 1)\ (2y + 1)$

Textbook Exercises 2.4

Q.1. Use suitable identities to find the following products:

(i) $(x + 4)\ (x + 10)$ (ii) $(x + 8)\ (x - 10)$

(iii) $(3x + 4)(3x - 5)$ (iv) $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right)$

(v) $(3 - 2x)\ (3 + 2x)$.

Sol. (i) $(x + 4)\ (x + 10)$

$= x^2 + (4 + 10)x + 4 \times 10$

$= x^2 + 14x + 40.$

(ii) $(x + 8)\ (x - 10)$

$= x^2 + [8 + (-10)]x + 8 \times (-10)$

$= x^2 - 2x - 80$

(iii) $(3x + 4)\ (3x - 5)$

$= (3x)^2 + [4 + (-5)] \times 3x + 4 \times (-5)$

$= 9x^2 - 3x - 20.$

(iv) $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right) = (y^2)^2 - \left(\frac{3}{2}\right)^2$

$= y^4 - \frac{9}{4}.$

(v) $(3 - 2x)\ (3 + 2x) = (3)^2 - (2x)^2$

Hence, $(3 - 2x)(3 + 2x) = 9 - 4x^2.$

Q.2. Evaluate the following products without multiplying directly :

(i) 103×107 (ii) 95×96

(iii) 104×96.

Sol. (i) $103 \times 107 = (100 + 3)(100 + 7)$

$= (100)^2 + (3 + 7) \times 100 + 3 \times 7$

$= 10000 + 10 \times 100 + 21$

$= 10000 + 1000 + 21$

$= 11021.$

(ii) $95 \times 96 = (100 - 5)(100 - 4)$

$= (100)^2 + [(-5) + (-4)] \times 100 + (-5) \times (-4)$

$= 10000 - 9 \times 100 + 20$

$= 10000 - 900 + 20$

$= 9120.$

(iii) $104 \times 96 = (100 + 4)(100 - 4)$

$= (100)^2 - (4)^2,$

$= 10000 - 16 = 9984.$

Q.3. Factorise the following using appropriate identities:

(i) $9x^2 + 6xy + y^2$ (ii) $4y^2 - 4y + 1$

(iii) $x^2 - \dfrac{y^2}{100}$.

Sol. (i) $9x^2 + 6xy + y^2$

$= (3x)^2 + 2 \times 3x \times y + y^2$

$= (3x + y)^2,$

$= (3x + y)(3x + y).$

(ii) $4y^2 - 4y + 1 = (2y)^2 - 2 \times 2y \times 1 + (1)^2$

$= (2y - 1)^2$

$= (2y - 1)(2y - 1).$

(iii) $x^2 - \dfrac{y^2}{100} = (x)^2 - \left(\dfrac{y}{10}\right)^2$

$= \left(x + \dfrac{y}{10}\right)\left(x - \dfrac{y}{10}\right)$

Q.4. Expand each of the following using suitable identities:

(i) $(x + 2y + 4z)^2$ (ii) $(2x - y + z)^2$

(iii) $(-2x + 3y + 2z)^2$ (iv) $(3a - 7b - c)^2$

(v) $(-2x + 5y - 3z)^2$ (vi) $\left[\dfrac{1}{4}a - \dfrac{1}{2}b + 1\right]^2$.

Sol. We will use the following identity in each

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

(i) $(x + 2y + 4z)^2 = x^2 + (2y)^2 + (4z)^2 + 2 \times x \times 2y + 2 \times 2y \times 4z + 2 \times 4z \times x$

$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8zx.$

(ii) $(2x - y + z)^2 = [2x + (-y) + z]^2$

$= (2x)^2 + (-y)^2 + z^2 + 2 \times 2x \times (-y) + 2 \times (-y) \times z + 2 \times z \times 2x$

$= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4zx.$

(iii) $(-2x + 3y + 2z)^2 = [(-2x) + 3y + 2z]^2$

$= (-2x)^2 + (3y)^2 + (2z)^2 + 2 \times (-2x) \times 3y + 2 \times 3y \times 2z + 2 \times 2z \times (-2x)$

$= 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8zx.$

(iv) $(3a - 7b - c)^2 = [3a + (-7b) + (-c)]^2$

$= (3a)^2 + (-7b)^2 + (-c)^2 + 2 \times 3a \times (-7b) + 2 \times (-7b) \times (-c) + 2 \times (-c) \times 3a$

$= 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ca.$

(v) $(-2x + 5y - 3z)^2 = [(-2x) + 5y + (-3z)]^2$

$= (-2x)^2 + (5y)^2 + (-3z)^2 + 2 \times (-2x) \times 5y + 2 \times 5y \times (-3z) + 2 \times (-3z) \times (-2x)$

$= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12zx.$

(vi) $\left[\dfrac{1}{4}a - \dfrac{1}{2}b + 1\right]^2 = \left[\dfrac{1}{4}a + \left(-\dfrac{1}{2}b\right) + 1\right]^2$

$= \left(\dfrac{1}{4}a\right)^2 + \left(-\dfrac{1}{2}b\right)^2 + (1)^2 + 2 \times \dfrac{1}{4}a \times \left(-\dfrac{1}{2}b\right) + 2 \times \left(-\dfrac{1}{2}b\right) \times 1 + 2 \times 1 \times \dfrac{1}{4}a$

$= \dfrac{1}{16}a^2 + \dfrac{1}{4}b^2 + 1 - \dfrac{1}{4}ab - b + \dfrac{a}{2}$

Q.5. Factorise :

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$.

Sol. (i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

$= (2x)^2 + (3y)^2 + (-4z)^2 + 2 \times (2x) \times (3y) + 2 \times (3y) \times (-4z) + 2 \times (-4z) \times (2x)$

$= [2x + 3y + (-4z)]^2$

$= (2x + 3y - 4z)^2$

$= (2x + 3y - 4z)(2x + 3y - 4z)$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

$= (-\sqrt{2}x)^2 + (y)^2 + (2\sqrt{2}z)^2 + 2\times(-\sqrt{2}x)\times(y)$

$+ 2\times(y)\times(2\sqrt{2}z) + 2\times(2\sqrt{2}z)\times(-\sqrt{2}x)$

$= [(-\sqrt{2}x) + (y) + (2\sqrt{2}z)]^2,$

$= (-\sqrt{2}x + y + 2\sqrt{2}z)^2$

Q.6. Write the following cubes in expanded form :

(i) $(2x + 1)^3$ (ii) $(2a - 3b)^3$

(iii) $\left[\frac{3}{2}x + 1\right]^3$ (iv) $\left[x - \frac{2}{3}y\right]^3$.

Sol. (i) $(2x + 1)^3 = (2x)^3 + 1^3 + 3 \times 2x \times 1(2x + 1)$

$= 8x^3 + 1 + 6x(2x + 1)$

$= 8x^3 + 1 + 12x^2 + 6x.$

(ii) $(2a - 3b)^3 = (2a)^3 - (3b)^3 - 3 \times 2a \times (3b)(2a - 3b)$

$= 8a^3 - 27b^3 - 18ab(2a - 3b)$

$= 8a^3 - 27b^3 - 36a^2b + 54ab^2.$

(iii) $\left[\frac{3}{2}x + 1\right]^3 = \left(\frac{3}{2}x\right)^3 + (1)^3 + 3\times\frac{3}{2}x\times1\left(\frac{3}{2}x+1\right)$

$= \frac{27}{8}x^3 + 1 + \frac{9x}{2}\left(\frac{3}{2}x + 1\right)$

$= \frac{27}{8}x^3 + 1 + \frac{27x^2}{4} + \frac{9x}{2}$

$= \frac{27}{8}x^3 + \frac{27x^2}{4} + \frac{9x}{2} + 1.$

(iv) $\left[x - \frac{2}{3}y\right]^3 = x^3 - \left(\frac{2}{3}y\right)^3 - 3\times x\times\frac{2}{3}y\left(x - \frac{2}{3}y\right)$

$= x^3 - \frac{8}{27}y^3 - 2xy\left(x - \frac{2}{3}y\right)$

$= x^3 - \frac{8}{27}y^3 - 2x^2y + \frac{4xy^2}{3}.$

Q.7. Evaluate the following using suitable identities :

(i) $(99)^3$ (ii) $(102)^3$ (iii) $(998)^3$.

Sol. (i) $(99)^3 = (100 - 1)^3$

$= (100)^3 - (1)^3 - 3 \times 100 \times 1(100 - 1)$

$= 1000000 - 1 - 300(99)$

$= 1000000 - 1 - 29700$

$= 970299.$

(ii) $(102)^3 = (100 + 2)^3$

$= (100)^3 + (2)^3 + 3 \times 100 \times 2(100 + 2)$

$= 1000000 + 8 + 600(102)$

$= 1000000 + 8 + 61200$

$= 1061208.$

(iii) $(998)^3 = (1000 - 2)^3$

$= (1000)^3 - (2)^3 - 3 \times 1000 \times 2(1000 - 2)$

$= 1000000000 - 8 - 6000 \times 998$

$= 1000000000 - 8 - 5988000$

$= 994011992.$

Hence, $(998)^3 = 994011992.$ **Ans.**

Q.8. Factorise each of the following :

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

(iii) $27 - 125a^3 - 135a + 225a^2$

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p.$

Sol. (i) $8a^3 + b^3 + 12a^2b + 6ab^2$

$= (2a)^3 + (b)^3 + 6ab(2a + b)$

$= (2a)^3 + (b)^3 + 3 \times 2a \times b(2a + b)$

$= (2a + b)^3$

$= (2a + b)(2a + b)(2a + b).$

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

$= (2a)^3 - (b)^3 - 6ab(2a - b)$

$= (2a)^3 - (b)^3 - 3 \times 2a \times b(2a - b)$

$= (2a - b)^3$

$= (2a - b)(2a - b)(2a - b).$

(iii) $27 - 125a^3 - 135a + 225a^2$

$= (3)^3 - (5a)^3 - 45a(3 - 5a)$

$= (3)^3 - (5a)^3 - 3 \times 3 \times 5a(3 - 5a)$

$= (3 - 5a)^3$

$= (3 - 5a)(3 - 5a)(3 - 5a).$

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

$= (4a)^3 - (3b)^3 - 36ab(4a - 3b)$

$= (4a)^3 - (3b)^3 - 3 \times 4a \times 3b(4a - 3b)$

$= (4a - 3b)^3$

$= (4a - 3b)(4a - 3b)(4a - 3b).$

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

$= (3p)^3 - \left(\frac{1}{6}\right)^3 - \frac{9p}{6}\left(3p - \frac{1}{6}\right)$

$= (3p)^3 - \left(\frac{1}{6}\right)^3 - 3 \times 3p \times \frac{1}{6}\left(3p - \frac{1}{6}\right)$

$= \left(3p - \frac{1}{6}\right)^3$

$= \left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right).$

Q.9. Verify : (i) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

(ii) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$.

Sol. (i) We have,

$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

R.H.S. $= (x + y)(x^2 - xy + y^2)$

$= x(x^2 - xy + y^2) + y(x^2 - xy + y^2)$

$= x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3$

$= x^3 + y^3 =$ L.H.S.

Hence, L.H.S. = R.H.S. **Verified**

(ii) We have, $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

R.H.S $= (x - y)(x^2 + xy + y^2)$

$= x(x^2 + xy + y^2) - y(x^2 + xy + y^2)$

$= x^3 + x^2y + xy^2 - x^2y - xy^2 - y^3$

$= x^3 - y^3 =$ L.H.S.

Hence, L.H.S. = R.H.S. **Verified**

Q.10. Factorise each of the following :

(i) $27y^3 + 125z^3$ (ii) $64m^3 - 343n^3$.

Sol. (i) $27y^3 + 125z^3 = (3y)^3 + (5z)^3$

$= (3y + 5z)[(3y)^2 - 3y \times 5z + (5z)^2]$

$= (3y + 5z)(9y^2 - 15yz + 25z^2).$

(ii) $64m^3 - 343n^3 = (4m)^3 - (7n)^3$

$= (4m - 7n)[(4m)^2 + 4m \times 7n + (7n)^2]$

$= (4m - 7n)(16m^2 + 28mn + 49n^2)$

Q.11. Factorise : $27x^3 + y^3 + z^3 - 9xyz$.

Sol. $27x^3 + y^3 + z^3 - 9xyz$

$= (3x)^3 + (y)^3 (z)^3 - 3 \times 3x \times y \times z$

$= (3x + y + z)[(3x)^2 + y^2 + z^2 - 3x \times y - y \times z - z \times 3x]$

$= (3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3zx).$

Q.12. Verify that :

$$x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2].$$

Sol. We have,

$$x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$$

R.H.S. $= \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$

$= \frac{1}{2}(x + y + z)(x^2 - 2xy + y^2 + y^2 - 2yz + z^2 + z^2 - 2xz + x^2)$

$= \frac{1}{2}(x + y + z)(2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2xz)$

$= \frac{1}{2} \times 2(x + y + z)(x^2 + y^2 + z^2 - xy - yz - xz)$

$= x^3 + y^3 + z^3 - 3xyz.$

$= x^3 + y^3 + z^3 - 3xyz]$

= L.H.S.

Hence, L.H.S. = R.H.S. **Verified**

Q.13. If $x + y + z = 0$, show that : $x^3 + y^3 + z^3 = 3xyz$.

Sol. We have, $x + y + z = 0$

$\Rightarrow \quad x + y = -z \quad$ ...(i)

Cubing both sides of equation (i), we get

$(x + y)^3 = (-z)^3$

$\Rightarrow \quad x^3 + y^3 + 3xy(x + y) = -z^3$

$\Rightarrow \quad x^3 + y^3 + z^3 + 3xy \times (-z) = 0,$ $[\because x + y = -z]$

$\Rightarrow \quad x^3 + y^3 + z^3 - 3xyz = 0$

$\Rightarrow \quad x^3 + y^3 + z^3 = 3xyz.$ **Proved**

Q.14. Without actually calculating the cubes, find the value of each of the following :

(i) $(-12)^3 + (7)^3 + (5)^3$,

(ii) $(28)^3 + (-15)^3 + (-13)^3$.

Sol. (i) We know that, if

$$x + y + z = 0, \text{ then}$$
$$x^3 + y^3 + z^3 = 3xyz$$

Since, $(-12) + 7 + 5 = -12 + 12 = 0$

Therefore,

$$(-12)^3 + (7)^3 + (5)^3 = 3 \times (-12) \times 7 \times 5 = -1260$$

(ii) We know that, If $x + y + z = 0$, then

$$x^3 + y^3 + z^3 = 3xyz$$

Since, $28 + (-15) + (-13) = 28 - 28 = 0$

Therefore, $(28)^3 + (-15)^3 + (-13)^3$

$$= 3 \times 28 \times (-15) \times (-13) = 16380.$$

Hence, $(28)^3 + (-15)^3 + (-13)^3 = 16380$.

Q.15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:

(i) Area : $25a^2 - 35a + 12$

(ii) Area : $35y^2 + 13y - 12$

Sol. (i) We have,

Area of rectangle $= 25a^2 - 35a + 12$

$\Rightarrow$ length $\times$ breadth $= 25a^2 - (20 + 15)a + 12$

$\Rightarrow$ length $\times$ breadth $= 25a^2 - 20a - 15a + 12$

$\Rightarrow$ length $\times$ breadth $= (25a^2 - 20a) - (15a - 12)$

$\Rightarrow$ length $\times$ breadth $= 5a(5a - 4) - 3(5a - 4)$

$\Rightarrow$ length $\times$ breadth $= (5a - 4)(5a - 3)$

$\Rightarrow$ length $\times$ breadth $= (5a - 3)(5a - 4)$

Hence, One possible answer is length $= (5a - 3)$ and breadth $= (5a - 4)$.

(ii) We have,

Area of rectangle $= 35y^2 + 13y - 12$

$\Rightarrow$ length $\times$ breadth $= 35y^2 + (28 - 15)y - 12$,

$\Rightarrow$ length $\times$ breadth $= 35y^2 + 28y - 15y - 12$

$\Rightarrow$ length $\times$ breadth $= (35y^2 + 28y) - (15y + 12)$

$\Rightarrow$ length $\times$ breadth $= 7y(5y + 4) - 3(5y + 4)$

$\Rightarrow$ length $\times$ breadth $= (5y + 4)(7y - 3)$

$\Rightarrow$ length $\times$ breadth $= (7y - 3)(5y + 4)$

Hence, one possible answer is : length $= (7y - 3)$ and breadth $= (5y + 4)$.

Q.16. What are the possible expressions for the dimensions of the cuboids whose volumes are given below ?

(i) Volume : $3x^2 - 12x$

(ii) Volume : $12ky^2 + 8ky - 20k$

Sol. (i) We have,

Volume of cuboid $= 3x^2 - 12x$

$\Rightarrow$ length $\times$ breadth $\times$ height $= 3x(x - 4)$

$\Rightarrow$ length $\times$ breadth $\times$ height $= 3 \times x \times (x - 4)$

Hence, possible dimensions of cuboid are 3, x and $(x - 4)$.

(ii) We have,

Volume of cuboid $= 12ky^2 + 8ky - 20k$

$\Rightarrow$ length $\times$ breadth $\times$ height $= 4k[3y^2 + 2y - 5]$

$$[\because 5 \times (-3) = -15, 5 - 3 = 2]$$

$\Rightarrow$ length $\times$ breadth $\times$ height $= 4k[3y^2 + (5 - 3)y - 5]$,

$\Rightarrow$ length $\times$ breadth $\times$ height $= 4k[3y^2 + 5y - 3y - 5]$

$\Rightarrow$ length $\times$ breadth $\times$ height $= 4k[(3y^2 + 5y) - (3y + 5)]$

$\Rightarrow$ length $\times$ breadth $\times$ height $= 4k[y(3y + 5) - 1(3y + 5)]$

$\Rightarrow$ length $\times$ breadth $\times$ height $= 4k \times (3y + 5)(y - 1)$

Hence, possible demensions of cuboid are $4k$, $(3y + 5)$ and $(y - 1)$.

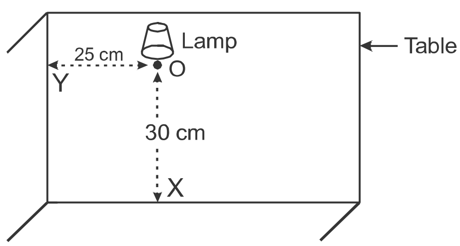
3 Coordinate Geometry

Textbook Exercises 3.1

Q.1. How will you describe the position of a table lamp on your study table to another person?

Sol. Consider, the lamp as a point and table as a plane. Choose any two perpendicular edges of the table, say OX and OY. Measure the distance of the lamp from longer edge *i.e.*, distance of P from longer edge OX.

Suppose it is 30 cm. Again measure the distance of the lamp from the shorter edge *i.e.*, distance of P from the shorter edge OY. Suppose it is 25 cm. You can write the position of the lamp as (25, 30) or (30, 25), depending on the order you fix.



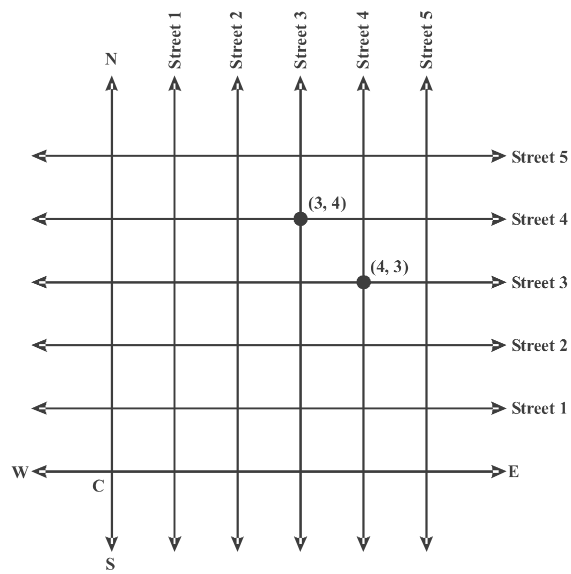
Q.2. (Street plan) : A city has two main roads which cross each other at the centre of the city. These two roads are along the North-South direction and East-West direction. All the other streets of the city run parallel to these roads and are 200 m apart. There are 5 streets in each direction. Using 1 cm = 200 m, draw a model of the city on your note-book. Represent the road/streets by single lines.

There are many cross streets in your model. A particular cross-street is made by two streets, one running in the North-South direction and another in the East-West direction. Each cross street is referred to in the following manner : If the 2nd street running in the North-South direction and 5th in the East-West direction meet at some crossing, then we will call this cross street (2, 5). Using this convention, find :

(i) how many cross-streets can be referred to as (4, 3) ?

(ii) how many cross-streets can be referred to as (3, 4) ?

Sol. The street plan is shown in the figure given below :



(i) There is only one cross street, which can be referred to as (4, 3).

(ii) There is only one cross street, which can be referred to as (3, 4).

Textbook Exercise 3.2

Q.1. Write the answer of each of the following questions :

(i) What is the name of horizontal and vertical lines drawn to determine the position of any point in the cartesian plane ?

(ii) What is the name of each part of the plane formed by these two lines ?

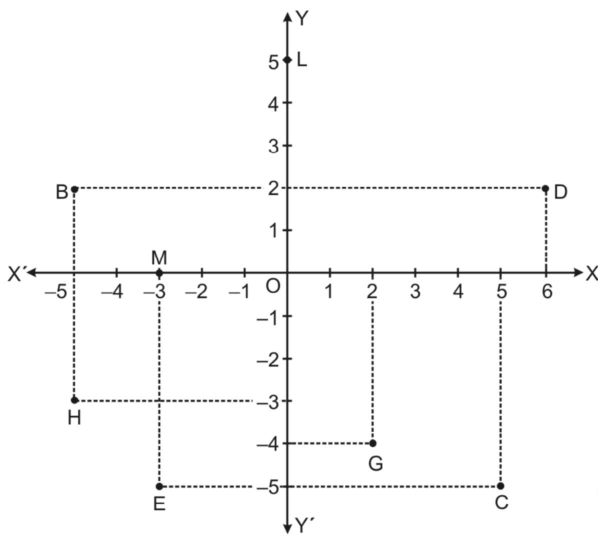
(iii) Write the name of the point where these two lines intersect.

Sol. (i) The name of the horizontal line is x-axis and vertical line is y-axis.

(ii) Each part of the plane formed by these two lines is known as quadrant.

(iii) Name of the point where these two lines intersect is origin. It is denoted by O.

Q.2. See figure, and write the following:



(i) The coordinates of B.

(ii) The coordinates of C.

(iii) The point identified by the coordinates (– 3, – 5).

(iv) The point identified by the coordinates (2, – 4).

(v) The abscissa of the point D.

(vi) The ordinate of the point H.

(vii) The coordinates of the point L.

(viii) The coordinates of the point M.

Sol. (i) Distance of the point B from the y-axis is – 5 units. Therefore, the x-coordinate or abscissa of the point B is – 5. The distance of the point B from the x-axis is 2 units. Therefore, the y-coordinate *i.e.*, the ordinate of the point B is 2. Hence, the coordinates of the point B are (– 5, 2).

(ii) Distance of the point C from the y-axis is 5 units and that of from x-axis is – 5 units.

Hence, the coordinates of point C are (5, – 5).

(iii) The point identified by the coordinates (– 3, – 5) is E.

(iv) The point identified by the coordinates (2, – 4) is G.

(v) Distance of the point D from the y-axis is 6 units, so the x-coordinate *i.e.,* the abscissa of the point D is 6.

(vi) Distance of the point H from x-axis is – 3 units. So, the y-coordinate *i.e.,* the ordinate of the point H is – 3.

(vii) The distance of the point L from y-axis is 0 unit and that of from x-axis is 5 units.

Hence, the coordinates of the point L are (0, 5).

(viii) The distance of the point M from y-axis is – 3 units and that of from x-axis is 0 unit.

Hence, the coordinates of the point M are (– 3, 0).

4 Linear Equations in Two Variables

Textbook Exercises 4.1

Q.1. The cost of a note-book is twice the cost of a pen. Write a linear equation in two variables to represent this statement.

Sol. Let the cost of a note-book be ₹. x and cost of a pen be ₹. y.

According to statement, the cost of a note-book is twice the cost of a pen.

So, the required linear equation in two variables to represent above statement is

$$x = 2y \text{ or } x - 2y = 0$$

Q.2. Express the following linear equations in the form $ax + by + c = 0$ and indicate the values of a, b and c in each case :

(i) $2x + 3y = 9{\cdot}3\overline{5}$ (ii) $x - \frac{y}{5} - 10 = 0$

(iii) $-2x + 3y = 6$ (iv) $x = 3y$

(v) $2x = -5y$ (vi) $3x + 2 = 0$

(vii) $y - 2 = 0$ (viii) $5 = 2x$.

Sol. (i) Writing the given equation $2x + 3y = 9{\cdot}3\overline{5}$ in the form $ax + by + c = 0$, we get

$$2x + 3y - 9{\cdot}3\overline{5} = 0 \quad \ldots\text{(i)}$$

Comparing the equation (i) with the standard form of the linear equation $ax + by + c = 0$, we get

$$a = 2, b = 3 \text{ and } c = -9{\cdot}3\overline{5}.$$

(ii) Writing the given equation $x - \frac{y}{5} - 10 = 0$ in the form $ax + by + c = 0$, we get

$$1.x - \frac{1}{5} \cdot y - 10 = 0 \quad \ldots\text{(i)}$$

Comparing the equation (i) with the standard form of the linear equation $ax + by + c = 0$, we get

$$a = 1, b = -\frac{1}{5} \text{ and } c = -10.$$

(iii) Writing the given equation $-2x + 3y = 6$ in the form $ax + by + c = 0$, we get

$$-2x + 3y - 6 = 0 \quad \ldots\text{(i)}$$

Comparing the equation (i) with the standard form of the linear equation $ax + by + c = 0$, we get

$$a = -2, b = 3 \text{ and } c = -6.$$

(iv) Writing the given equation $x = 3y$ in the form $ax + by + c = 0$, we get

$$1.x - 3y + 0 = 0 \quad \ldots\text{(i)}$$

Comparing the equation (i) with the standard form of the linear equation $ax + by + c = 0$, we get

$$a = 1, b = -3 \text{ and } c = 0.$$

(v) Writing the given equation $2x = -5y$ in the form $ax + by + c = 0$, we get

$$2x + 5y + 0 = 0 \quad \ldots\text{(i)}$$

Comparingm the equation (i) with the standard form of the linear equation $ax + by + c = 0$, we get

$$a = 2, b = 5 \text{ and } c = 0.$$

(vi) Writing the given equation $3x + 2 = 0$ in the form $ax + by + c = 0$, we get

$$3x + 0.y + 2 = 0 \quad \ldots\text{(i)}$$

Comparing the equation (i) with the standard form of the linear equation $ax + by + c = 0$, we get

$$a = 3, b = 0 \text{ and } c = 2.$$

(vii) Writing the given equation $y - 2 = 0$ in the form $ax + by + c = 0$, we get

$$0.x + 1.y - 2 = 0 \quad \ldots\text{(i)}$$

Comparing the equation (i) with the standard form of the linear equation $ax + by + c = 0$, we get

$$a = 0, b = 1 \text{ and } c = -2.$$

(viii) Writing the given equation $5 = 2x$ in the form $ax + by + c = 0$, we get

$$-2x + 0.y + 5 = 0 \quad \ldots\text{(i)}$$

Comparing the equation (i) with the standard form of the linear equation $ax + by + c = 0$, we get

$$a = -2, b = 0 \text{ and } c = 5.$$

Textbook Exercise 4.2

Q.1. Which one of the following options is true, and why ?

$y = 3x + 5$ has :

(i) a unique solution

(ii) only two solutions

(iii) infinitely many solutions.

Sol. (iii) is true, because $y = 3x + 5$ is a linear equation in two variables. We know that a linear equation in two variables has infinitely many solutions.

Q.2. Write four solutions for each of the following equations :

(i) $2x + y = 7$ (ii) $px + y = 9$

(iii) $x = 4y$.

Sol. (i) We have,

$$2x + y = 7 \quad \ldots\text{(i)}$$

Substituting $x = 0$ in the equation (i), we get

$$2 \times 0 + y = 7$$

$$\Rightarrow \quad 0 + y = 7 \Rightarrow y = 7$$

$\therefore$ (0, 7) is a solution of the given equation.

Substituting $x = 1$ in the equation (i), we get

$$2 \times 1 + y = 7$$

$$\Rightarrow \quad 2 + y = 7$$

$$\Rightarrow \quad y = 7 - 2 = 5$$

$\therefore$ (1, 5) is a solution of the given equation.

Substituting $x = 2$ in equation (i), we get

$$2 \times 2 + y = 7$$

$$\Rightarrow \quad 4 + y = 7$$

$$\Rightarrow \quad y = 7 - 4 = 3$$

$\therefore$ (2, 3) is a solution of the given equation.

Substituting $x = 3$ in the equation (i), we get

$$2 \times 3 + y = 7$$

$$\Rightarrow \quad 6 + y = 7$$

$$\Rightarrow \quad y = 7 - 6 = 1$$

$\therefore$ (3, 1) is a solution of the given equation.

Hence, four solutions of the given equation are (0, 7), (1, 5), (2, 3) and (3, 1).

(ii) We have, $\pi x + y = 9 \quad \ldots$(i)

Substituting $x = 0$ in the equation (i), we get

$$\pi \times 0 + y = 9$$

$$\Rightarrow \quad 0 + y = 9 \Rightarrow y = 9$$

$\therefore$ (0, 9) is a solution of the given equation.

Substituting $x = 1$ in the equation (i), we get

$$\pi \times 1 + y = 9$$

$$\Rightarrow \quad \pi + y = 9 \Rightarrow y = 9 - \pi$$

$\therefore$ $(1, 9 - \pi)$ is a solution of the given equation.

Substituting $x = \frac{9}{\pi}$ in the equation (i), we get

$$\pi \times \frac{9}{\pi} + y = 9$$

$$\Rightarrow \quad 9 + y = 9 \Rightarrow y = 9 - 9 = 0$$

$\therefore \left(\frac{9}{\pi}, 0\right)$ is a solution of the given equation.

Substituting $x = -1$ in the equation (i), we get

$$\pi \times (-1) + y = 9$$

$$\Rightarrow \quad -\pi + y = 9 \Rightarrow y = 9 + \pi$$

$\therefore$ $(-1, 9 + \pi)$ is a solution of the given equation.

Hence, four solutions of the given equation are (0, 9), $(1, 9 - \pi)$, $\left(\frac{9}{\pi}, 0\right)$ and $(-1, 9 + \pi)$.

(iii) We have, $x = 4y \quad \ldots$(i)

Substituting $x = 0$ in the equation (i), we get

$$0 = 4y \quad \Rightarrow \frac{0}{4} = y \quad \Rightarrow \quad y = 0$$

$\therefore$ (0, 0) is a solution of the given equation.

Substituting $x = 4$ in the equation (i), we get

$$4 = 4y \quad \Rightarrow \frac{4}{4} = y \quad \Rightarrow \quad y = 1$$

$\therefore$ (4, 1) is a solution of the given equation.

Substituting $x = -4$ in the equation (i), we get

$$-4 = 4y \Rightarrow \frac{-4}{4} = y$$

$$\Rightarrow \quad -1 = y \quad \Rightarrow y = -1$$

$\therefore$ $(-4, -1)$ is a solution of the given equation.

Substituting $x = 2$ in the equation (i), we get

$$2 = 4y \Rightarrow \frac{2}{4} = y \Rightarrow y = \frac{1}{2}$$

$\therefore$ $\left(2, \frac{1}{2}\right)$ is a solution of the given equation.

Hence, four solutions of the given equation are (0, 0), (4, 1), (– 4, – 1) and $\left(2, \frac{1}{2}\right)$.

Q.3. Check which of the following are solutions of the equation $x - 2y = 4$ and which are not :

(i) (0, 2) (ii) (2, 0) (iii) (4, 0)

(iv) $(\sqrt{2}, 4\sqrt{2})$ (v) (1, 1).

Sol. We have, $x - 2y = 4$ ...(i)

(i) Substituting $x = 0, y = 2$ in the L.H.S. of the equation (i), we get

$$\text{L.H.S.} = 0 - 2 \times 2 = -4 \neq \text{R.H.S.}$$

$\because$ L.H.S. $\neq$ R.H.S.

$\therefore$ (0, 2) is not a solution of the given equation.

(ii) Substituting $x = 2, y = 0$ in the L.H.S. of the equation (i), we get

$$\text{L.H.S.} = 2 - 2 \times 0 = 2 - 0 = 2 \neq \text{R.H.S.}$$

$\because$ L.H.S. $\neq$ R.H.S.

$\therefore$. (2, 0) is not a solution of the given equation.

(iii) Substituting $x = 4, y = 0$ in the L.H.S. of equation (i), we get

$$\text{L.H.S.} = 4 - 2 \times 0 = 4 - 0 = 4 = \text{R.H.S.}$$

$\because$ L.H.S. = R.H.S.

$\therefore$ (4, 0) is a solution of the given equation.

(iv) Substituting $x = \sqrt{2}$, $y = 4\sqrt{2}$ in the L.H.S. of equation (i), we get

$$\text{L.H.S.} = \sqrt{2} - 2 \times 4\sqrt{2} = \sqrt{2} - 8\sqrt{2} = -7\sqrt{2} \neq \text{R.H.S.}$$

$\because$ L.H.S. $\neq$ R.H.S.

$\therefore$ $(\sqrt{2}, 4\sqrt{2})$ is not a solution of the given equation.

(v) Substituting $x = 1, y = 1$ in the L.H.S. of equation (i), we get

$$\text{L.H.S.} = 1 - 2 \times 1 = 1 - 2 = -1 \neq \text{R.H.S.}$$

$\because$ L.H.S. $\neq$ R.H.S.

$\therefore$ (1, 1) is not a solution of the given equation.

Q.4. Find the value of k, if $x = 2$, $y = 1$ is a solution of the equation $2x + 3y = k$.

Sol. Since $x = 2$, $y = 1$ is a solution of the equation

$$2x + 3y = k$$

Therefore, $x = 2$, $y = 1$ will satisfy the given equation.

$$\Rightarrow \quad 2 \times 2 + 3 \times 1 = k$$

$$\Rightarrow \quad 4 + 3 = k \Rightarrow 7 = k$$

Hence, $k = 7$

5 Introduction to Euclid's Geometry

Textbook Exercises 5.1

Q.1. Which of the following statements are true and which are false ? Give reasons for your answers :

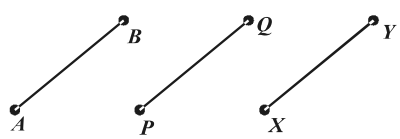
(i) Only one line can pass through a single point.

(ii) There are an infinite number of lines which pass through two distinct points.

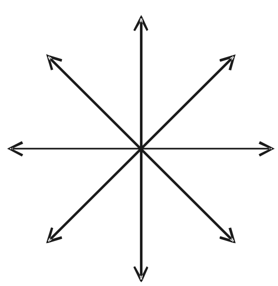
(iii) A terminated line can be produced indefinitely on both the sides.

(iv) If two circles are equal, then their radii are equal.

(v) In figure, if $AB = PQ$ and $PQ = XY$, then $AB = XY$.



Sol. (i) False, because an infinite number of lines can be drawn passing through a given point see in the figure.



(ii) False, because by Euclid's postulate 1, a straight line may drawn from any one point to any other point.

(iii) True, because by Euclid's postulate 2, a terminated line can be produced indefinitely on both the sides.

(iv) True, because, if you superimpose the region bounded by one circle on the other, then they coincide, so their centres and boundaries coincide. Therefore, their radii will be equal.

(v) True, because by Axiom 1, things which are equal to the same things are equal to one another. **Ans.**

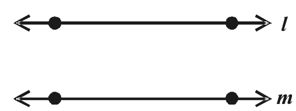
Q.2. Give definition for each of the following terms. Are there other terms that need to be defined first ? What are they, and how might you define them ?

(i) Parallel lines (ii) Perpendicular lines

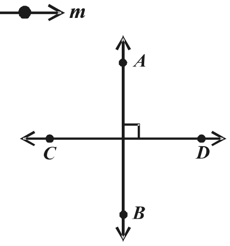
(iii) Line segment (iv) Radius of a circle

(v) Square.

Sol. (i) Parallel lines : The straight lines, which lie in the same plane and do not meet at any point on producing on either side, are called parallel lines. In the given figure, lines l and m are parallel lines. We write $l \parallel m$.



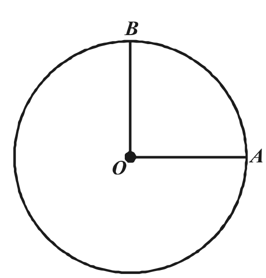
(ii) Perpendicular lines : If two lines AB and CD intersect at right angle, then they are known as perpendicular lines. We write $AB \perp CD$.



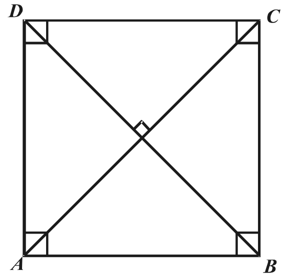
(iii) Line segment : Given two distinct points A and B on the line l, the connected part (segment) of the line with end points at A and B, is called the line segment AB.

Line segment AB is denoted by the symbol $\overline{AB}$.

(iv) Radius of a circle : The distance from the centre to any point on the circle is called the radius of the circle. In the figure OA and OB are same radii of the circle.



(v) Square : A plane figure bounded by four straight lines having all its four interior angles right angles and all its sides of equal length. It's diagonals bisect one another at right angles.



In the figure, $ABCD$ is a square.

Q.3. Consider two postulates given below :

(i) Given any two distinct points A and B, there exists a third point C which is in between A and B.

(ii) There exist at least three points that are not on the same line.

Do these postulates contain any undefined terms ? Are these postulates consistent ?

Do they follow from Euclid's postulates ? Explain.

Sol. There are several undefined terms which the students should list. They are consistent, because they deal with two different situations.

(i) Says that the given two points A and B, there is a point C lying on the line in between them.

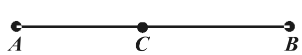
(ii) Says that given A and B, you can take C not lying on the line through A and B.

These 'postulates' do not follow from Euclid's postulates. However, they follow from axiom 5·1 given two distinct points, there is a unique line that passes through them.

Q.4. If a point C lies between two points A and B such that $AC = BC$, then prove that $AC = \frac{1}{2}AB$. Explain by drawing the figure.

Sol. We have a point C lying between A and B, such that

$$AC = BC$$



$\therefore \quad AC + CB = AB, \quad$ [By axiom 4]

But $\quad AC = BC$

$\therefore \quad AC + AC = AB \Rightarrow 2AC = AB$

$\Rightarrow \quad AC = \frac{1}{2}AB.$ **Hence proved**

Q.5. In question 4, point C is called a mid point of line segment AB. Prove that every line segment has one and only one mid-point.

Sol. Let D be another mid-point of AB.

$\therefore \quad AD = DB \quad \ldots$(i)

But C is the mid point of AB, (given)

$\therefore \quad AC = CB \quad \ldots$(ii)

Subtracting (ii) from (i), we get

$$AD - AC = BD - CB$$

$\Rightarrow \quad CD = -CD$

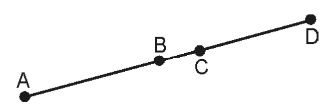
$\Rightarrow \quad CD + CD = 0 \Rightarrow 2CD = 0$

$\Rightarrow \quad CD = 0$

$\therefore$ C and D coincide.

Hence, every line segment has one and only one mid point. **Proved**

Q.6. In figure, if $AC = BD$, then prove that $AB = CD$.



Sol. We have, $AC = BD \quad \ldots$(i)

Since, point B lies between A and C.

Therefore, $\quad AC = AB + BC \quad \ldots$(ii)

and point C lies between B and D.

$\therefore \quad BD = BC + CD \quad \ldots$(iii)

From (i), (ii) and (iii), we get

$$AB + BC = BC + CD$$

$\Rightarrow AB + BC - BC = BC + CD - BC,$

(subtracting BC from both sides)

$\Rightarrow \quad AB = CD.$ **Hence proved**

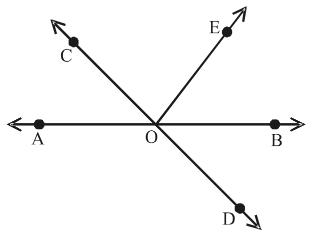
Q.7. Why is Axiom 5, in the list of Euclid's axioms, considered a 'universal truth' ? (Note that the question is not about the fifth postulate).

Sol. Axiom 5, in the list of Euclid's axiom's is true for any thing in any part of the world, this is a universal truth.

6 Lines and Angles

Textbook Exercises 6.1

Q.1. In figure, lines AB and CD intersect at O. If $\angle AOC + \angle BOE = 70°$ and $\angle BOD = 40°$, find $\angle BOE$ and reflex $\angle COE$.



Sol. $\angle AOC = \angle BOD,$ (Vertically opposite angles)

$\Rightarrow \quad \angle AOC = 40°,$

$\because \quad \angle BOD = 40°$, given ...(i)

Now, $\angle AOC + \angle BOE = 70°,$ (Given)

$\Rightarrow \quad 40° + \angle BOE = 70°$

$\Rightarrow \quad \angle BOE = 70° - 40° = 30°$ ...(ii)

Ray OC stands on line AB.

$\therefore \quad \angle AOC + \angle COB = 180°,$ (Linear pair axiom)

$\Rightarrow \angle AOC + \angle COE + \angle BOE = 180°,$

$\because \quad \angle COB = \angle COE + \angle BOE$

$\Rightarrow \quad 40° + \angle COE + 30° = 180°,$

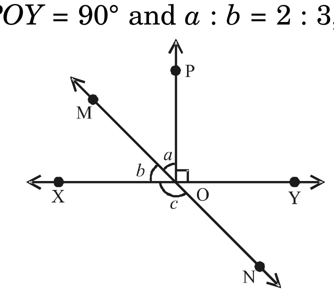
[From (i) and (ii), we have $\angle AOC = 40°$, $\angle BOE = 30°$]

$\Rightarrow \quad \angle COE + 70° = 180°$

$\Rightarrow \quad \angle COE = 180° - 70° = 110°$

So, Reflex $\angle COE = 360° - 110° = 250°$

Q.2. In figure, lines XY and MN intersect at O. If $\angle POY = 90°$ and $a : b = 2 : 3$, find c.



Sol. Since, ray OP stands on line XY. Therefore

$\angle POY + \angle POX = 180°,$ (Linear pair axiom)

$\Rightarrow \quad 90° + \angle POX = 180°,$ [$\because \angle POY = 90°$, given]

$\Rightarrow \quad \angle POX = 180° - 90° = 90°$

$\Rightarrow \quad a + b = 90°$

But $\quad a : b = 2 : 3$ (given)

Sum of ratios $= 2 + 3 = 5$

$\therefore \quad a = \dfrac{2}{5} \times 90° = 36°$

and $\quad b = \dfrac{3}{5} \times 90° = 54°$

Now, MN is a line.

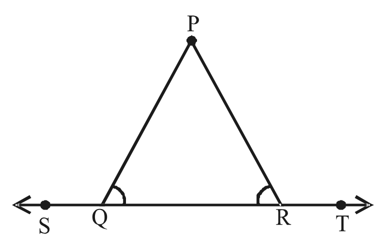
Since, ray OX stands on line MN.

Therefore, $\angle MOX + \angle XON = 180°,$ (Linear pair axiom)

$\Rightarrow \quad b + c = 180° \Rightarrow 54° + c = 180°$

$\Rightarrow \quad c = 180° - 54° = 126°$

Q.3. In figure, $\angle PQR = \angle PRQ$, then prove that $\angle PQS = \angle PRT$.



Sol. Since, ray QP stands on line SQT.

Therefore, $\angle PQS + \angle PQR = 180°,$ (Linear pair axiom) ...(i)

Since, ray RP stands on line SQT.

Therefore, $\angle PRT + \angle PRQ = 180°,$ (Linear pair axiom) ...(ii)

From (i) and (ii), we get

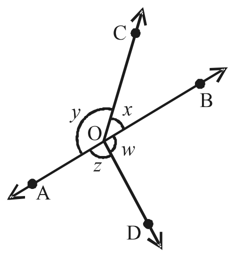
$\angle PQS + \angle PQR = \angle PRT + \angle PRQ$

$\Rightarrow \quad \angle PQS + \angle PQR = \angle PRT + \angle PQR,$ [$\because \angle PRQ = \angle PQR$, given]

$\Rightarrow \quad \angle PQS = \angle PRT,$ [Subtracting $\angle PQR$ from both sides]

Hence proved

Q.4. In figure, if $x + y = w + z$, then prove that AOB is a line.



Sol. Since, sum of all the angles round a point is equal to 360°.

Therefore,

$$\angle AOC + \angle BOC + \angle BOD + \angle AOD = 360°$$

$\Rightarrow \quad y + x + w + z = 360°$

$\Rightarrow \quad x + y + x + y = 360°,$ [Given $w + z = x + y$]

$\Rightarrow \quad 2x + 2y = 360° \Rightarrow 2(x + y) = 360°$

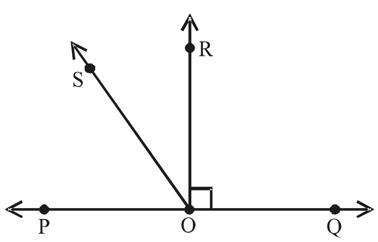
$\Rightarrow \quad x + y = \dfrac{360°}{2} = 180°$

If the sum of two adjacent angles is 180°, then non-common arms of the angles form a line.

Hence, AOB is a line. **Hence Proved**

Q.5. In figure, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that :

$$\angle ROS = \frac{1}{2}(\angle QOS - \angle POS).$$



Sol. Since, ray $OR \perp$ line PQ.

$\angle ROQ + \angle ROP = 180°,$ (Linear pair axiom)

$\Rightarrow \quad 90° + \angle ROP = 180°$ [$\because \angle ROQ = 90°$]

$\Rightarrow \angle ROP = 180° - 90° = 90°$

$\Rightarrow \quad \angle ROS + \angle POS = 90°,$

$\because$ [$\angle ROP = \angle ROS + \angle POS$]

$\Rightarrow \quad \angle POS = 90° - \angle ROS$ ...(i)

$\angle QOS = \angle ROS + \angle ROQ$

$\Rightarrow \quad \angle QOS = \angle ROS + 90°$ ...(ii)

Subtracting (i) from (ii), we get

$\angle QOS - \angle POS = \angle ROS + 90° - (90° - \angle ROS)$

$\Rightarrow \angle QOS - \angle POS = \angle ROS + 90° - 90° + \angle ROS$

$\Rightarrow \quad \angle QOS - \angle POS = 2\angle ROS$

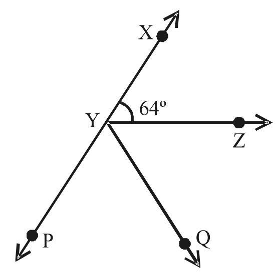
$\Rightarrow \quad \dfrac{1}{2}(\angle QOS - \angle POS) = \angle ROS$

$\Rightarrow \quad \angle ROS = \dfrac{1}{2}(\angle QOS - \angle POS).$

Hence proved

Q.6. It is given that $\angle XYZ = 64°$ and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects $\angle ZYP$, find $\angle XYQ$ and reflex $\angle QYP$.

Sol. It is given that XY produced to P. So, XP is a straight line.



Now, ray YZ stands on line XP.

$\therefore \quad \angle XYZ + \angle ZYP = 180°$ (Linear pair axiom)

$\Rightarrow \quad 64° + \angle ZYP = 180°,$ [$\because \angle XYZ = 64°$, given]

$\Rightarrow \quad \angle ZYP = 180° - 64° = 116°$

Since, ray YQ bisects $\angle ZYP$.

$$\angle ZYQ = \angle QYP = \frac{116°}{2} = 58°$$

Now, $\quad \angle XYQ = \angle XYZ + \angle ZYQ = 64° + 58° = 122°$

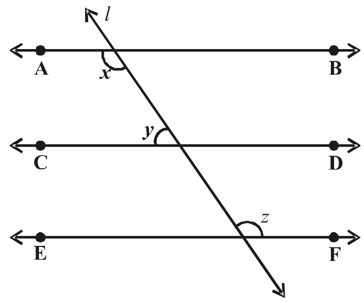
Reflex $\angle QYP = 360° - \angle QYP = 360° - 58° = 302°$

Hence, $\quad \angle XYQ = 122°$

and $\quad$ Reflex $\angle QYP = 302°$.

Textbook Exercises 6.2

Q.1. In figure, if $AB \parallel CD$, $CD \parallel EF$ and $y : z = 3 : 7$, find x.



Sol. Since, $CD \parallel EF$ and transversal line l intersects CD at Q and EF at R.

$$\angle CQR = \angle QRF,$$
(Alternate interior angles)

$\Rightarrow$ $\angle CQR = z$

$\angle PQC + \angle CQR = 180°$,
(Linear pair axiom)

$\Rightarrow$ $y + z = 180°$

Now, $y : z = 3 : 7$

Sum of ratios $= 3 + 7 = 10$

$\therefore$ $y = \dfrac{3}{10} \times 180° \Rightarrow y = 54°$

and $z = \dfrac{7}{10} \times 180° \Rightarrow z = 126°$

Since $AB \parallel CD$ and transversal line l intersects them at P and Q respectively. Therefore,

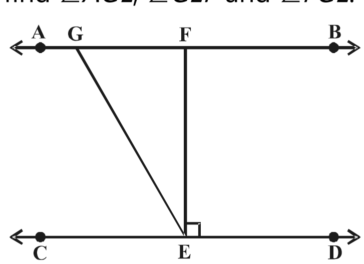
$\angle APQ + \angle PQC = 180°$,
[Co-interior angles are supplementary]

$\Rightarrow$ $x + y = 180°$

$\Rightarrow$ $x + 54° = 180°$ $[\because y = 54°]$

$\Rightarrow$ $x = 180° - 54° \Rightarrow x = 126°$

Q.2. In figure, if $AB \parallel CD$, $EF \perp CD$ and $\angle GED = 126°$, find $\angle AGE$, $\angle GEF$ and $\angle FGE$.



Sol. Since, $AB \parallel CD$ and transversal GE cuts them at G and E respectively.

$\angle AGE = \angle GED$,
(Alternate interior angles)

$\Rightarrow$ $\angle AGE = 126°$
$[\because \angle GED = 126°$ given$]$

$\angle GED = 126°$,

and $\angle FED = 90°$, $(EF \perp CD)$

Now, $\angle GEF = \angle GED - \angle FED$

$\Rightarrow$ $\angle GEF = 126° - 90°$

$\Rightarrow$ $\angle GEF = 36°$

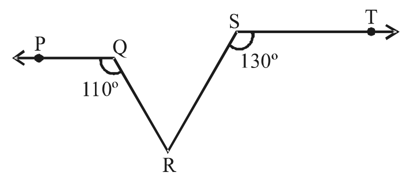
$\angle FGE + \angle GED = 180°$,
(Co-interior angles are supplementary)

$\Rightarrow$ $\angle FGE + 126° = 180°$

$\Rightarrow$ $\angle FGE = 180° - 126°$

$\Rightarrow$ $\angle FGE = 54°$

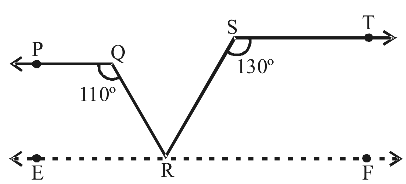
Q.3. In figure, if $PQ \parallel ST$, $\angle PQR = 110°$ and $\angle RST = 130°$, find $\angle QRS$.



Sol. Draw a line $EF \parallel ST$ through point R.

$\angle TSR + \angle FRS = 180°$
(Co-interior angles are supplementary)

$\Rightarrow$ $130° + \angle FRS = 180°$,
$(\because \angle TSR = 130°)$



$\Rightarrow$ $\angle FRS = 180° - 130°$

$\Rightarrow$ $\angle FRS = 50°$

$PQ \parallel ST$, (Given)

$EF \parallel ST$, (By construction)

$PQ \parallel EF$,

$\angle PQR + \angle ERQ = 180°$
(Co-interior angles are supplementary)

$\Rightarrow \quad 110° + \angle ERQ = 180°,$

$(\because \angle PQR = 110°)$

$\Rightarrow \quad \angle ERQ = 180° - 110°$

$\Rightarrow \quad \angle ERQ = 70°$

Now, $\angle ERQ + \angle QRF = 180°,$

(Linear pair axiom)

$\Rightarrow \angle ERQ + \angle QRS + \angle FRS = 180°$

$\Rightarrow 70° + \angle QRS + 50° = 180°$

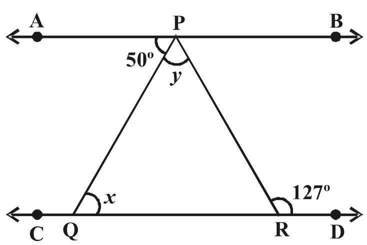
$(\because \angle ERQ = 70° \text{ and } \angle FRS = 50°)$

$\Rightarrow \quad 120° + \angle QRS = 180°$

$\Rightarrow \quad \angle QRS = 180° - 120°$

$\Rightarrow \quad \angle QRS = 60°.$ **Ans.**

Q.4. In figure, if $AB \parallel CD$, $\angle APQ = 50°$ and $\angle PRD = 127°$, find x and y.



Sol. $AB \parallel CD$ and transversal PQ intersects them at P and Q respectively.

$\therefore \quad \angle PQR = \angle APQ$

(Alternate interior angles)

$\Rightarrow \quad x = 50°, (\because \angle APQ = 50°)$

Also, $AB \parallel CD$ and transversal PR intersects them at P and R respectively.

$\therefore \quad \angle APR = \angle PRD$

$\Rightarrow \angle APQ + \angle QPR = 127°,$

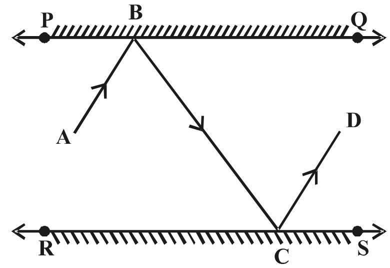
$(\because \angle PRD = 127°)$

$\Rightarrow \quad 50° + y = 127°,$

$(\because \angle APQ = 50°, \angle QPR = y)$

$\Rightarrow \quad y = 127° - 50° \Rightarrow y = 77°$

Q.5. In figure, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that $AB \parallel CD$.



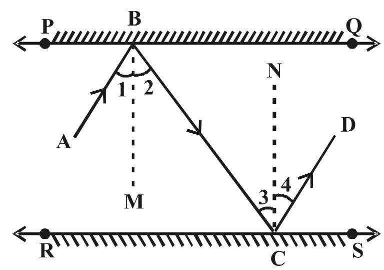
Sol. Given : Two mirrors PQ and RS are placed such that $PQ \parallel RS$. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD.

To prove : $AB \parallel CD$.

Construction : Draw normals BM on PQ and CN on RS.

Proof : $BM \perp PQ$ and $CN \perp RS$.

$\Rightarrow \quad BM \parallel CN$



Now, $BM \parallel CN$ and BC is the transversal.

$\therefore \quad \angle 2 = \angle 3,$

(Alternate interior angles) ...(i)

$\angle 1 = \angle 2$ and $\angle 3 = \angle 4,$

(By laws of reflection) ...(ii)

From (i), we have

$2\angle 2 = 2\angle 3$

$\Rightarrow \quad \angle 2 + \angle 2 = \angle 3 + \angle 3$

$\Rightarrow \quad \angle 1 + \angle 2 = \angle 3 + \angle 4,$

[From (ii), $\angle 1 = \angle 2$, $\angle 3 = \angle 4$]

$\Rightarrow \quad \angle ABC = \angle BCD$

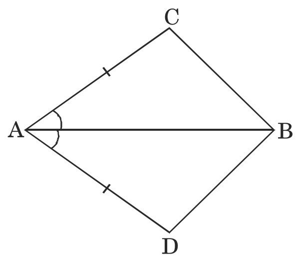
Thus, a pair of alternate interior angles $\angle ABC$ and $\angle BCD$ are equal.

$AB \parallel CD$. **Hence proved**

7 Triangles

Textbook Exercises 7.1

Q.1. In quadrilateral $ACBD$, $AC = AD$ and AB bisects $\angle A$ **(see figure)**.



Show that $\Delta ABC \cong \Delta ABD$. What can you say about BC and BD?

Sol. In ΔABC and ΔABD, we have

$AC = AD$, (Given)

$\angle CAB = \angle DAB$, (AB bisects $\angle A$)

and $AB = AB$, (Common)

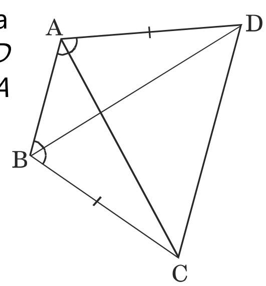
$\therefore$ $\Delta ABC \cong \Delta ABD$,

(By SAS congruence rule)

$\Rightarrow$ $BC = BD$, (CPCT)

Hence, BC and BD are equal. **Ans.**

Q.2. $ABCD$ is a quadrilateral in which $AD = BC$ and $\angle DAB = \angle CBA$ **(see figure)**.



Prove that :

(i) $\Delta ABD \cong \Delta BAC$

(ii) $BD = AC$

(iii) $\angle ABD = \angle BAC$.

Sol. (i) In ΔABD and ΔBAC,

we have $AD = BC$, (Given)

$\angle DAB = \angle CBA$, (Given)

and $AB = AB$, (Common)

$\Delta ABD \cong \Delta BAC$,

(By SAS congruence rule)

Hence proved

(ii) $\because$ $\Delta ABD \cong \Delta BAC$, (As proved above)

$\Rightarrow$ $BD = AC$, (CPCT)

Hence proved

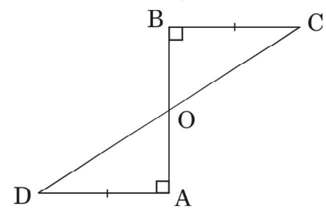
(iii) $\because$ $\Delta ABD \cong \Delta BAC$,

(As proved above)

$\Rightarrow$ $\angle ABD = \angle BAC$, (CPCT)

Hence proved

Q.3. AD and BC are equal perpendiculars to a line segment AB **(see figure).**



Show that CD bisects AB.

Sol. In ΔAOD and ΔBOC, we have

$\angle OAD = \angle OBC$, (Each = 90°)

$\angle AOD = \angle BOC$,

(Vertically opposite angles)

and $AD = BC$, (Given)

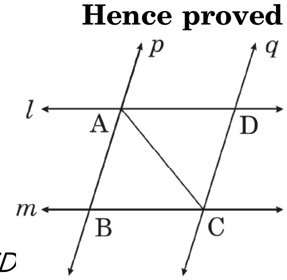
$\therefore$ $\Delta AOD \cong \Delta BOC$,

(By AAS congruence rule)

$\Rightarrow$ $OA = OB$, (CPCT)

or CD bisects AB. **Hence proved**

Q.4. l and m are two parallel lines intersected by another pair of parallel lines p and q **(see figure)**.



Show that $\Delta ABC \cong \Delta CD$

Sol. Since two parallel lines l and m intersected by another pair of parallel lines p and q.

Therefore $AD \parallel BC$ and $AB \parallel DC$

$\therefore$ $AD \parallel BC$ and AC is the transversal.

$\angle ACB = \angle DAC$

(A pair of alternate interior angles)

Again, $AB \parallel DC$ and AC is the transversal.

$\angle BAC = \angle ACD$,

(A pair of alternate interior angles)

Now in ΔABC and ΔCDA, we have Fig.

$\angle ACB = \angle DAC,$ (As proved above)

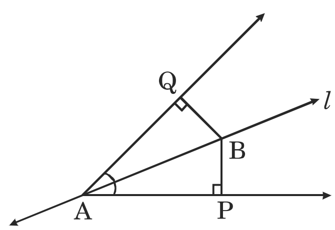
$AC = AC,$ (common)

and $\angle BAC = \angle ACD,$ (As proved above)

$\therefore$ $\Delta ABC \cong \Delta CDA$ [By ASA congruence rule]

Hence proved

Q.5. Line l is the bisector of an angle A and B is any point on l. BP and BQ are perpendiculars from B to the arms of $\angle A$ (see figure).



Show that :

(i) $\Delta APB \cong \Delta AQB$

(ii) $BP = BQ$ or B is equidistant from the arms of $\angle A$.

Sol. (i) In ΔAPB and ΔAQB, we have

$\angle APB = \angle AQB,$ (Each = 90°)

$\angle PAB = \angle QAB,$ (Line l bisects $\angle A$)

and $AB = AB,$ (Common)

$\therefore$ $\Delta APB \cong \Delta AQB,$ (By AAS congruence rule)

Hence proved

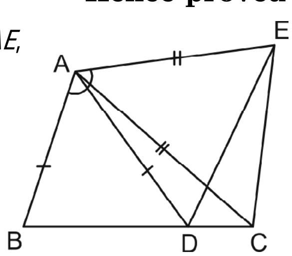
(ii) $\because$ $\Delta APB \cong \Delta AQB,$ (As proved above)

$\Rightarrow$ $BP = BQ,$ (CPCT)

or B is equidistant from the arms of $\angle A$.

Hence proved

Q.6. In figure, $AC = AE$, $AB = AD$ and $\angle BAD = \angle EAC$. Show that $BC = DE$.



Sol. We have

$\angle BAD = \angle EAC$ (Given)

$\Rightarrow$ $\angle BAD + \angle CAD = \angle EAC + \angle CAD,$ (Adding $\angle CAD$ on both sides)

$\Rightarrow$ $\angle BAC = \angle DAE$ ...(i)

Now in ΔABC and ΔADE, we have

$AB = AD,$ (Given)

$\angle BAC = \angle DAE,$ (As proved above)

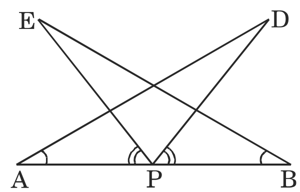
and $AC = AE,$ (Given)

$\therefore$ $\Delta ABC \cong \Delta ADE,$ (By SAS congruence rule)

$\Rightarrow$ $BC = DE,$ (CPCT)

Hence proved

Q.7. AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that $\angle BAD = \angle ABE$ and $\angle EPA = \angle DPB$ (see figure).



Show that :

(i) $\Delta DAP \cong \Delta EBP$ (ii) $AD = BE$.

Sol. (i) We have

$\angle EPA = \angle DPB$ (Given)

$\Rightarrow$ $\angle EPA + \angle EPD = \angle DPB + \angle EPD.$ (Adding $\angle EPD$ on both sides)

$\Rightarrow$ $\angle APD = \angle BPE$ ...(i)

Now, ΔDAP and ΔEBP, we have

$\angle BAD = \angle ABE$ (Given)

$\Rightarrow$ $\angle DAP = \angle EBP,$

$AP = PB,$ (P is the mid point of AB)

and $\angle APD = \angle BPE,$ (As proved above (i))

$\therefore$ $\Delta DAP \cong \Delta EBP$ (By ASA congruence rule)

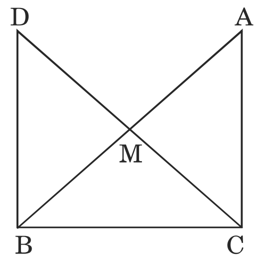
Hence proved

(ii) $\because$ $\Delta DAP \cong \Delta EBP,$ (As proved above)

$\Rightarrow$ $AD = BE,$ (CPCT)

Hence proved

Q.8. In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that $DM = CM$. Point D is joined to point B (see figure).



Show that:

(i) $\Delta AMC \cong \Delta BMD$

(ii) $\angle DBC$ is a right angle

(iii) $\Delta DBC \cong \Delta ACB$

(iv) $CM = \frac{1}{2} AB$.

Sol. (i) In ΔBMD and ΔAMC, we have

$BM = AM$, (M is the mid point of AB)

$DM = CM$, (Given)

and $\angle BMD = \angle AMC$, (Vertically opposite angles)

$\therefore$ $\Delta BMD \cong \Delta AMC$, (By SAS congruence rule)

Hence proved

(ii) $\Delta BMD \cong \Delta AMC$, [As proved above in solution (i)]

$\Rightarrow$ $BD = AC$, ...(1) (CPCT)

and $\angle DBM = \angle CAM$, (CPCT)

But these are alternate interior angles.

$DB \parallel AC$, [By theorem 6·3]

Now $DB \parallel AC$

and BC is the transversal.

$\angle DBC + \angle ACB = 180°$,

(A pair of allied angles are supplementary)

$\Rightarrow$ $\angle DBC + 90° = 180°$,

$\because$ $\angle ACB = 90°$ (Given)

$\Rightarrow$ $\angle DBC = 180° - 90° = 90°$

$\Rightarrow$ So, $\angle DBC$ is a right angle.

Hence proved

(iii) In ΔDBC and ΔACB, we have

$DB = AC$, [From (i)]

$\angle DBC = \angle ACB$, [Each = 90°]

and $BC = BC$, (common)

$\Delta DBC \cong \Delta ACB$, (By SAS congruence rule)

Hence proved

(iv) $\Delta DBC \cong ACB$

(As proved above in solution (iii)] (CPCT)

$\Rightarrow$ $DC = AB$,

$\Rightarrow$ $\frac{1}{2} DC = \frac{1}{2} AB$

(Multiply by 1/2 on both sides)

$\Rightarrow$ $CM = \frac{1}{2} AB$

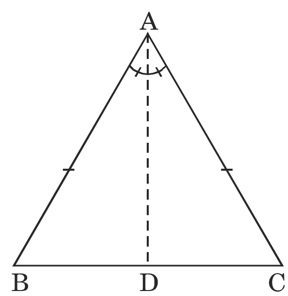
[$\because$ M is the mid point of DC and AB respectively]

Hence proved

Theorem 7·2 : Angles opposite to equal sides of an isosceles triangle are equal.

Given : In ΔABC, $AB = AC$.

To prove : $\angle B = \angle C$.



Construction : Draw AD, the bisector of $\angle A$ which meets BC on D.

Proof : In ΔABD and ΔACD, we have

$AB = AC$, (Given)

$\angle BAD = \angle CAD$, (By construction)

and $AD = AD$, (Common)

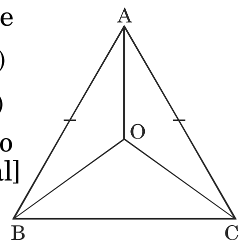
$\therefore$ $\Delta ABD \cong \Delta ACD$, (By SAS congruence rule)

$\Rightarrow$ $\angle B = \angle C$. **Hence proved.**

Textbook Exercises 7.2

Q.1. In an isosceles triangle ABC with $AB = AC$, the bisectors of $\angle B$ and $\angle C$ intersect each other at O. Join A to O Show that :

(i) $OB = OC$ (ii) AO bisects $\angle A$.



Sol. (i) In ΔABC, we have

$AB = AC$ (Given)

$\Rightarrow$ $\angle B = \angle C$ ...(i)

[$\because$ Angles opposite to equal sides are equal]

Since BO is the bisector of $\angle B$.

$\therefore$ $\angle ABO = \angle CBO$ ...(ii)

and CO is the bisector of $\angle C$.

$\therefore$ $\angle ACO = \angle BCO$ ...(iii)

From (i), (ii) and (iii), we get

$\angle ABO = \angle ACO$ ...(iv)

and $\angle CBO = \angle BCO$

$\Rightarrow$ $BO = CO$ ...(v)

($\because$ Sides opposite to equal angles are equal)

(ii) In ΔABO and ΔACO, we have

$AB = AC,$ (Given)

$\angle ABO = \angle ACO,$ [From (iv)]

and $BO = CO,$ [From (v)]

$\therefore$ $\Delta ABO \cong \Delta ACO,$

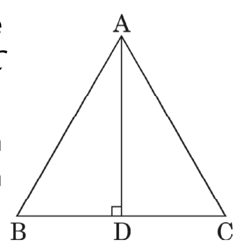
(By SAS congruence rule)

$\Rightarrow$ $\angle OAB = \angle OAC,$ (CPCT)

OR AO bisects $\angle A$. **Hence Proved**

Q.2. In ΔABC, AD is the perpendicular bisector of BC (see figure).

Show that ΔABC is an isosceles triangle in which $AB = AC$.



Sol. In ΔADB and ΔADC,

we have $BD = CD,$

($\because$ AD is the bisector of BC)

$\angle ADB = \angle ADC,$

($\because$ $AD \perp BC$, $\therefore$ Each = 90°)

and $AD = AD,$ (Common)

$\therefore$ $\Delta ADB \cong \Delta ADC,$

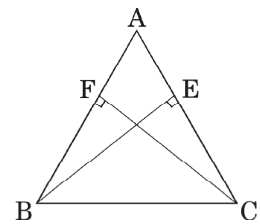
(By SAS congruence rule)

$\Rightarrow$ $AB = AC,$ (CPCT)

Or ΔABC is an isosceles triangle.

Hence Proved

Q.3. ABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respect-ively (see figure).



Show that these altitudes are equal.

Sol. In ΔABC, we have

$AB = AC,$

($\because$ ΔABC is an isosceles triangle)

$\Rightarrow$ $\angle B = \angle C,$

($\because$ Angles opposite to equal sides are equal)

Now, in ΔBFC and ΔCEB, we have

$\angle B = \angle C,$

(As proved above)

$\angle BFC = \angle CEB$

($\because$ $CF \perp AB$ and $BE \perp AC$, $\therefore$ Each = 90°)

and $BC = BC,$ (Common)

$\therefore$ $\Delta BFC \cong \Delta CEB$

(By AAS congruence rule)

$\Rightarrow$ $CF = BE$ (CPCT)

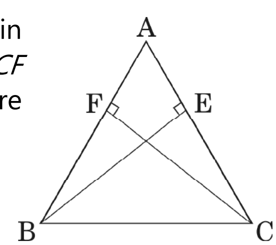
Hence proved

Q.4. ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal (see figure).

Show that :

(i) $\Delta ABE \cong \Delta ACF$

(ii) $AB = AC$, *i.e.*, ABC is an isosceles triangle.



Sol. (i) In ΔABE and ΔACF,

we have

$$\angle AEB = \angle AFC, \quad \left[\begin{array}{l}\because BE \perp AC \text{ and } CF \perp AB \\ \therefore \text{Each} = 90^\circ\end{array}\right]$$

$$\angle BAE = \angle CAF, \quad \text{(Common)}$$

and $$BE = CF, \quad \text{(Given)}$$

$$\Delta ABE \cong \Delta ACF,$$

(By AAS congruence rule)

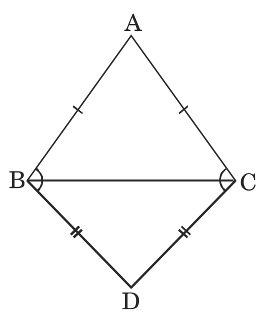
(ii) $$\Delta ABE \cong \Delta ACF \text{ (As proved above)}$$

$$\Rightarrow \quad AB = AC, \quad \text{(CPCT)}$$

Or ABC is an isosceles triangle.

Hence proved

Q.5. *ABC* and *DBC* are two isosceles triangles on the same base *BC* (see figure).



Show that $\angle ABD = \angle ACD$.

Sol. In ΔABC, we have

$$AB = AC$$

($\because \Delta ABC$ is an isosceles triangle)

$$\Rightarrow \quad \angle ABC = \angle ACB \quad \ldots\text{(i)}$$

($\because$ Angles opposite to equal sides are equal)

Again in ΔDBC, we have

$$DB = DC$$

($\because \Delta DBC$ is an isosceles triangle)

$$\Rightarrow \quad \angle DBC = \angle DCB, \quad \ldots\text{(ii)}$$

($\because$ Angles opposite to equal sides are equal)

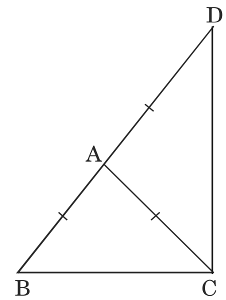
Adding (i) and (ii), we get

$$\angle ABC + \angle DBC = \angle ACB + \angle DCB$$

$$\Rightarrow \quad \angle ABD = \angle ACD.$$

Hence proved.

Q.6. ΔABC is an isosceles triangle in which $AB = AC$. Side *BA* is produced to *D* such that $AD = AB$ (see figure).



Show that $\angle BCD$ is a right angle.

Sol. Given : ΔABC in which $AB = AC$. Side BA is produced to D such that

$$AD = AB.$$

To prove : $\angle BCD = 90^\circ$.

Construction : Join C to D.

Proof : In ΔABC, we have

$$AB = AC \quad \text{(Given)}$$

$$\Rightarrow \quad \angle ABC = \angle ACB \quad \ldots\text{(i)}$$

($\because$ Angles opposite to equal sides are equal)

$$AB = AD, \quad \text{(Given)}$$

$$\therefore \quad AD = AC, \quad (\because AB = AC)$$

$$\Rightarrow \quad \angle ADC = \angle ACD \quad \ldots\text{(ii)}$$

($\because$ Angles opposite to equal sides are equal)

Adding (i), (ii), we get

$$\angle ABC + \angle ADC = \angle ACB + \angle ACD$$

$$\Rightarrow \angle ABC + \angle ADC = \angle BCD$$

$$\Rightarrow \quad \angle DBC + \angle BDC = \angle BCD$$

(from fig. $\angle ABC = \angle DBC$ and $\angle ADC = \angle BDC$)

Now in ΔBCD, we have

$$\angle DBC + \angle BDC + \angle BCD = 180^\circ,$$

($\because$ Sum of interior angles of a triangle = 180°)

$$\Rightarrow \quad \angle BCD + \angle BCD = 180^\circ, \quad \text{[Using (iii)]}$$

$$\Rightarrow \quad 2\angle BCD = 180^\circ$$

$$\Rightarrow \quad \angle BCD = \frac{180^\circ}{2} = 90^\circ.$$

Or $\angle BCD$ is a right angle.

Hence Proved

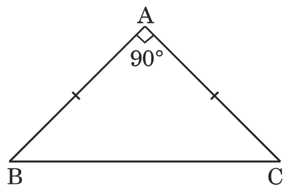
Q.7. *ABC* is a right angled triangle in which $\angle A = 90^\circ$ and $AB = AC$. Find $\angle B$ and $\angle C$.

Sol. In ΔABC, we have

$\angle A = 90°$ and $AB = AC$

$\Rightarrow \quad \angle B = \angle C \quad ...(i)$

($\because$ Angles opposite to equal sides are equal)



Now in ΔABC, we have

$\angle A + \angle B + \angle C = 180°$,

($\because$ Sum of interior angles of a triangle = 180°)

$\Rightarrow \quad 90° + \angle B + \angle B = 180°$,

[$\because$ From (i), $\angle C = \angle B$]

$\Rightarrow \quad 2\angle B = 180° - 90°$,

$\Rightarrow \quad 2\angle B = 90° \Rightarrow \angle B = \frac{90°}{2} = 45°$

Hence, $\quad \angle B = \angle C = 45°$.

Q.8. Show that the angles of an equilateral triangle are 60° each.

Sol. Let ΔABC be an equilateral triangle.

So, $\quad AB = BC = AC$

Now, $\quad AB = AC$

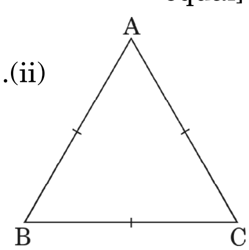
$\Rightarrow \quad \angle B = \angle C \quad ...(i)$

[Angles opposite to equal sides are equal]

Again, $\quad BC = AC$

$\Rightarrow \quad \angle B = \angle A \quad ...(ii)$

[Angles opposite to equal sides are equal]



From (i) and (ii), we get

$\angle A = \angle B = \angle C \quad ...(iii)$

Now $\angle A + \angle B + \angle C = 180°$

[$\because$ Sum of interior angles of a Δ = 180°]

$\Rightarrow \quad \angle A + \angle A + \angle A = 180° \quad$ [Using (iii)]

$\Rightarrow \quad 3\angle A = 180°$

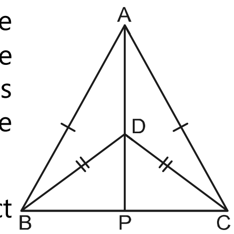
$\Rightarrow \quad \angle A = \frac{180°}{3} = 60°$

$\therefore \quad \angle A = \angle B = \angle C = 60°$

Hence, each angle of an equilateral triangle is 60°. **Hence Proved**

Textbook Exercise 7.3

Q.1. ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (see figure).



If AD is extended to intersect BC at P, show that:

(i) $\Delta ABD \cong \Delta ACD$,

(ii) $\Delta ABP \cong \Delta ACP$,

(iii) AP bisects $\angle A$ as well as $\angle D$,

(iv) AP is the perpendicular bisector of BC.

Sol. (i) In ΔABD and ΔACD, we have

$AB = AC$,

($\because$ ΔABC is an isosceles triangle)

$DB = DC$,

($\because$ ΔDBC is an isosceles triangle)

and $\quad AD = AD$, $\quad$ (Common)

$\therefore \quad \Delta ABD \cong \Delta ACD$,

(By SSS congruence rule)

$\Rightarrow \quad \angle BAD = \angle CAD$, (CPCT) ...(i)

Hence proved

(ii) In ΔABP and ΔACP, we have

$AB = AC$, $\quad$ (Given)

$\angle BAP = \angle CAP$,

[from (i), $\angle BAD = \angle CAD$]

and $\quad AP = AP$, $\quad$ (Common)

$\therefore \quad \Delta ABP \cong \Delta ACP$, $\quad$ ...(ii)

(by SAS congruence rule)

$\Rightarrow \quad BP = CP \quad$ (CPCT) ...(iii)

Hence proved

(iii) From (i), we have

$\angle BAD = \angle CAD \Rightarrow \angle BAP = \angle CAP$

$\Rightarrow AP$ is the bisector of $\angle A$.

In ΔBPD and ΔCPD, we have

$BD = CD$, $\quad$ (Given)

$BP = CP$, $\quad$ [from (iii)]

and $\quad PD = PD$, $\quad$ (Common)

$\therefore \quad \Delta BPD \cong \Delta CPD$,

(by SSS congruence rule)

$\Rightarrow \quad \angle BDP = \angle CDP,$ (CPCT)

$\Rightarrow DP$ is the bisector of $\angle D$.

Hence, AP is the bisector of $\angle A$ as well as $\angle D$. **Hence proved**

(iv) From (ii), we have

$\Delta ABP \cong \Delta ACP$ [from (iii)]

$\Rightarrow \quad BP = CP,$ [From (iii)]

and $\quad \angle APB = \angle APC,$ (CPCT) ...(iv)

$\angle APB + \angle APC = 180°,$ (linear pair)

$\Rightarrow \angle APB + \angle APB = 180°,$ [Using (iv)]

$\Rightarrow \quad 2\angle APB = 180°$

$\Rightarrow \quad \angle APB = \dfrac{180°}{2} = 90°$

Hence, AP is the perpendicular bisector of BC. **Hence proved**

Q.2. *AD* is an altitude of an isosceles triangle *ABC* in which *AB* = *AC*. Show that :

(i) *AD* bisects *BC* (ii) *AD* bisects $\angle A$.

Sol. In ΔADB and ΔADC, we have

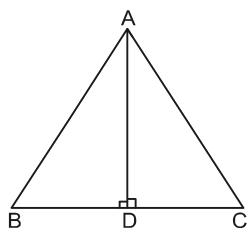
$\angle ADB = \angle ADC,$ (Each = 90°)

Hyp. AB = Hyp. $AC,$ (Given)

and $\quad AD = AD,$ (Common)

$\therefore \quad \Delta ADB \cong \Delta ADC,$

(by RHS congruence rule)



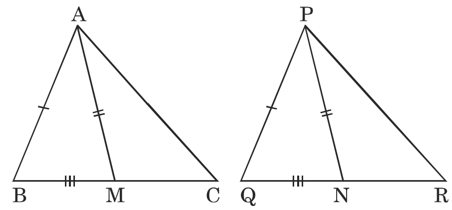
(i) $\Rightarrow \quad DB = DC,$ (CPCT)

OR AD bisects BC. **Hence proved**

(ii) $\quad \angle BAD = \angle CAD,$ (CPCT)

Or AD bisects $\angle A$. **Hence proved**

Q.3. Two sides *AB* and *BC* and median *AM* of one triangle *ABC* are respectively equal to sides *PQ* and *QR* and median *PN* of ΔPQR (see figure).



Show that :

(i) $\Delta ABM \cong \Delta PQN,$ (ii) $\Delta ABC \cong \Delta PQR.$

Sol. We have,

AM is the median of ΔABC.

$\Rightarrow \quad BM = CM$ ...(i)

PN is the median of ΔPQR.

$\because \quad QN = RN$ ...(ii)

$BC = QR$ (Given)

$\Rightarrow \quad \dfrac{1}{2}BC = \dfrac{1}{2}QR$

(Multiplying by 1/2 on both sides)

$\Rightarrow \quad BM = QN$ ...(iii)

(i) In ΔABM and ΔPQN, we have

$AB = PQ,$ (Given)

$BM = QN,$ [from (iii)]

and $\quad AM = PN,$ (Given)

$\therefore \quad \Delta ABM \cong \Delta PQN,$

(by SSS congruence rule)

Hence proved

$\Rightarrow \quad \angle B = \angle Q,$ (CPCT) ...(iv)

(ii) In ΔABC and ΔPQR, we have

$AB = PQ,$ (Given)

$\angle B = \angle Q,$ [from (iv)]

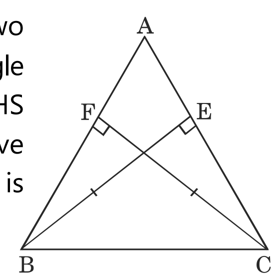
and $\quad BC = QR$ (Given)

$\therefore \quad \Delta ABC \cong \Delta PQR,$

(by SAS congruence rule)

Hence proved

Q.4. *BE* and *CF* are two equal altitudes of a triangle *ABC*. Using RHS congruence rule, prove that the triangle *ABC* is isosceles.



Sol. We have,

$BE \perp AC$ and $CF \perp AB$

i.e., $\angle CFB = 90°$ and $\angle BEC = 90°$ ...(i)

In ΔBFC and ΔCEB, we have

$\angle CFB = \angle BEC$, [from (i), Each = 90°]

Hyp. BC = Hyp. BC, (Common)

and $CF = BE$, (Given)

$\therefore$ $\Delta BFC \cong \Delta CEB$, (by RHS congruence rule)

$\Rightarrow$ $\angle FBC = \angle ECB$, (CPCT)

$\Rightarrow$ $\angle B = \angle C$

Now in ΔABC, we have

$\angle B = \angle C$, (As proved above)

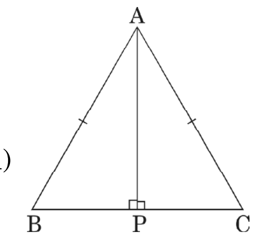
$\Rightarrow$ $AB = AC$, ($\because$ Sides opposite to equal angles are equal)

Or ΔABC is an isosceles triangle.

Hence proved

Q.5. *ABC* is an isosceles triangle with $AB = AC$. Draw $AP \perp BC$ to show that $\angle B = \angle C$.

Sol. In ΔAPB and ΔAPC, we have



$\angle APB = \angle APC$, ($\because AP \perp BC$, $\therefore$ Each = 90°)

Hyp. AB = Hyp. AC, (Given)

and $AP = AP$ (common)

$\therefore$ $\Delta APB \cong \Delta APC$, (by RHS congruence rule)

$\Rightarrow$ $\angle B = \angle C$, (CPCT)

Hence proved

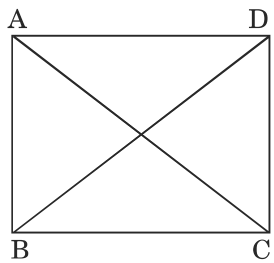
8 Quadrilaterals

Textbook Exercises 8.1

Q.1. If the diagonals of a parallelogram are equal, then show that it is a rectangle.

Sol. Given : A parallelogram $ABCD$ in which diagonal AC = diagonal BD.

To prove : $ABCD$ is a rectangle.



Proof : In ΔABC and ΔDCB, we have

$AB = DC$, (Opposite sides of a parallelogram)

$BC = BC$, (Common)

and $AC = BD$, (Given)

$\therefore$ $\Delta ABC \cong \Delta DCB$, (By SSS congruence rule)

$\Rightarrow$ $\angle ABC = \angle DCB$, (CPCT) ...(i)

But $AB \parallel CD$ and BC intersects them.

$\angle ABC + \angle DCB = 180°$, [$\because$ Sum of co-interior angles is 180°]

$\Rightarrow$ $\angle ABC + \angle ABC = 180°$, [Using (i)]

$\Rightarrow$ $2\angle ABC = 180°$

$$\Rightarrow \quad \angle ABC = \frac{180°}{2} = 90°$$

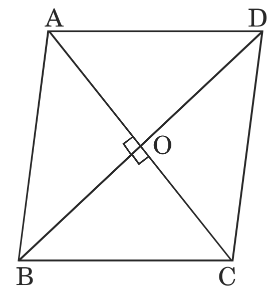
$\therefore$ $\angle ABC = \angle DCB = 90°$

Thus, $ABCD$ is a parallelogram in which one angle is 90°.

Hence, $ABCD$ is a rectangle. **Proved**

Q.2. Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

Sol. Given : A quadrilateral $ABCD$ in which diagonals AC and BD bisect each other at right angles *i.e.*, $AO = OC$, $OB = OD$ and $\angle AOB = \angle BOC = 90°$.



To prove : $ABCD$ is a rhombus.

Proof : In ΔAOB and ΔCOB, we have

$AO = OC$, (Given)

$\angle AOB = \angle COB$, (Each = 90°)

and $BO = BO$, (Common)

$\therefore$ $\Delta AOB \cong \Delta COB$, (By SAS congruence rule)

$\Rightarrow$ $AB = BC$, (CPCT) ...(i)

Similarly, $\Delta BOC \cong \Delta DOC$, (By SAS congruence rule)

$\Rightarrow$ $BC = CD$, (CPCT) ...(ii)

and $\Delta COD \cong \Delta AOD$, (By SAS congruence rule)

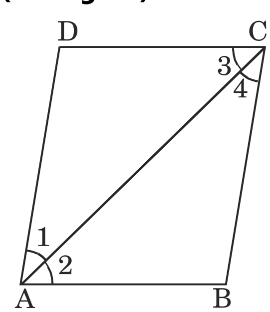
$\Rightarrow$ $D = AD$, (CPCT) ...(iii)

From (i), (ii) and (iii), we get

$AB = BC = CD = AD$

Hence, $ABCD$ is a rhombus. **Proved**

Q.3. Diagonal AC of a parallelogram $ABCD$ bisects $\angle A$ **(see figure)**.



Show that :

(i) it bisect $\angle C$ also.

(ii) $ABCD$ is a rhombus.

Sol. (i) Since $ABCD$ is a parallelogram.

$AD \parallel BC$ and AC intersects them.

$\Rightarrow \quad \angle 1 = \angle 4$ ...(i)

(Alternate interior angles)

Again, $AB \parallel CD$ and AC intersects them.

$\Rightarrow \quad \angle 2 = \angle 3$ ...(ii)

(Alternate interior angles)

But, $\angle 1 = \angle 2$, (Given) ...(iii)

From (i), (ii) and (iii), we get

$\angle 3 = \angle 4$

Hence AC bisects $\angle C$. **Proved**

(ii) $\angle 2 = \angle 3$, [From (ii)] ...(iv)

$\angle 4 = \angle 3$, [As proved above] ...(v)

From, (iv) and (v), we get

$\angle 2 = \angle 4 \Rightarrow AB = BC$

Thus, adjacent sides of a parallelogram are equal.

Hence, $ABCD$ is a rhombus. **Proved**

Q.4. $ABCD$ is a rectangle in which diagonal AC bisects $\angle A$ as well as $\angle C$. Show that :

(i) $ABCD$ is a square.

(ii) diagonal BD bisects $\angle B$ as well as $\angle D$.

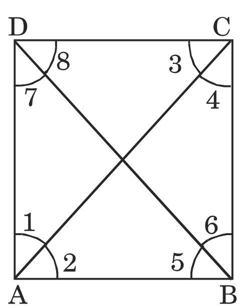
Sol. Given : A rectangle $ABCD$ in which diagonal AC bisects $\angle A$ as well as $\angle C$

i.e., $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$.

To prove : (i) $ABCD$ is a square.

(ii) diagonal BD bisects $\angle B$ as well as $\angle D$

i.e., $\angle 5 = \angle 6$ and $\angle 7 = \angle 8$.



Proof : (i) Since AC bisects $\angle A$ as well as $\angle C$.

$\angle 1 = \angle 2$ ...(i)

But, $ABCD$ is a rectangle.

$AD \parallel BC$ and AC intersects them.

$\angle 1 = \angle 4$, ...(ii)

(Alternate interior angles)

From (i) and (ii), we get

$\angle 2 = \angle 4 \Rightarrow AB = BC$

(Sides opposite to equal angles are equal)

Thus, adjacent sides of a rectangle are equal.

Hence, $ABCD$ is a square. **Proved**

(ii) In ΔABD, $AB = AD$,

(Sides of a square)

$\Rightarrow \quad \angle 7 = \angle 5$ ...(iii)

(Angles opposite to equal sides are equal)

But $AD \parallel BC$ and BD intersects them.

$\therefore \quad \angle 7 = \angle 6$, ...(iv)

(Alternate interior angles)

From (iii) and (iv), we get

$\angle 5 = \angle 6$

In ΔBCD, $BC = CD$ (Sides of a square)

$\Rightarrow \quad \angle 8 = \angle 6$ ...(v)

[Angles opposite to equal sides are equal]

But, $AD \parallel BC$ and BD intersects them.

$\therefore \quad \angle 7 = \angle 6$ ...(vi)

(Alternate interior angles)

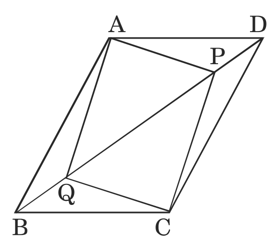
From (v) and (vi), we get

$\angle 7 = \angle 8$

Thus, $\angle 5 = \angle 6$ and $\angle 7 = \angle 8$

Hence, diagonal BD bisects $\angle B$ as well as $\angle D$. **Hence Proved**

Q.5. In a parallelogram $ABCD$, two points P and Q are taken on diagonal BD such that $DP = BQ$ **(see figure)**.



Show that :

(i) $\Delta APD \cong \Delta CQB$ (ii) $AP = CQ$

(iii) $\Delta AQB \cong \Delta CPD$ (iv) $AQ = CP$

(v) $APCQ$ is a parallelogram.

Sol. (i) Since $ABCD$ is a parallelogram.

$BC \parallel AD$ and BD intersects them.

$\therefore \quad \angle ADB = \angle CBD,$ (Alternate interior angles)

$\Rightarrow \quad \angle ADP = \angle CBQ$ ...(i)

In ΔAPD and ΔCQB, we have

$DP = BQ,$ (Given)

$\angle ADP = \angle CBQ,$ [From (i)]

and $AD = BC,$ (Opposite sides of a parallelogram)

$\therefore \quad \Delta APD \cong \Delta CQB,$ (By SAS congruence rule)

Hence proved

(ii) $\because \quad \Delta APD \cong \Delta CQB,$

$\Rightarrow \quad AP = CQ,$ (CPCT) ...(ii)

Hence proved

(iii) $AB \parallel CD$ and BD intersects them.

$\angle ABD = \angle CDB,$ (Alternate interior angles)

$\Rightarrow \quad \angle ABQ = \angle CDP$ ...(iii)

Now in ΔAQB and ΔCPD, we have

$BQ = DP,$ (Given)

$\angle ABQ = \angle CDP,$ [From (iii)]

and $AB = CD,$ (Opposite sides of a parallelogram)

$\therefore \quad \Delta AQB \cong \Delta CPD,$ (By SAS congruence rule)

Hence proved

(iv) $\because \quad \Delta AQB \cong \Delta CPD$

$\Rightarrow \quad AQ = CP,$ (CPCT) ...(iv)

Hence proved

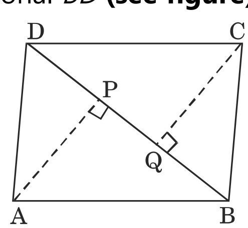
(v) From (ii) and (iv), we have

$AP = CQ$ and $AQ = CP,$

Hence, $APCQ$ is a parallelogram.

Hence proved

Q.6. $ABCD$ is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD **(see figure)**.



Show that :

(i) $\Delta APB \cong \Delta CQD.$ (ii) $AP = CQ.$

Sol. (i) Since, $ABCD$ is a parallelogram.

$AB \parallel CD$ and BD intersects them.

$\therefore \quad \angle ABD = \angle CDB,$ (Alternate interior angles)

$\Rightarrow \quad \angle ABP = \angle CDQ$ ...(i)

In ΔAPB and ΔCQD, we have

$\angle APB = \angle CQD,$ (Each = 90°)

$\angle ABP = \angle CDQ,$ [From (i)]

and $AB = CD,$ (Opposite sides of a parallelogram)

$\therefore \quad \Delta APB \cong \Delta CQD,$ (By AAS congruence rule)

(ii) $\because \quad \Delta APB \cong \Delta CQD$

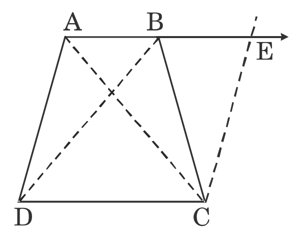
$\Rightarrow \quad AP = CQ,$ (CPCT)

Hence proved

Q.7. $ABCD$ is a trapezium in which $AB \parallel CD$ and $AD = BC$ **(see figure)**.

Show that:

(i) $\angle A = \angle B.$
(ii) $\angle C = \angle D.$
(iii) $\Delta ABC \cong \Delta BAD.$
(iv) diagonal AC = diagonal $BD.$



Sol. Given : $ABCD$ is a trapezium in which $AB \parallel CD$ and $AD = BC$.

To prove : (i) $\angle A = \angle B$, (ii) $\angle C = \angle D$, (iii) $\Delta ABC \cong \Delta BAD$, (iv) diagonal AC = diagonal BD.

Construction : Extend AB and draw a line through C parallel to DA intersecting AB produced at E. Join AC and BD.

Proof : (i) Since $AD \parallel CE$ and AE intersects them.

$\therefore \quad \angle A + \angle E = 180°$ ...(i)

[$\because$ Sum of co-interior angles is 180°]

Now $AE \parallel CD,$ (Given)

and $AD \parallel CE,$ (By construction)

$\therefore AECD$ is a parallelogram.

$\Rightarrow \quad AD = CE$ (Opposite sides of parallelogram)

But, $AD = BC$ (Given)

$\therefore$ $BC = CE$

$\Rightarrow$ $\angle CBE = \angle CEB$ ...(ii)

(Angles opposite to equal sides are equal)

Now, $\angle B + \angle CBE = 180°$, (Linear pair)

$\Rightarrow$ $\angle B + \angle CEB = 180°$, [Using (ii)]

$\Rightarrow$ $\angle B + \angle E = 180°$ ...(iii)

From (i) and (iii), we get

$\angle A + \angle E = \angle B + \angle E$

$\Rightarrow$ $\angle A = \angle B$ ...(iv)

Hence proved

(ii) Since $AB \parallel CD$ and AD intersects them.

$\angle A + \angle D = 180°$, ...(v)

[$\because$ Sum of co-interior angles is 180°]

Again, $AB \parallel CD$ and BC intersects them.

$\angle B + \angle C = 180°$ ...(vi)

From (v) and (vi), we have

$\angle A + \angle D = \angle B + \angle C$

$\Rightarrow$ $\angle A + \angle D = \angle A + \angle C$, [Using (iv)]

$\Rightarrow$ $\angle D = \angle C$. **Hence proved**

(iii) In ΔABC and ΔBAD, we have

$AB = BA$, (Common)

$\angle B = \angle A$,

[As proved above in (iv)]

and $BC = AD$, (Given)

$\therefore$ $\Delta ABC \cong \Delta BAD$,

(By SAS congruence rule)

Proved

(iv) $\Delta ABC \cong \Delta BAD$

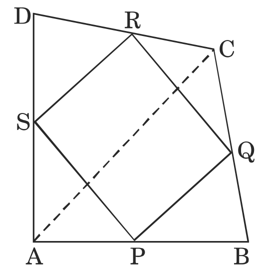
$\Rightarrow$ $AC = BD$. (CPCT)

Hence, diagonal AC = diagonal BD.

Proved

Textbook Exercises 8.2

Q.1. *ABCD* is a quadrilateral in which *P*, *Q*, *R* and *S* are mid points of the sides *AB*, *BC*, *CD* and *DA* **(see figure)**. *AC* is a diagonal.



Show that :

(i) $SR \parallel AC$ and $SR = \frac{1}{2}AC$.

(ii) $PQ = SR$.

(iii) *PQRS* is a parallelogram.

Sol. Given : $ABCD$ is a quadrilateral in which P, Q, R and S are mid points of the sides AB, BC, CD and DA respectively. AC is a diagonal.

To prove : (i) $SR \parallel AC$ and $SR = \frac{1}{2}AC$,

(ii) $PQ = SR$

(iii) $PQRS$ is a parallelogram.

Proof : (i) In ΔACD, we have

$\because$ S and R are the mid points of AD and CD respectively.

$\therefore$ $SR \parallel AC$ and $SR = \frac{1}{2}AC$ ...(i)

(ii) In ΔABC, we have

P and Q are the mid points of AB and BC respectively.

$\therefore$ $PQ \parallel AC$ and $PQ = \frac{1}{2}AC$, ...(ii)

From (i) and (ii), we get

$PQ \parallel SR$ and $PQ = SR$.

(iii) $\because$ $PQ \parallel SR$ and $PQ = SR$,

(As proved above)

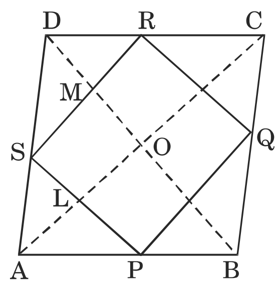
$\therefore$ $PQRS$ is a parallelogram.

Hence proved

Q.2. *ABCD* is a rhombus and *P*, *Q*, *R* and *S* are mid points of the sides *AB*, *BC*, *CD* and *DA* respectively. Show that the quadrilateral *PQRS* is a rectangle.

Sol. Given : A rhombus $ABCD$ in which P, Q, R and S are mid points of the sides AB, BC, CD and DA respectively.

To prove : Quadrilateral $PQRS$ is a rectangle.



Construction : Join AC and BD.

Proof : In ΔADC, we have

S and R are the mid points of AD and CD respectively.

$\therefore \quad SR \parallel AC$ and $SR = \frac{1}{2}AC$ ...(i)

and in ΔABC, we have

P and Q are the mid points of AB and BC respectively.

$\therefore \quad PQ \parallel AC$ and $PQ = \frac{1}{2}AC,$ ...(ii)

From (i) and (ii), we get

$$PQ \parallel SR \text{ and } PQ = SR$$

$\therefore$ $PQRS$ is a parallelogram.

Again in ΔDAB, we have

S and P are the mid points of AD and AB respectively.

$\therefore \quad SP \parallel BD$

$\Rightarrow \quad SL \parallel MO$

and $\quad SR \parallel AC$ [From (i)]

$\Rightarrow \quad SM \parallel LO$

$\therefore$ $SLOM$ is a parallelogram.

Since, diagonals of a rhombus bisect each other at 90°.

$\therefore \quad \angle AOD = 90°$

$\Rightarrow \quad \angle MOL = 90°$

But, $\quad \angle LSM = \angle MOL,$

(Opposite angles of a parallelogram)

$\Rightarrow \quad \angle LSM = 90°$

$\Rightarrow \quad \angle PSR = 90°$

Since, one angle of a parallelogram $PQRS$ is 90°.

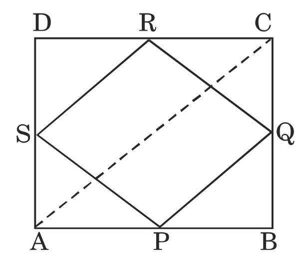
Therefore, quadrilateral $PQRS$ is a rectangle.

Hence proved

Q.3. $ABCD$ is a rectangle and P, Q, R and S are mid points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral $PQRS$ is a rhombus.

Sol. Given : A rectangle $ABCD$ in which P, Q, R and S are mid points of the sides AB, BC, CD and DA respectively.

To prove : Quadrilateral $PQRS$ is a rhombus.



Construction : Join AC.

Proof : In ΔADC, S and R are the mid points of AD and CD respectively.

$\therefore \quad SR \parallel AC$ and $SR = \frac{1}{2}AC$ ...(i)

In ΔABC, P and Q are the mid points of AB and BC respectively.

$\therefore \quad PQ \parallel AC$ and $PQ = \frac{1}{2}AC$ ...(ii)

From (i) and (ii), we get

$$PQ \parallel RQ \text{ and } PQ = SR$$

$\therefore$ $PQRS$ is a parallelogram.

$\because$ $ABCD$ is a rectangle.

$$\angle A = \angle B = 90° \quad \text{...(iii)}$$

and $\quad AD = BC,$

(Opposite sides of a rectangle)

$\Rightarrow \quad \frac{1}{2}AD = \frac{1}{2}BC \Rightarrow AS = BQ,$

[$\because$ S is the mid point of AD and Q is the mid point of BC

$\therefore AS = \frac{1}{2}AD$ and $BQ = \frac{1}{2}BC$]

Now in ΔSAP and ΔQBP, we have

$AS = BQ,$ [As proved above]

$\angle SAP = \angle QBP,$

[From (iii), Each = 90°]

and $AP = BP,$

[$\because$ P is the mid point of AB]

$\therefore$ $\Delta SAP \cong \Delta QBP,$

(By SAS congruence rule)

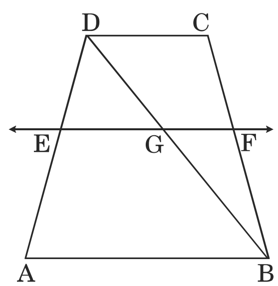
$\Rightarrow$ $SP = PQ,$ (CPCT)

Thus, adjacent sides of a parallelogram $PQRS$ are equal.

Therefore, $PQRS$ is a rhombus.

Hence proved

Q.4. *ABCD* is a trapezium in which $AB \parallel CD$, *BD* is a diagonal and *E* is the mid point of *AD*. A line is drawn through *E* parallel to *AB* intersecting *BC* at *F* **(see figure)**.



Show that *F* is the mid point of *BC*.

Sol. Given : $ABCD$ is a trapezium in which $AB \parallel CD$, $EF \parallel AB$ and E is the mid point of AD.

To prove : F is the mid point of BC.

Proof : Let line EF intersecting BD at G.

In ΔDAB, we have

$EG \parallel AB$ ($\because$ $EF \parallel AB$)

and E is the mid point of AD.

$\therefore$ G is the mid point of BD.

Now, $AB \parallel CD$ and $AB \parallel EF,$ (Given)

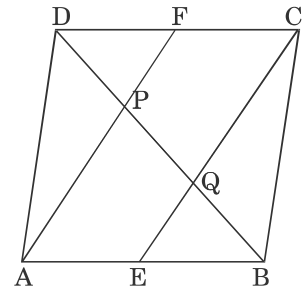
$\Rightarrow$ $EF \parallel CD \Rightarrow GF \parallel CD$

and G is the mid point of BD. (As proved above)

$\therefore$ F is the mid point of BC.

Hence proved

Q.5. In a parallelogram *ABCD*, *E* and *F* are the mid points of sides *AB* and *CD* respectively (see figure).



Show that the line segments *AF* and *EC* trisect the diagonal *BD*.

Sol. Since, $ABCD$ is a parallelogram.

$\therefore$ $AB = CD,$

(Opposite sides of parallelogram)

$\Rightarrow$ $\frac{1}{2}AB = \frac{1}{2}CD \Rightarrow AE = FC,$

[$\because$ E and F are mid points of AB and CD respectively

$\therefore$ $AE = \frac{1}{2}AB$ and $FC = \frac{1}{2}CD$]

and $AB \parallel CD \Rightarrow AE \parallel FC$

$\therefore$ $AECF$ is a parallelogram.

$\Rightarrow$ $AF \parallel EC$ ...(i)

In ΔABP, we have

E is the mid point of AB.

and $EQ \parallel AP$ [$\because$ $AF \parallel EC$]

$\therefore$ Q is the mid point of BP.

$\Rightarrow$ $BQ = PQ$ ...(ii)

In ΔCQD, we have

F is the mid point of CD.

and $PF \parallel CQ$ [$\because$ $AF \parallel EC$]

$\therefore$ P is the mid point of DQ.

$\Rightarrow$ $DP = PQ$ ...(iii)

From (ii) and (iii), we get

$DP = PQ = BQ$

Hence, line segments AF and EC trisect the diagonal BD. **Hence proved**

Q.6. *ABC* is a triangle right angled at *C*. A line through the mid point *M* of hypotenuse *AB* and parallel to *BC* intersects *AC* at *D*. Show that:

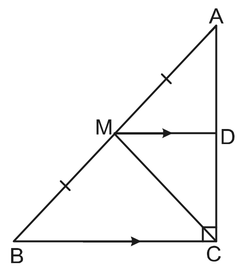
(i) *D* is the mid point of *AC*,

(ii) $MD \perp AC$ (iii) $CM = MA = \frac{1}{2} AB.$

Sol. Given : A ΔABC in which $\angle C = 90°$, M is the mid point of AB and $MD \parallel BC$.

To prove : (i) D is the mid point of AC.

(ii) $MD \perp AC$ (iii) $CM = MA = \frac{1}{2} AB$.



Proof : (i) In ΔABC, M is the mid point of AB.

and $MD \parallel BC$, (Given)

$\therefore$ D is the mid point of AC.

(ii) Since, $MD \parallel BC$ and $\angle ACB = 90°$

$\therefore$ $\angle ADM = 90°$,

(Corresponding angles) ...(i)

(iii) But, $\angle ADM + \angle CDM = 180°$,

(Linear pair)

$\Rightarrow$ $90° + \angle CDM = 180°$, [Using (i)]

$\Rightarrow$ $\angle CDM = 180° - 90° = 90°$ ...(ii)

Now, ΔADM and ΔCDM, we have

$AD = CD$, (As proved above, D is the mid point of AC)

$\angle ADM = \angle CDM$,

[From (i) and (ii), Each = 90°]

and $MD = MD$, (Common)

$\therefore$ $\Delta ADM \cong \Delta CDM$,

(By SAS congruence rule)

$\Rightarrow$ $AM = CM$, (CPCT)

But, $AM = BM$,

($\because$ M is the mid point of AB)

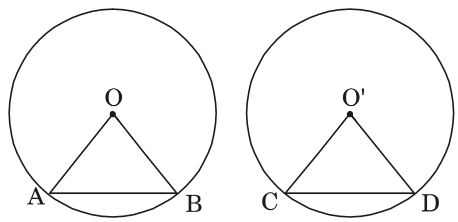
$\therefore$ $AM = CM = \frac{1}{2} AB$. **Hence proved**

9 Circles

Textbook Exercises 9.1

Q.1. Recall that two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centres.

Sol. Given : AB and CD are two equal chords of two congruent circles $C(O, r)$ and $C(O', r)$ respectively.



To prove :

$$\angle AOB = \angle CO'D.$$

Proof : In ΔAOB and $\Delta CO'D$, we have

$OA = O'C$ [Equal radii of congruent circles]

$OB = O'D$ [Equal radii of congruent circles]

$AB = CD$ (given)

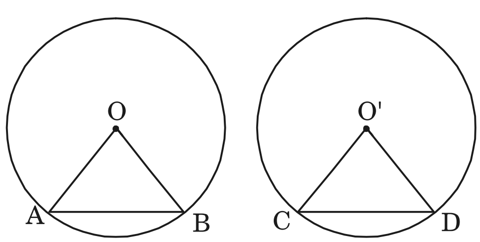
$\therefore \quad \Delta AOB \cong \Delta CO'D$ (By SSS congruence rule)

$\Rightarrow \quad \angle AOB = \angle CO'D$ (CPCT)

Hence proved

Q.2. Prove that if chords of congruent circles subtend equal angles at their centres, then the chords are equal.

Sol. Given : AB and CD are two chords of congruent circles $C(O, r)$ and $C(O', r)$ such that $\angle AOB = \angle CO'D$.



To prove : $AB = CD$.

Proof : In ΔAOB and $\Delta CO'D$, we have

$AO = O'C$ [Equal radii of congruent circles]

$\angle AOB = \angle CO'D$ (given)

$OB = O'D$ [Equal radii of congruent circles]

$\therefore \quad \Delta AOB \cong \Delta CO'D$ [By SAS congruence rule]

$\Rightarrow \quad AB = CD$ (CPCT)

Hence proved

Textbook Exercises 9.2

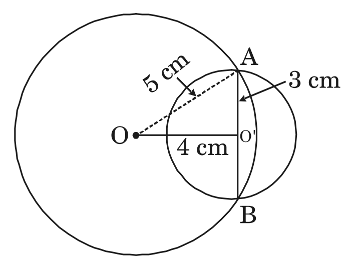
Q.1. Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.

Sol. Let AB be the common chord of the two circles having their centres at O and O', then

$OO' = 4$ cm, $OA = 5$ cm and $AO' = 3$ cm

we have $\quad 5^2 = 4^2 + 3^2$

$\Rightarrow \quad 25 = 16 + 9 \Rightarrow 25 = 25$



Therefore, OA is the hypotenuse and $AO'O$ is the right triangle right angled at O'.

Now, $\quad OO' \perp AB$

$$AO' = O'B = \frac{1}{2}AB$$

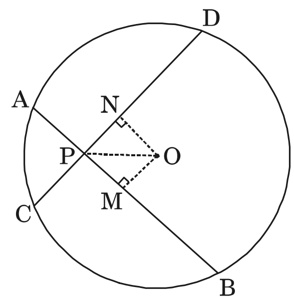
$$\Rightarrow \quad 3 = \frac{1}{2}AB \Rightarrow AB = 6 \text{ cm}$$

Hence, $\quad AB = 6$ cm.

Q.2. If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.

Sol. Given : AB and CD are two equal chords of a circle $C(O, r)$. AB and CD intersect at P.

To prove : (i) $BP = DP$, (ii) $AP = CP$.



Construction : Draw $OM \perp AB$ and $ON \perp CD$ and join OP.

Proof : Since, $OM \perp AB$

$\therefore \quad AM = MB = \frac{1}{2}AB \quad \ldots$(i)

and $\quad ON \perp CD$

$\therefore \quad CN = ND = \frac{1}{2}CD \quad \ldots$(ii)

$AB = CD \quad$ (given) $\ldots$(iii)

From (i), (ii) and (iii), we get

$AM = CN$

and $\quad MB = ND \quad \ldots$(iv)

In right ΔOMP and ΔONP, we have

$\angle OMP = \angle ONP \quad$ [Each is 90°]

Hyp. OP = Hyp. $OP \quad$ (Common)

$OM = ON$

[$\because$ Equal chords of a circle are equidistant from the centre]

$\therefore \quad \Delta OMP \cong \Delta ONP$

[By RHS congruence rule]

$\Rightarrow \quad PM = PN \quad$ (CPCT) $\ldots$(v)

Adding (iv) and (v), we get

$MB + PM = ND + PN$

$\Rightarrow \quad BP = PD$

Subtracting (v) from (iv), we get

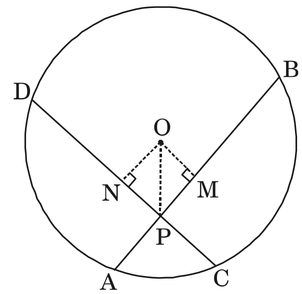
$AM - PM = CN - PN \Rightarrow AP = CP$

Hence, $\quad BP = PD$ and $AP = CP$.

Proved

Q.3. If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.

Sol. Given : AB and CD are two equal chords of a circle with centre O.



To prove : $\angle OPM = \angle OPN$.

Construction : Draw $OM \perp AB$ and $ON \perp CD$ and join OP.

Proof : In right ΔOMP and ΔONP, we have

$\angle OMP = \angle ONP \quad$ [Each is 90°]

Hyp. OP = Hyp. $OP \quad$ (Common)

$OM = ON$

[$\because$ Equal chords are equidistant from the centre]

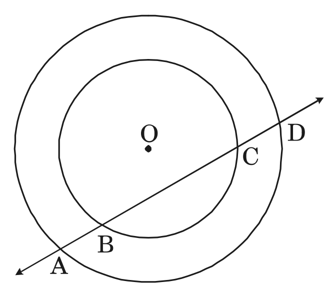
$\therefore \quad \Delta OMP \cong \Delta ONP$

[By RHS congruence rule]

$\Rightarrow \quad \angle OPM = \angle OPN \quad$ (CPCT)

Hence proved

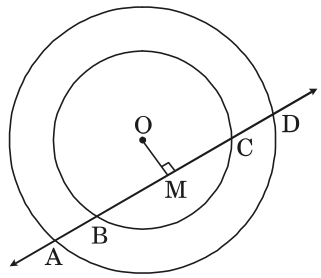
Q.4. If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that $AB = CD$ (see Fig.)



Sol. Draw $OM \perp AD$.

Now, AD is a chord of larger circle and $OM \perp AD$.

$\therefore \quad AM = MD \quad \ldots$(i)



Again BC is a chord of smaller circle and $OM \perp BC$.

$\therefore \quad BM = MC \quad \ldots$(ii)

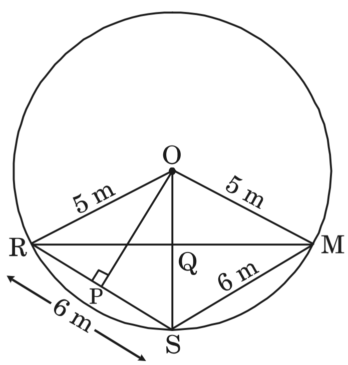
Subtracting (ii) from (i), we get

$$AM - BM = MD - MC$$

$\Rightarrow \quad AB = CD$. **Hence proved**

Q.5. Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5 m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6 m each, what is the distance between Reshma and Mandip?

Sol. Let the position of Reshma, Salma and Mandip be at R, S and M respectively on the circumference of the circular park.



$\therefore \quad RS = SM = 6$ m

Draw $\quad OP \perp RS$

$\therefore \quad RP = PS = \frac{1}{2} RS$

$\Rightarrow \quad RP = \frac{1}{2} \times 6 = 3$ m

In right ΔOPR, we have

$$OR^2 = RP^2 + OP^2$$

$\Rightarrow \quad 5^2 = 3^2 + OP^2$

$\Rightarrow \quad OP^2 = 5^2 - 3^2$

$\Rightarrow \quad OP^2 = (5 + 3)(5 - 3)$

$\Rightarrow \quad OP^2 = 8 \times 2 = 16$

$\Rightarrow \quad OP = \sqrt{16} = \sqrt{4 \times 4} = 4$ m

Area of $\Delta ORS = \frac{1}{2} \times RS \times OP$

$\Rightarrow$ Area of $\Delta ORS = \frac{1}{2} \times 6 \times 4 = 12$ m^2...(i)

In ΔROS and ΔMOS, we have

$RO = MO$ [Radii of a same circle]

$RS = SM$ [Each is equal to 6 m]

$OS = OS$ (Common)

$\therefore \quad \Delta ROS \cong \Delta MOS$ (By SSS congruence rule)

$\Rightarrow \quad \angle ROS = \angle MOS$ (CPCT)

$\Rightarrow \quad \angle ROQ = \angle MOQ \quad \ldots$(ii)

Now, in ΔOQR and ΔOQM, we have

$OR = OM$ [Radii of a same circle]

$\angle ROQ = \angle MOQ$ [As proved above in (ii)]

$OQ = OQ$ (Common)

$\therefore \quad \Delta OQR \cong \Delta OQM$ (By SAS congruence rule)

$\Rightarrow \quad \angle OQR = \angle OQM$ (CPCT) ...(iii)

But $\angle OQR + \angle OQM = 180°$ (By linear pair axiom)

$\Rightarrow \quad \angle OQR + \angle OQR = 180°$ [Using (iii)]

$\Rightarrow \quad 2\angle OQR = 180°$

$\Rightarrow \quad \angle OQR = \frac{180°}{2} = 90°$

$\therefore \quad RQ \perp OS$

Again, area of $\Delta ORS = \frac{1}{2} \times OS \times RQ$

$\Rightarrow \quad 12 = \frac{1}{2} \times 5 \times RQ$

$\Rightarrow \quad \frac{12 \times 2}{5} = RQ \Rightarrow RQ = 4{\cdot}8$ m

$OQ \perp RM$ and RM is a chord.

$\therefore \quad RQ = \frac{1}{2}RM \Rightarrow 4{\cdot}8 = \frac{1}{2}RM$

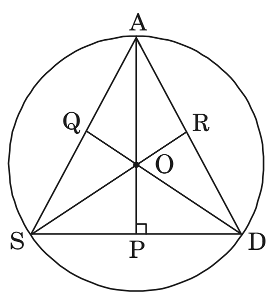
$RM = 4{\cdot}8 \times 2 = 9{\cdot}6$ m

Hence, the distance between Reshma and Mandip = 9·6 m.

Q.6. A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

Sol. Let position of Ankur, Syed and David be A, S and D respectively at the boundary of circular park.

Since, three boys are sitting at equal distance at boundary of circular park.



$\therefore \quad \widehat{AS} = \widehat{SD} = \widehat{AD}$

$\Rightarrow$ chord AS= chord SD = chord AD

$\Rightarrow ASD$ is an equilateral triangle.

We know that centroid of an equilateral ΔASD concides with its circumcentre *i.e.,* centroid of ΔASD is O and radius of circle is 20 m

$\Rightarrow \quad SO = DO = AO = 20$ m

Since, centroid divides the median in the ratio 2 : 1.

Therefore, $AO : OP = 2 : 1$

$$\Rightarrow \quad \frac{AO}{OP} = \frac{2}{1} \Rightarrow \frac{20}{OP} = \frac{2}{1}$$

$$\Rightarrow \quad OP = \frac{20}{2} = 10 \text{ m}$$

In right ΔOPS, we have

$$OS^2 = OP^2 + SP^2$$

(By Pythagoras theorem)

$\Rightarrow \quad 20^2 = 10^2 + SP^2$

$\Rightarrow \quad SP^2 = 20^2 - 10^2$

$\Rightarrow \quad SP^2 = (20 + 10)(20 - 10)$

$\Rightarrow \quad SP^2 = 30 \times 10$

$\Rightarrow \quad SP = \sqrt{30 \times 10} = \sqrt{3 \times 10 \times 10}$

$\Rightarrow \quad SP = 10\sqrt{3}$ m

We know that in an equilateral triangle, median is perpendicular to corresponding side of the triangle. Therefore $AP \perp SD$.

Now $OP \perp SD$ and SD is a chord.

$$\therefore \quad SP = \frac{1}{2}SD$$

$$\Rightarrow \quad 10\sqrt{3} = \frac{1}{2}SD \Rightarrow SD = 20\sqrt{3} \text{ m}$$

Hence, length of each string of phone is $20\sqrt{3}$ m.

Alter : Let $\quad AS = SD = AD = x$ m

In right ΔAPS, we have

$$AS^2 = SP^2 + AP^2$$

(By Pythagoras theorem)

$$\Rightarrow \quad x^2 = \left(\frac{x}{2}\right)^2 + AP^2, \quad \left[\because SP = \frac{1}{2}SD = \frac{x}{2}\right]$$

$$\Rightarrow \quad AP^2 = x^2 - \frac{x^2}{4}$$

$$\Rightarrow \quad AP^2 = \frac{3x^2}{4} \Rightarrow AP = \frac{x\sqrt{3}}{2}$$

$$\therefore \quad OP = AP - AO$$

$$\Rightarrow \quad OP = \frac{\sqrt{3}x}{2} - 20 = \frac{\sqrt{3}x - 40}{2}$$

In right ΔOPS, we have

$$OS^2 = SP^2 + OP^2$$

$$\Rightarrow \quad 20^2 = \left(\frac{x}{2}\right)^2 + \left(\frac{\sqrt{3}x - 40}{2}\right)^2$$

$$\Rightarrow \quad 400 = \frac{x^2}{4} + \frac{3x^2 + 1600 - 80\sqrt{3}x}{4}$$

$$\Rightarrow \quad 1600 = x^2 + 3x^2 + 1600 - 80\sqrt{3}x$$

$$\Rightarrow \quad 0 = 4x^2 - 80\sqrt{3}x$$

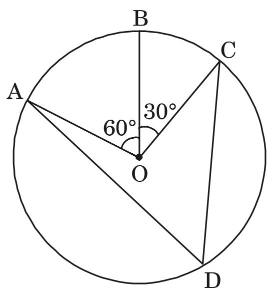
$$\Rightarrow \quad 80\sqrt{3}x = 4x^2 \Rightarrow \frac{80\sqrt{3}}{4} = \frac{x^2}{x}$$

$$\Rightarrow \quad 20\sqrt{3} = x \Rightarrow x = 20\sqrt{3} \text{ m}$$

Hence, length of each string of phone is $20\sqrt{3}$ m.

Textbook Exercises 9.3

Q.1. In Fig., A, B and C are three points on a circle with centre O such that $\angle BOC = 30°$ and $\angle AOB = 60°$. If D is a point on the circle other than the arc ABC, find $\angle ADC$.



Sol. $\angle AOC = \angle AOB + \angle BOC$

$\Rightarrow \quad \angle AOC = 60° + 30°$

$\Rightarrow \quad \angle AOC = 90°$

$\angle AOC$ is an angle subtended by an arc AC on centre and $\angle ADC$ on the remaining part of the circle.

$\therefore \quad \angle AOC = 2 \times \angle ADC$

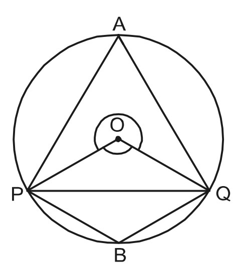
$\Rightarrow \quad 90° = 2 \times \angle ADC$

$\Rightarrow \quad \angle ADC = \dfrac{90°}{2} = 45°$

Hence, $\quad \angle ADC = 45°$.

Q.2. A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.

Sol. Let AB be a chord which is equal to radius. Join OA and OB, where O is the centre of the circle.



$\because \quad AO = OB = AB$ (given)

$\therefore$ AOB is an equilateral triangle.

$\Rightarrow \quad \angle AOB = 60°$

Reflex $\angle AOB = 360° - 60° = 300°$

$\angle AOB = 2 \times \angle ACB$

$\Rightarrow \quad 60° = 2 \times \angle ACB$

$\Rightarrow \quad \angle ACB = \dfrac{60°}{2} = 30°$

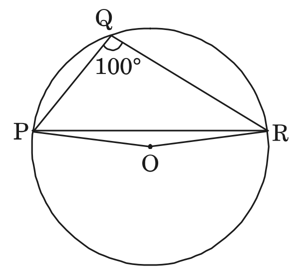
and Reflex $(\angle AOB) = 2 \times \angle APB$

$\Rightarrow \quad 300° = 2 \times \angle APB$

$\Rightarrow \quad \angle APB = \dfrac{300°}{2} = 150°$

Hence, angle subtended by the chord at a point on the minor arc is 150° and major arc is 30°.

Q.3. In Fig., $\angle PQR = 100°$, where P, Q and R are points on a circle with centre O. Find $\angle OPR$.



Sol. Since, reflex $\angle POR$ is an angle subtended by an arc PR at the centre and $\angle PQR$ at the remaining part of the circle.

$\therefore \quad$ Reflex $\angle POR = 2 \times \angle PQR$

$\Rightarrow \quad$ Reflex $\angle POR = 2 \times 100° = 200°$

$\angle POR = 360° -$ Reflex $\angle POR$

$\Rightarrow \quad \angle POR = 360° - 200° = 160°$

In ΔPOR, $\quad OP = OR$

[Radii of the same circle]

$\Rightarrow \quad \angle OPR = \angle ORP$

[Angles opposite to equal sides are equal]

Let $\quad \angle OPR = \angle ORP = x°$

In ΔPOR, we have

$\angle OPR + \angle ORP + \angle POR = 180°$

[$\because$ Sum of angles of a triangle is 180°]

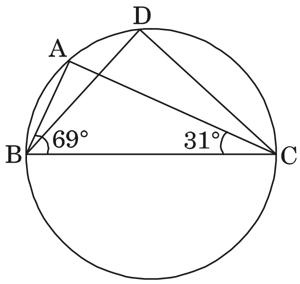
$\Rightarrow \quad x° + x° + 160° = 180°$

$\Rightarrow \quad 2x° = 180° - 160° = 20°$

$\Rightarrow \quad x° = \dfrac{20°}{2} = 10°$

Hence, $\quad \angle OPR = 10°$.

Q.4. In Fig., $\angle ABC = 69°$, $\angle ACB = 31°$, find $\angle BDC$.



Sol. In ΔBAC, we have

$$\angle ABC + \angle BCA + \angle BAC = 180°$$

[$\because$ Sum of angles of a triangle = 180°]

$\Rightarrow$ $69° + 31° + \angle BAC = 180°$

$\Rightarrow$ $100° + \angle BAC = 180°$

$\Rightarrow$ $\angle BAC = 180° - 100°$

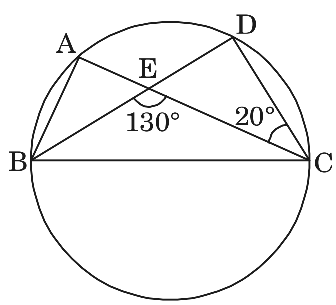
$\Rightarrow$ $\angle BAC = 80°$

$\angle BDC = \angle BAC$

[$\because$ Angles in the same segment are equal]

Hence $\angle BDC = 80°$.

Q.5. In Fig., *A*, *B*, *C* and *D* are four points on a circle. *AC* and *BD* intersect at a point *E* such that $\angle BEC$ = 130° and $\angle ECD$ = 20°. Find $\angle BAC$.



Sol. In ΔECD, we have

$$\angle BEC = \angle EDC + \angle ECD$$

[$\because$ Exterior angle is equal to sum of its opposite two interior angles]

$\Rightarrow$ $130° = \angle EDC + 20°$

$\Rightarrow$ $\angle EDC = 130° - 20°$

$\Rightarrow$ $\angle EDC = 110°$

$\Rightarrow$ $\angle BDC = 110°$

$\angle BAC = \angle BDC$

[$\because$ Angles in the same segment are equal]

Hence, $\angle BAC = 110°$.

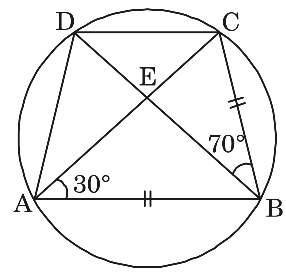
Q.6. *ABCD* is a cyclic quadrilateral whose diagonals intersect at a point *E*. If $\angle DBC$ = 70°, $\angle BAC$ is 30°, find $\angle BCD$. Further, if *AB* = *BC*, find $\angle ECD$.

Sol. In ΔABC,

$AB = BC$ (given)

$\Rightarrow$ $\angle BAC = \angle BCA$

[$\because$ Angles opp. to equal sides are equal]



$\Rightarrow$ $30° = \angle BCA \Rightarrow \angle BCA = 30°$

In ΔABC, we have

$$\angle BAC + \angle BCA + \angle ABC = 180°$$

$\Rightarrow$ $30° + 30° + 70° + \angle ABE = 180°$

$\Rightarrow$ $130° + \angle ABE = 180°$

$\Rightarrow$ $\angle ABE = 180° - 130° = 50°$

$\Rightarrow$ $\angle ABD = 50°$

$\angle ACD = \angle ABD$

[$\because$ Angles in the same segment of a circle are equal]

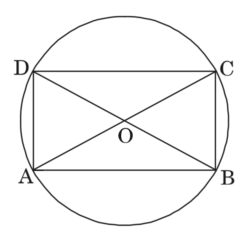
$\Rightarrow$ $\angle ACD = 50° \Rightarrow \angle ECD = 50°$

$\angle BCD = \angle BCA + \angle ACD$

$\Rightarrow$ $\angle BCD = 30° + 50° \Rightarrow \angle BCD = 80°$

Q.7. If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.

Sol. Given : Diagonals *AC* and *BD* of cyclic quadrilateral are the diameters of the circle through the vertices of the quadrilateral *ABCD*.



To prove : Quadrilateral *ABCD* is a rectangle.

Proof : Since *BD* is a diameter.

$\angle C = \angle A = 90°$

[Angle in a semicircle is 90°]

Similarly, AC is a diameter.

$\angle B = \angle D = 90°$

[Angle in a semicircle is 90°]

Thus, in a quadrilateral $ABCD$

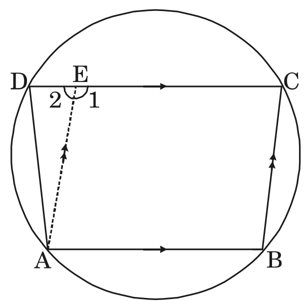
$\angle A = \angle B = \angle C = \angle D = 90°$

Hence, quadrilateral $ABCD$ is a rectangle

Proved

Q.8. If the non-parallel sides of a trapezium are equal, prove that it is cyclic.

Sol. A trapezium $ABCD$ in which $AB \parallel CD$ and $AD = BC$.



To prove : Trapezium $ABCD$ is cyclic.

Construction : Draw $AE \parallel BC$.

Proof : Since, $AB \parallel CD$ (given)

and $AE \parallel BC$ (By construction)

$\therefore$ $AE = BC$ ...(i)

(Opposite sides of a parallelogram)

$AD = BC$ (given) ...(ii)

From (i) and (ii), we get

$AE = AD$

$\Rightarrow$ $\angle 2 = \angle D$ ...(iii)

[Angles opposite to equal sides are equal]

$\angle B = \angle 1$ ...(iv)

[Opposite angles of a parallelogram]

But, $\angle 1 + \angle 2 = 180°$

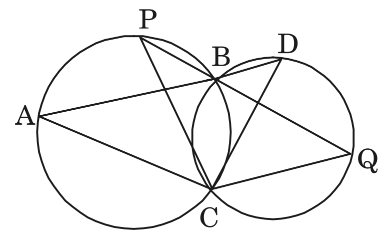
[By linear pair axiom]

$\Rightarrow$ $\angle B + \angle D = 180°$

[Using (iii) and (iv)]

$\Rightarrow$ $ABCD$ is a cyclic.

Q.9. Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively (see Fig.).



Prove that $\angle ACP = \angle QCD$.

Sol. $\angle ABP = \angle ACP$ ...(i)

[Angles in a same segment are equal]

$\angle QBD = \angle QCD$ ...(ii)

[Angles in a same segment are equal]

$\angle ABP = \angle QBD$ ...(iii)

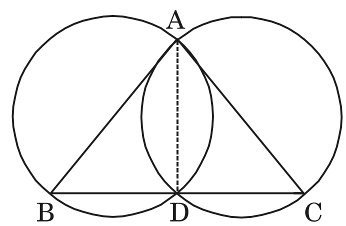
(Vertically opposite angles)

From (i), (ii) and (iii), we get

$\angle ACP = \angle QCD$. **Hence proved**

Q.10. If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.

Sol. Given : Two circles are drawn with sides AB and AC of ΔABC as diameters. The circles intersect each other at D.



To prove : Point D lies on BC *i.e.*, BDC is a straight line.

Construction : Join AD.

Proof : Since AB and AC are diameters of two circles.

$\therefore$ $\angle ADB = 90°$ ...(i)

[Angle in semicircle is 90°]

and $\angle ADC = 90°$ ...(ii)

[Angle in a semicircle is 90°]

Adding (i) and (ii), we get

$\angle ADB + \angle ADC = 90° + 90°$

$\Rightarrow$ $\angle ADB + \angle ADC = 180°$

$\Rightarrow$ BDC is a straight line.

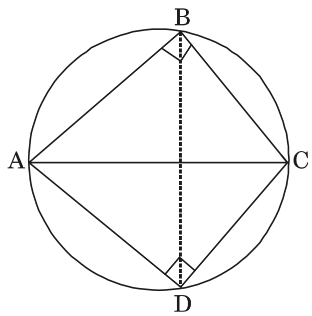
Hence, D lies on BC. **Proved.**

Q.11. ABC and ADC are two right triangles with common hypotenuse AC. Prove that $\angle CAD = \angle CBD$.

Sol. $\angle ABC = \angle ADC = 90°$ (given)

In a quadrilateral $ABCD$,

$$\angle ABC + \angle ADC = 90° + 90° = 180°$$



Therefore, $ABCD$ is a cyclic quadrilateral.

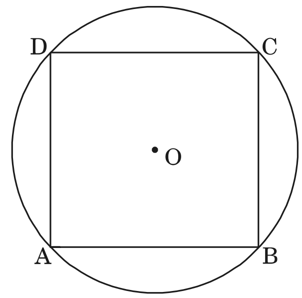
$$\angle CAD = \angle CBD$$

[Angles in a same segment are equal]

Hence proved

Q.12. Prove that a cyclic parallelogram is a rectangle.

Sol. Given : A cyclic parallelogram $ABCD$.



To prove : $ABCD$ is a rectangle.

Proof : Since $ABCD$ is a parallelogram. (given)

$\therefore$ $\angle A = \angle C$

(Opposite angles of a ||gm) ...(i)

But, $\angle A + \angle C = 180°$ [By theorem]

$\Rightarrow$ $\angle A + \angle A = 180°$ [Using (i)]

$\Rightarrow$ $2\angle A = 180°$

$\Rightarrow$ $\angle A = \dfrac{180°}{2} = 90°$

So, $\angle A = \angle C = 90°$

Thus, $ABCD$ is a parallelogram in which

$\angle A = \angle C = 90°$.

Hence, $ABCD$ is a rectangle. **Proved**

10 Heron's Formula

Textbook Exercises 10.1

Q.1. A traffic signal board, indicating 'SCHOOL AHEAD' is an equilateral triangle with side 'a'. Find the area of the signal board, using Heron's formula. If its perimeter is 180 cm, what will be the area of the signal board ?

Sol. Let ABC be the signal board is shape of an equilateral triangle with side a.

$$s = \frac{a+a+a}{2} \Rightarrow s = \frac{3a}{2}$$

By Heron's formula, we have

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{\frac{3a}{2}\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)}$$

$$= \sqrt{\frac{3a}{2}\times\frac{a}{2}\times\frac{a}{2}\times\frac{a}{2}} = \frac{a}{2}\times\frac{a}{2}\times\sqrt{3}$$

$$= \frac{a^2}{4}\times\sqrt{3} = \frac{\sqrt{3}}{4}a^2 \text{ square units}$$

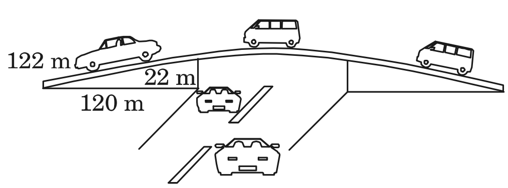
Perimeter = 180 cm

$$\Rightarrow \quad 3a = 180 \Rightarrow a = \frac{180}{3} = 60$$

$$\therefore \text{Area of triangle} = \frac{\sqrt{3}}{4} \times (60)^2$$

$$= 900\sqrt{3} \text{ cm}^2$$

Q.2. The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 122 m, 22 m and 120 m (see Fig.). The advertisements yield an earning of Rs. 5000 per m^2 per year. A company hired one of its walls for 3 months. How much rent did it pay ?



Sol. Sides of a triangular wall of a flyover are 122 m, 22 m and 120 m.

Here, $a = 122$ m, $b = 22$ m

and $c = 120$ m

$$s = \frac{122+22+120}{2}$$

$$\Rightarrow \quad s = \frac{264}{2} = 132 \text{ m}$$

By Heron's formula, we have

Area of side walls of flyover

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{132(132-122)(132-22)(132-120)}$$

$$= \sqrt{132\times10\times110\times12}$$

$$= \sqrt{11\times12\times10\times10\times11\times12}$$

$$= 10 \times 11 \times 12 = 1320 \text{ m}^2$$

$\because$ Rent paid by a company in 12 months for

1 m^2 = ₹ 5000

$\therefore$ Rent paid by a company in 1 month for

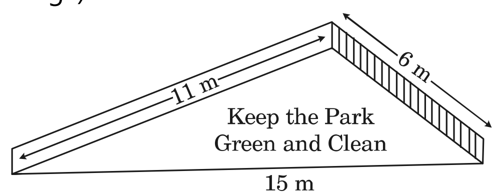
1 m^2 = ₹ $\frac{5000}{12}$

$\therefore$ Rent paid by company in 3 months for

1320 m^2 = ₹ $\frac{5000\times3\times1320}{12}$

= ₹ 1250 × 1320 = ₹ 1650000

Q.3. There is a slide in a park. One of its side walls has been painted in some colour with a message **'keep the park green and clean''** (see Fig.).



If the sides of the wall are 15 m, 11 m and 6 m, find the area painted in colour.

Sol. Sides of the triangular park are 15 m, 11 m and 6 m.

Here, $a = 15$ m, $b = 11$ m

and $c = 6$ m

$$s = \frac{15+11+6}{2} \Rightarrow s = \frac{32}{2}$$

$\Rightarrow$ $s = 16$ m

By Heron's formula, we have

Area of triangular park

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{16(16-15)(16-11)(16-6)}$$
$$= \sqrt{16\times1\times5\times10} = \sqrt{4\times4\times1\times5\times5\times2}$$
$$= 4\times5\sqrt{2} = 20\sqrt{2} \text{ m}^2$$

Q.4. Find the area of a triangle two sides of which are 18 cm and 10 cm and the perimeter is 42 cm.

Sol. We have,

Two sides of a triangle are a = 18 cm and b = 10 cm

andperimeter of triangle $(2s)$ = 42 cm

We know that,

Perimeter $(2s) = a + b + c$

$\Rightarrow$ $42 = 18 + 10 + c$

$\Rightarrow$ $42 = 28 + c$

$\Rightarrow$ $c = 42 - 28 = 14$ cm

Now $2s = 42$

$\Rightarrow$ $s = \frac{42}{2} = 21$ cm

By Heron's formula, we have

Area of a triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{21(21-18)(21-10)(21-14)}$$
$$= \sqrt{21\times3\times11\times7} = \sqrt{3\times7\times3\times11\times7}$$
$$= 3\times7\sqrt{11} = 21\sqrt{11} \text{ cm}^2$$

Q.5. Sides of a triangle are in the ratio of 12 : 17 : 25 and its perimeter is 540 cm. Find its area.

Sol. Ratio of sides of a triangle = 12 : 17 : 25 and perimeter is 540 cm.

Let sides of a triangle be $12x$ cm, $17x$ cm and $25x$ cm.

Perimeter = Sum of the sides of a triangle

$\Rightarrow$ $540 = 12x + 17x + 25x$

$\Rightarrow$ $540 = 54x \Rightarrow x = \frac{540}{54} = 10$

So,sides of the triangle= 12 × 10, 17 × 10, 25 × 10 = 120 cm, 170 cm, 250 cm

Here, a = 120 cm, b = 170 cm

and c = 250 cm

$\therefore$ $s = \frac{120+170+250}{2}$

$\Rightarrow$ $s = \frac{540}{2} \Rightarrow s = 270$ cm

By Heron's formula, we have

Area of a triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{270(270-120)(270-170)(270-250)}$$
$$= \sqrt{270\times150\times100\times20}$$
$$= \sqrt{3\times3\times30\times30\times5\times5\times20\times20}$$
$$= 3 \times 5 \times 20 \times 30 = 9000 \text{ cm}^2$$

Q.6. An isosceles triangle has perimeter 30 cm and each of the equal sides is 12 cm. Find the area of the triangle.

Sol. Two equal sides of an isosceles triangle are 12 cm and 12 cm

and perimeter = 30 cm

Perimeter of a triangle = Sum of sides of a triangle

$\Rightarrow$ 30 = 12 + 12 + IIIrd side

$\Rightarrow$ 30 = 24 + IIIrd side

$\Rightarrow$ IIIrd side = 30 – 24 = 6 cm

Here, a = 12 cm, b = 12 cm and c = 6 cm

$$s = \frac{12+12+6}{2} \Rightarrow s = \frac{30}{2} = 15 \text{ cm}$$

By Heron's formula, we have

Area of a triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{15(15-12)(15-12)(15-6)}$$
$$= \sqrt{15\times3\times3\times9} = \sqrt{15\times3\times3\times3\times3}$$
$$= 3\times3\sqrt{15} = 9\sqrt{15} \text{ cm}^2$$

11 Surface Areas and Volumes

Textbook Exercises 11.1

Q.1. Diameter of the base of a cone is 10·5 cm and its slant height is 10 cm. Find its curved surface area.

Sol. We have,

Slant height of the cone $(l) = 10$ cm

and diameter of the base of a cone = 10·5 cm

$\therefore$ Radius of the base of a cone

$$(r) = \frac{10\cdot5}{2} = 5\cdot25 \text{ cm}$$

$\therefore$ Curved surface area of the cone $= \pi rl$

$$= \frac{22}{7} \times 5\cdot25 \times 10 = 165 \text{ cm}^2$$

Q.2. Find the total surface area of a cone, if its slant height is 21 m and diameter of its base is 24 m.

Sol. We have,

Slant height of a cone $(l) = 21$ m

and diameter of base of a cone = 24 m

$\therefore$ Radius of base of a cone (r)

$$= \frac{24}{2} = 12 \text{ m}$$

$\therefore$ Total surface area of a cone (r)

$$= \pi r(l+r) = \frac{22}{7} \times 12(21 + 12)$$

$$= \frac{22}{7} \times 12 \times 33$$

$$= 1244\cdot57 \text{ m}^2 \text{ (approx.)}.$$

Q.3. Curved surface area of a cone is 308 cm^2 and its slant height is 14 cm. Find :

(i) radius of the base and

(ii) total surface area of the cone.

Sol. We have,

Slant height of a cone $(l) = 14$ cm

Let radius of the base of a cone be r cm.

(i) curved surface area of a cone = 308 cm^2

$$\Rightarrow \quad \pi rl = 308$$

$$\Rightarrow \quad \frac{22}{7} \times r \times 14 = 308$$

$$\Rightarrow \quad 44r = 308 \Rightarrow r = \frac{308}{44} = 7 \text{ cm.}$$

(ii) Total surface area of the cone $= \pi r(l + r)$

$$= \frac{22}{7} \times 7(14 + 7)$$

$$= 22 \times 21 = 462 \text{ cm}^2$$

Q.4. A conical tent is 10 m high and the radius of its base is 24 m. Find :

(i) slant height of the tent.

(ii) cost of the canvas required to make the tent, if the cost of 1 m^2 canvas is ₹ 70.

Sol. We have,

Height of the conical tent $(h) = 10$ m

and radius of the base of conical tent

$(r) = 24$ m

(i) Let slant height of the cone be l m.

$$\therefore \quad l = \sqrt{h^2+r^2} = \sqrt{10^2+24^2}$$

$$\Rightarrow \quad = \sqrt{100+576} = \sqrt{676} = 26 \text{ m}$$

(ii) Required area of canvas to make the conical tent

= Curved surface area of the cone

$$= \pi rl = \frac{22}{7} \times 24 \times 26 = \frac{13728}{7} \text{ m}^2$$

$\because$ 1 m^2 canvas cost = ₹ 70

$\therefore$ $\frac{13728}{7}$ m^2 canvas cost

$$= \frac{70 \times 13728}{7} = ₹\ 137280$$

(ii) Cost of canvas = ₹ 137280.

Q.5. What length of tarpaulin 3 m wide will be required to make conical tent of height 8 m and base radius 6 m ? Assume that the extra length of material that will be required for stitching margins and wastage in cutting is approximately 20 cm (Use $\pi = 3\cdot14$)

Sol. We have,

Height of the conical tent $(h) = 8$ m

Radius of base of conical tent $(r) = 6$ m

$$\therefore \quad l = \sqrt{h^2+r^2} = \sqrt{8^2+6^2}$$

$$\Rightarrow \quad = \sqrt{64+36} = \sqrt{100} = 10 \text{ m}$$

Area of tarpaulin required to make the conical tent

= Curved surface area of the cone

$= \pi rl = 3{\cdot}14 \times 6 \times 10$

$= 188{\cdot}4 \text{ m}^2$

$\because$ Width of tarpaulin = 3 m

$\therefore$ Length of tarpaulin

$$= \frac{\text{Area of tarpaulin}}{\text{Width of tarpaulin}} = \frac{188{\cdot}4}{3} = 62{\cdot}8 \text{ m}$$

Wastage in cutting = 20 cm = 0·20 m

Total length of tarpaulin = 62·8 + 0·20

= 63 m

Q.6. The slant height and base diameter of a conical tomb are 25 m and 14 m respectively. Find the cost of white-washing its curved surface at the rate of Rs. 210 per 100 m^2.

Sol. We have,

Slant height of a conical tomb (l) = 25 m

and diameter of a conical tomb = 14 m

$\therefore$ Radius of a conical tomb (r) $= \frac{14}{2} = 7$ m

$\therefore$ Curved surface area of the conical tomb

$$= \pi rl = \frac{22}{7} \times 7 \times 25 = 550 \text{ m}^2$$

$\because$ Cost of white washing of 100 m^2 of tomb

= ₹ 210

$\therefore$ Cost of white washing of 550 m^2 of tomb

$$= ₹\ \frac{210 \times 550}{100} = ₹\ 1155$$

Q.7. A joker's cap is in the form of a right circular cone of base radius 7 cm and height 24 cm. Find the area of the sheet required to make 10 such caps.

Sol. We have,

Radius of conical cap (r) = 7 cm

and height of the conical cap (h) = 24 cm

$\therefore \quad l = \sqrt{h^2 + r^2}$

$\Rightarrow \quad l = \sqrt{24^2 + 7^2} \Rightarrow l = \sqrt{576 + 49}$

$\Rightarrow \quad l = \sqrt{625} \quad \Rightarrow l = \pm 25$ cm

$\Rightarrow \quad l = 25$ cm

$\therefore$ Curved surface area of a cap $= \pi rl$

$$= \frac{22}{7} \times 7 \times 25 = 550 \text{ cm}^2$$

Curved surface area of 10 such caps

$= 550 \times 10 = 5500 \text{ cm}^2$

Q.8. A bus stop is barricaded from the remaining part of the road, by using 50 hollow cones made of recycled cardboard. Each cone has a base diameter of 40 cm and height 1 m. If the outer side of each of the cone is to be painted and the cost of painting is Rs. 12 per m^2, what will be the cost of painting all these cones ? (Use π = 3.14 and take $\sqrt{1.04}$ = 1.02)

Sol. We have,

Height of a cone (h) = 1 m = 100 cm

and diameter of a cone (d) = 40 cm

$\therefore$ Radius of a cone (r) $= \frac{40}{2} = 20$ cm

$l = \sqrt{h^2 + r^2}$

$\Rightarrow \quad l = \sqrt{100^2 + 20^2}$

$\Rightarrow \quad l = \sqrt{10000 + 400}$

$\Rightarrow \quad l = \sqrt{10400} \quad \Rightarrow l = 101{\cdot}98$

$\Rightarrow \quad l = 102$ (approx.)

Outer surface area of 1 cone $= \pi rl$

$= 3{\cdot}14 \times 20 \times 102 = 6405{\cdot}6 \text{ cm}^2$

Outer surface area of 50 cones

$= 6405{\cdot}6 \times 50 = 320280 \text{ cm}^2$

$= \frac{320280}{10000} \text{ m}^2$

$= 32{\cdot}028 \text{ m}^2$

Cost of painting of 1 m^2 = ₹ 12

Cost of painting of 32·028 m^2

= ₹ 12 × 32·028

= ₹ 384·336 = ₹ 384·34

Textbook Exercises 11.2

Q.1. Find the surface area of a sphere of radius:

(i) 10·5 cm (ii) 5·6 cm (iii) 14 cm.

Sol. (i) We have,

Radius of sphere (r) = 10·5 cm

$\therefore$ Surface area of a sphere $= 4\pi r^2$

$$= 4 \times \frac{22}{7} \times (10\cdot5)^2$$

$$= \frac{9702}{7} = 1386 \text{ cm}^2$$

(ii) We have,

Radius of sphere $(r) = 5\cdot6$ cm

$\therefore$ Surface area of a sphere $= 4\pi r^2$

$$= 4 \times \frac{22}{7} \times (5\cdot6)^2$$

$$= \frac{2759\cdot68}{7} = 394\cdot24 \text{ cm}^2$$

(iii) We have,

Radius of sphere $(r) = 14$ cm

$\therefore$ Surface area of a sphere $= 4\pi r^2$

$$= 4 \times \frac{22}{7} \times (14)^2$$

$$= \frac{17248}{7} = 2464 \text{ cm}^2$$

Q.2. Find the surface area of a sphere of diameter :

(i) 14 cm (ii) 21 cm (iii) 3·5 m.

Sol. (i) We have,

Diameter of a sphere $(d) = 14$ cm

$\therefore$ Radius of a sphere $(r) = \frac{14}{2} = 7$ cm

$\therefore$ Surface area of a sphere $= 4\pi r^2$

$$= 4 \times \frac{22}{7} \times (7)^2 = \frac{4312}{7} = 616 \text{ cm}^2$$

(ii) We have,

Diameter of a sphere $(d) = 21$ cm

$\therefore$ Radius of a sphere $(r) = \frac{21}{2} = 10\cdot5$ cm

$\therefore$ Surface area of a sphere $= 4\pi r^2$

$$= 4 \times \frac{22}{7} \times (10\cdot5)^2 = \frac{9702}{7} = 1386 \text{ cm}^2$$

(iii) We have,

Diameter of a sphere $(d) = 3\cdot5$ m

$\therefore$ Radius of a sphere $(r) = \frac{3\cdot5}{2} = 1\cdot75$ m

$\therefore$ Surface area of a sphere $= 4\pi r^2$

$$= 4 \times \frac{22}{7} \times (1\cdot75)^2 = \frac{269\cdot5}{7} = 38\cdot5 \text{ m}^2$$

Q.3. Find the total surface area of a hemisphere of radius 10 cm. [Use $\pi = 3\cdot14$]

Sol. We have,

Radius of a hemisphere $(r) = 10$ cm

$\therefore$ Total surface area of a hemisphere

$$= 3\pi r^2 = 3 \times 3\cdot14 \times (10)^2$$

$$= 942 \text{ cm}^2$$

Q.4. The radius of a spherical balloon increases from 7 cm to 14 cm as air is being pumped into it. Find the ratio of surface areas of the balloon in the two cases.

Sol. We have,

Radius of a spherical balloon $(r_1) = 7$ cm

$\therefore$ Surface area of a spherical balloon (S_1)

$$= 4\pi(r_1)^2 = 4 \times \frac{22}{7} \times (7)^2 = 616 \text{ cm}^2$$

and increased radius of a spherical balloon

$(r_2) = 14$ cm

$\therefore$ Surface area of a spherical balloon in this case $(S_2) = 4\pi(r_2)^2$

$$= 4 \times \frac{22}{7} \times (14)^2 = 2464 \text{ cm}^2$$

Now, $\frac{S_1}{S_2} = \frac{616}{2464} \Rightarrow \frac{S_1}{S_2} = \frac{1}{4}$

$\Rightarrow$ $S_1 : S_2 = 1 : 4$

Q.5. A hemispherical bowl made of brass has inner diameter 10·5 cm. Find the cost of tin-plating it on the inside at the rate of Rs. 16 per 100 cm^2.

Sol. We have,

Diameter of hemispherical bowl

$(d) = 10\cdot5$ cm

$\therefore$ Radius of the hemispherical bowl

$$(r) = \frac{10\cdot5}{2} = 5\cdot25 \text{ cm}$$

$\therefore$ Inner curved surface area of the bowl

$$= 2\pi r^2 = 2 \times \frac{22}{7} \times (5\cdot25)^2$$

$$= \frac{44 \times 27\cdot5625}{7} = 173\cdot25 \text{ cm}^2$$

Rate of tin-plating = Rs. 16 per 100 cm^2

$\therefore$ Cost of tin-plating inside of bowl

$$= \frac{173\cdot25 \times 16}{100} = ₹\ 27\cdot72$$

Q.6. Find the radius of a sphere whose surface area is 154 cm^2.

Sol. Let the radius of a sphere be r cm.

Surface area of a sphere = 154 cm^2, (given)

$$\Rightarrow \quad 4\pi r^2 = 154$$

$$\Rightarrow \quad 4 \times \frac{22}{7} \times r^2 = 154$$

$$\Rightarrow \quad \frac{88}{7} \times r^2 = 154$$

$$\Rightarrow \quad r^2 = \frac{154 \times 7}{88} \Rightarrow r^2 = \frac{7 \times 7}{4}$$

$$\Rightarrow \quad r = \sqrt{\frac{7 \times 7}{2 \times 2}} \Rightarrow r = \frac{7}{2} = 3{\cdot}5 \text{ cm}$$

Q.7. The diameter of the Moon is approximately one fourth of the diameter of the Earth. Find the ratio of their surface areas.

Sol. Let diameter of the Earth be $2x$.

$\therefore$ Radius of the Earth $(R) = x$

$\therefore$ Surface area of the Earth

$$(S_1) = 4\pi R^2 = 4\pi x^2$$

According to question,

Diameter of the Moon

$$= \frac{1}{4} \times \text{diameter of the Earth}$$

$$= \frac{1}{4} \times 2x = \frac{x}{2}$$

$$\therefore \text{ Radius of the Moon } (r) = \frac{\frac{x}{2}}{2} = \frac{x}{4}$$

$\therefore$ Surface area of the Moon $(S_2) = 4\pi r^2$

$$= 4\pi\left(\frac{x}{4}\right)^2 = 4\pi \times \frac{x^2}{16} = \frac{\pi x^2}{4}$$

$$\therefore \quad \frac{S_1}{S_2} = \frac{4\pi x^2}{\frac{\pi x^2}{4}} \Rightarrow \frac{S_1}{S_2} = \frac{16}{1}$$

$$\Rightarrow \quad \frac{S_2}{S_1} = \frac{1}{16} \Rightarrow S_2 : S_1 = 1 : 16$$

Q.8. A hemispherical bowl is made of steel, 0·25 cm thick. The inner radius of the bowl is 5 cm. Find the outer curved surface area of the bowl.

Sol. We have,

Inner radius of the bowl = 5 cm

and thickness of the bowl = 0·25 cm

$\therefore$ Outer radius of the bowl (r)

$$= 5 + 0{\cdot}25 = 5{\cdot}25 \text{ cm}$$

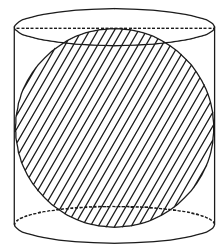
$\therefore$ Outer curved surface area of the bowl

$$= 2\pi r^2 = 2 \times \frac{22}{7} \times (5{\cdot}25)^2$$

$$= \frac{44 \times 27{\cdot}5625}{7}$$

$$= 44 \times 3{\cdot}9375 = 173{\cdot}25 \text{ cm}^2$$

Q.9. A right circular cylinder just encloses a sphere of radius r **(see Fig.)**.



Find :

(i) surface area of the sphere,

(ii) curved surface area of the cylinder,

(iii) ratio of the areas obtained in (i) and (ii).

Sol. We have,

(i) Radius of sphere = r

$\therefore$ Surface area of the sphere $(S_1) = 4\pi r^2$.

(ii) Radius of the cylinder = radius of the sphere = r

Height of the cylinder $(h) = 2 \times r = 2r$

$\therefore$ Curved surface area of the cylinder

$$(S_2) = 2\pi rh = 2\pi \times r \times 2r$$

$$= 4\pi r^2$$

(iii) $$\frac{\text{Surface area of the sphere } (S_1)}{\text{Curved surface area of the cylinder } (S_2)} = \frac{4\pi r^2}{4\pi r^2}$$

$$\Rightarrow \quad \frac{S_1}{S_2} = \frac{1}{1} \Rightarrow S_1 : S_2 = 1 : 1$$

Textbook Exercises 11.3

Q.1. Find the volume of the right circular cone with :

(i) radius 6 cm, height 7 cm

(ii) radius 3·5 cm, height 12 cm.

Sol. (i) We have,

Radius of cone (r) = 6 cm

Height of the cone (h) = 7 cm

$\therefore$ Volume of the cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\times\frac{22}{7}\times 6^2\times 7 = 264 \text{ cm}^3.$$

(ii) We have,

Radius of the cone $(r) = 3{\cdot}5$ cm

Height of the cone $(h) = 12$ cm

$\therefore$ Volume of the cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\times\frac{22}{7}\times(3{\cdot}5)^2\times 12$$

$$= 154 \text{ cm}^3$$

Q.2. Find the capacity in litres of a conical vessel with :

(i) radius 7 cm, slant height 25 cm

(ii) height 12 cm, slant height 13 cm.

Sol. (i) We have,

Radius of the conical vessel $(r) = 7$ cm

Slant height of the conical vessel $(l) = 25$ cm

Let the height of the conical vessel be h cm.

$\therefore \quad l^2 = h^2 + r^2$

$\Rightarrow \quad 25^2 = h^2 + 7^2$

$\Rightarrow \quad h^2 = 25^2 - 7^2$

$\Rightarrow \quad h^2 = (25 + 7)(25 - 7)$

$\Rightarrow \quad h^2 = 32 \times 18 \Rightarrow h^2 = 576$

$\Rightarrow \quad h = \sqrt{576} \Rightarrow h = 24$ cm

$\therefore$ Volume of the conical vessel $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\times\frac{22}{7}\times 7^2\times 24$$

$$= 1232 \text{ cm}^3$$

$$= \frac{1232}{1000} = 1{\cdot}232 \text{ litres.}$$

(ii) We have,

Slant height of the conical vessel $(l) = 13$ cm

Height of the conical vessel $(h) = 12$ cm

Let the radius of the conical vessel be r cm.

$\therefore \quad l^2 = h^2 + r^2$

$\Rightarrow \quad 13^2 = 12^2 + r^2$

$\Rightarrow \quad r^2 = 13^2 - 12^2$

$\Rightarrow \quad r^2 = (13 + 12)(13 - 12)$

$\Rightarrow \quad r^2 = 25 \quad \Rightarrow r = \sqrt{25}$

$\Rightarrow \quad r = 5$ cm

$\therefore$ Volume of the conical vessel $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\times\frac{22}{7}\times 5^2\times 12 = \frac{2200}{7} \text{ cm}^3$$

$$= \frac{2200}{7\times 1000} = \frac{11}{35} \text{ litres}$$

Q.3. The height of a cone is 15 cm. If its volume is 1570 cm^3, find the radius of the base. (Use $\pi = 3{\cdot}14$)

Sol. Let the radius of base of a cone be r cm.

We have, Height of the cone $(h) = 15$ cm

Volume of the cone $= 1570 \text{ cm}^3$

$\Rightarrow \quad \frac{1}{3}\pi r^2 h = 1570$

$\Rightarrow \quad \frac{1}{3}\times 3{\cdot}14\times r^2\times 15 = 1570$

$\Rightarrow \quad 15{\cdot}7\times r^2 = 1570 \quad \Rightarrow r^2 = \frac{1570}{15{\cdot}7}$

$\Rightarrow \quad r^2 = 100 \quad \Rightarrow r = \sqrt{100}$

$\Rightarrow \quad r = 10$ cm

Q.4. If the volume of a right circular cone of height 9 cm is $48\pi \text{ cm}^3$, find the diameter of its base.

Sol. Let the radius of the base of the cone be r cm.

We have, Height of the cone $(h) = 9$ cm

Volume of the cone $= 48\pi \text{ cm}^3$

$\Rightarrow \quad \frac{1}{3}\pi r^2 h = 48\pi$

$\Rightarrow \quad \frac{1}{3}\pi\times r^2\times 9 = 48\pi$

$\Rightarrow \quad 3\pi r^2 = 48\pi$

$\Rightarrow \quad r^2 = \frac{48\pi}{3\pi} \quad \Rightarrow r^2 = 16$

$\Rightarrow \quad r = \sqrt{16} \quad \Rightarrow r = 4$ cm

$\therefore$ Diameter of the base of the cone $= 2 \times 4 = 8$ cm

Q.5. A conical pit of top diameter 3·5 m is 12 m deep. What is its capacity in kilolitres ?

Sol. We have,

Depth of conical pit (h) = 12 m

Diameter of conical pit (d) = 3·5 m

$\therefore$ Radius of conical pit $(r) = \frac{3 \cdot 5}{2} = 1 \cdot 75$ m

$\therefore$ Capacity of the conical pit $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times (1 \cdot 75)^2 \times 12$

$= 38 \cdot 5$ m^3 = 38·5 kilolitres

($\because$ 1 m^3 = 1 kilolitres)

Q.6. The volume of a right circular cone is 9856 cm^3. If the diameter of the base is 28 cm, find :

(i) height of the cone

(ii) slant height of the cone

(iii) curved surface area of the cone.

Sol. We have,

The volume of the cone = 9856 cm^3

The diameter of the base of the cone

(d) = 28 cm

$\therefore$ Radius of the base of the cone

$(r) = \frac{28}{2}$ cm = 14 cm

(i) Let the height of the cone be h cm.

Volume of the cone = 9856 cm^3

$\Rightarrow \quad \frac{1}{3}\pi r^2 h = 9856$

$\Rightarrow \quad \frac{1}{3} \times \frac{22}{7} \times 14^2 \times h = 9856$

$\Rightarrow \quad \frac{616}{3} \times h = 9856$

$\Rightarrow \quad h = \frac{9856 \times 3}{616} \Rightarrow h = 48$ cm.

(ii) Let the slant height of the cone be l cm, then

$l^2 = h^2 + r^2$

$\Rightarrow \quad l^2 = 48^2 + 14^2$

$\Rightarrow \quad l^2 = 2304 + 196$

$\Rightarrow \quad l^2 = 2500 \Rightarrow l = \sqrt{2500}$

$\Rightarrow \quad l = 50$ cm.

(iii) Curved surface of the cone $= \pi r l$

$= \frac{22}{7} \times 14 \times 50 = 2200$ cm^2.

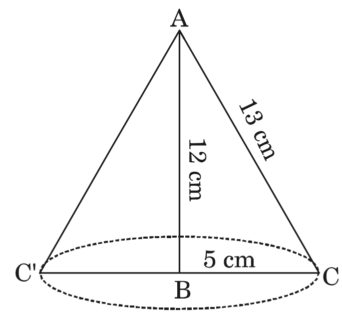
Q.7. A right triangle ABC with sides 5 cm, 12 cm and 13 cm is revolved about the side 12 cm. Find the volume of the solid so obtained.

Sol. We have,

Sides of the triangle ABC are AB = 12 cm, BC = 5 cm and AC = 13 cm

Since ΔABC is revolved about the side AB (= 12 cm)

$\therefore$ We get a cone $(AC'C)$ as shown in the figure.



Then, radius of the base of the cone

(r) = 5 cm

and height of the cone (h) = 12 cm

$\therefore \quad$ Volume of the cone (solid) $= \frac{1}{3}\pi r^2 h$

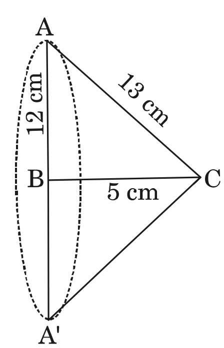
$= \frac{1}{3} \times \pi \times 5^2 \times 12$

$= 100\pi$ cm^3

Q.8. If the triangle ABC in the Question 7 above is revolved about the side 5 cm, then find the volume of the solid so obtained. Find also the ratio of the volumes of the two solids obtained in Questions 7 and 8.

Sol. Since ΔABC is revolved about BC(= 5 cm).

$\therefore$ We get a cone (CAA') as shown in the figure.



Then,radius of the base of the cone

$(r) = 12$ cm

and height of the cone $(h) = 5$ cm

Volume of the cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 12^2 \times 5 = 240\pi$ cm^3

Ratio of the volumes of the two solids obtained in Q. 7 and Q. 8

$= 100\pi : 240\pi = \frac{100\pi}{240\pi}$

$= \frac{5}{12} = 5 : 12$

Hence, volume of the cone (solid)

$= 240\pi$ cm^3,

and required ratio of their volumes

$= 5 : 12$.

Q.9. A heap of wheat is in the form of a cone whose diameter is 10·5 m and height is 3 m. Find its volume. The heap is to be covered by canvas to protect it from rain. Find the area of the canvas required.

Sol. We have,

Height of the conical heap $(h) = 3$ m

Diameter of the base of the conical heap

$(d) \quad = 10{\cdot}5$ m

$\therefore$ Radius of the base of the conical heap

$(d) = \frac{10{\cdot}5}{2} = 5{\cdot}25$ m

$\therefore$ Slant height of a conical heap

$(l) = \sqrt{h^2 + r^2} = \sqrt{3^2 + (5{\cdot}25)^2}$

$= \sqrt{9 + 27{\cdot}56}$

$= \sqrt{36{\cdot}56} = 6{\cdot}05$ m

$\therefore$ Volume of the conical heap $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times (5{\cdot}25)^2 \times 3$

$= \frac{606{\cdot}375}{7} = 86{\cdot}625$ m^3

Required area of canvas = Curved surface area of the conical heap $= \pi rl$

$= \frac{22}{7} \times 5{\cdot}25 \times 6{\cdot}05$

$= 16{\cdot}5 \times 6{\cdot}05$

$= 99{\cdot}825$ m^2

Textbook Exercises 11.4

Q.1. Find the volume of a sphere whose radius is :

(i) 7 cm (ii) 0·63 m.

Sol. (i) We have,

Radius of sphere $(r) = 7$ cm

$\therefore$ Volume of sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times (7)^3 = \frac{88 \times 49}{3}$

$= \frac{4312}{3} = 1437\frac{1}{3}$ cm^3

(ii) We have,

Radius of the sphere $(r) = 0{\cdot}63$ m

$\therefore$ Volume of the sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times (0{\cdot}63)^3$

$= 88 \times 0{\cdot}03 \times 0{\cdot}3969$

$= 1{\cdot}0478$ m^3 $= 1{\cdot}05$ m^3

Q.2. Find the amount of water displaced by a solid spherical ball of diameter :

(i) 28 cm (ii) 0·21 m.

Sol. (i) We have,

Diameter of spherical ball = 28 cm

$\therefore$ Radius of the spherical ball $(r) = \frac{28}{2}$

$= 14$ cm

Amount of water displaced by spherical ball = Volume of spherical ball

$= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (14)^3$

$= \frac{4}{3} \times \frac{22}{7} \times 14 \times 14 \times 14$

$= \frac{88 \times 2 \times 196}{3} = \frac{34496}{3} = 11498\frac{2}{3}$ cm^3

(ii) We have,

Diameter of spherical ball = 0·21 m

$\therefore$ Radius of spherical ball $(r) = \frac{0{\cdot}21}{2}$

$= 0{\cdot}105$ m

Amount of water displaced by spherical ball

= Volume of spherical ball

$= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (0 \cdot 105)^3$

$= \frac{4}{3} \times \frac{22}{7} \times 0 \cdot 105 \times 0 \cdot 105 \times 0 \cdot 105$

$= 88 \times 0 \cdot 005 \times 0 \cdot 011025$

$= 0 \cdot 004851 \text{ m}^3$

Q.3. The diameter of a metallic ball is 4·2 cm. What is the mass of the ball, if the density of the metal is 8·9 g per cm^3 ?

Sol. We have,

Diameter of ball = 4·2 cm

$\therefore$ Radius of the ball $(r) = \frac{4 \cdot 2}{2} = 2 \cdot 1$ cm

Since, ball is in the shape of sphere, therefore we need to calculate the volume of sphere for its mass.

$\therefore$ Volume of spherical ball $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times (2 \cdot 1)^3 = \frac{88}{21} \times 2 \cdot 1 \times 2 \cdot 1 \times 2 \cdot 1$

$= 8 \cdot 8 \times 4 \cdot 41 = 38 \cdot 808 \text{ cm}^3$

Mass of the ball = volume × density

$= 38 \cdot 808 \times 8 \cdot 9$

$= 345 \cdot 39$ grams (approx.)

Q.4. The diameter of the moon is approximately one-fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon ?

Sol. Let the diameter of the earth be $2x$.

$\therefore$ Radius of the earth $(R) = \frac{2x}{2} = x$

According to question,

Diameter of the moon

$= \frac{1}{4}$ of diameter of the earth

$= \frac{1}{4} \times 2x = \frac{x}{2}$

$\therefore$ Radius of the moon $(r) = \frac{x}{4}$

Volume of the earth $= \frac{4}{3}\pi R^3 = \frac{4}{3}\pi x^3$

Volume of the moon $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \left(\frac{x}{4}\right)^3$

$= \frac{4}{3}\pi \frac{x^3}{64}$

$$\therefore \quad \frac{\text{Volume of the moon}}{\text{Volume of the earth}} = \frac{\frac{4}{3}\pi \frac{x^3}{64}}{\frac{4}{3}\pi x^3} = \frac{1}{64}$$

$\Rightarrow$ Volume of the moon $= \frac{1}{64} \times$ volume of the earth

Q.5. How many litres of milk can a hemispherical bowl of diameter 10·5 cm hold.

Sol. We have,

Diameter of hemispherical bowl = 10·5 cm

$\therefore$ Radius of hemispherical bowl (r)

$= \frac{10 \cdot 5}{2} = 5 \cdot 25$ cm

$\therefore$ Volume of the hemispherical bowl

$= \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times (5 \cdot 25)^3$

$= \frac{2}{3} \times \frac{22}{7} \times 5 \cdot 25 \times 5 \cdot 25 \times 5 \cdot 25$

$= 44 \times 0 \cdot 25 \times 27 \cdot 5625$

$= 303 \cdot 1875 \text{ cm}^3 = 0 \cdot 3031875$ litres

$= 0 \cdot 303$ litres (approx.)

Q.6. A hemispherical tank is made up of an iron sheet 1 cm thick. If the inner radius is 1 m, then find the volume of the iron used to make the tank.

Sol. We have,

Inner radius of the hemispherical tank

$(r) = 1$ m

Thickness of the iron sheet = 1 cm

$= 0 \cdot 01$ m

$\therefore$ Outer radius of the hemispherical tank

$(R) = 1 + 0 \cdot 01 = 1 \cdot 01$ m

Volume of the iron used to make the tank

= External volume – Internal volume

$= \frac{2}{3}\pi R^3 - \frac{2}{3}\pi r^3 = \frac{2}{3}\pi[R^3 - r^3]$

$= \frac{2}{3} \times \frac{22}{7} [(1 \cdot 01)^3 - (1)^3]$

$= \frac{44}{21}(1 \cdot 030301 - 1) = \frac{44}{21} \times 0 \cdot 030301$

$= 0 \cdot 06348 \text{ m}^3$ (approx.)

Q.7. Find the volume of a sphere whose surface area is 154 cm^2.

Sol. Let the radius of sphere be r cm.

Surface area of the sphere = 154 cm^2 (given)

$$\Rightarrow \quad 4\pi r^2 = 154$$

$$\Rightarrow \quad 4 \times \frac{22}{7} \times r^2 = 154 \quad \Rightarrow r^2 = \frac{154 \times 7}{4 \times 22}$$

$$\Rightarrow \quad r^2 = \frac{7 \times 7}{4} \quad \Rightarrow r = \sqrt{\frac{7 \times 7}{2 \times 2}} = \frac{7}{2} \text{ cm}$$

Volume of the sphere $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2}$$

$$= \frac{11 \times 49}{3} = \frac{539}{3} = 179\frac{2}{3} \text{ cm}^3$$

Q.8. A dome of a building is in the form of a hemisphere. From inside, it was white-washed at the cost of ₹ 4989·60. If the cost of the white-washing is ₹ 20.00 per square metre, find the :

(i) inside surface area of the dome,

(ii) volume of the air inside the dome.

Sol. Let inner radius of dome be r m.

We have,

Rate of white washed = ₹ 20·00 per m^2

Total cost of white washed = ₹ 4989·60

(i) Inside curved surface area of the dome

$$= \frac{\text{Total cost}}{1 \text{ m}^2 \text{ cost}} = \frac{4989{\cdot}60}{2} = 2494{\cdot}60 \text{ m}^2$$

(ii) We have,

Inside C.S.A. of the dome = 2494·60

$$\Rightarrow \quad 2\pi r^2 = 2494{\cdot}60$$

$$\Rightarrow 2 \times \frac{22}{7} \times r^2 = 2494{\cdot}60$$

$$\Rightarrow \quad r^2 = \frac{2494{\cdot}60 \times 7}{22 \times 2} \Rightarrow r^2 = 396{\cdot}90$$

$$\Rightarrow \quad r = \sqrt{396{\cdot}90} \quad \Rightarrow r = 19{\cdot}92 \text{ m.}$$

Volume of air inside the dome $= \frac{2}{3}\pi r^3$

$$= \frac{2}{3} \times \frac{22}{7} \times (19{\cdot}92)^3$$

$$= \frac{2 \times 22}{21} \times 19{\cdot}92 \times 19{\cdot}92 \times 19{\cdot}92$$

= 16561·56 m^3 (approx.)

(ii) Volume of air inside the dome

= 16561·56 m^3 (approx.)

Q.9. Twenty seven solid iron spheres, each of radius r and surface area S are melted to form a sphere with surface area S'. Find the :

(i) radius r' of the new sphere,

(ii) radio of S and S'.

Sol. We have,

Twenty seven spheres, each of radius r and surface area S are melted to form a sphere with surface area S'.

(i) Then $\quad S = 4\pi r^2 \quad$ ...(i)

Volume of one sphere $= \frac{4}{3}\pi r^3$

Volume of 27 spheres $= 27 \times \frac{4}{3}\pi r^3$

Volume of new sphere

= Volume of 27 spheres

$$\Rightarrow \quad \frac{4}{3}\pi r'^3 = 27 \times \frac{4}{3}\pi r^3$$

$$\Rightarrow \quad r'^3 = \frac{27 \times \frac{4}{3}\pi r^3}{\frac{4}{3}\pi}$$

$$\Rightarrow \quad r'^3 = 27r^3 = (3r)^3 \quad \Rightarrow r' = 3r$$

(ii) Surface area of new sphere $(S') = 4\pi r'^2$

$= 4\pi \times (3r)^2$

$= 4\pi \times 9r^2$

$= 36\pi r^2$

Now, $\quad \frac{S}{S'} = \frac{4\pi r^2}{36\pi r^2} \Rightarrow \frac{S}{S'} = \frac{1}{9}$

$\Rightarrow \quad S : S' = 1 : 9$

Q.10. A capsule of medicine is in the shape of a sphere of diameter 3·5 mm. How much medicine (in mm^3) is needed to fill this capsule?

Sol. We have,

Diameter of spherical capsule = 3·5 mm

$\therefore$ Radius of spherical capsule $(r) = \frac{3{\cdot}5}{2}$

= 1·75 mm

Volume of the capsule $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times (1{\cdot}75)^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 1{\cdot}75 \times 1{\cdot}75 \times 1{\cdot}75$$

$$= \frac{88 \times 0{\cdot}25 \times 3{\cdot}0625}{3} = \frac{67{\cdot}375}{3}$$

= 22·46 mm^3 (approx.)

Statistics

Textbook Exercises 12.1

Q.1. A survey conducted by an organisation for the cause of illness and death among the women between the ages 15 – 44 (in years) worldwide, found the following figures (in %) :

S. No.	Causes	Female fatality rate (%)
1.	Reproductive health conditions	31·8
2.	Neuropsychiatric conditions	25·4
3.	Injuries	12·4
4.	Cardiovascular conditions	4·3
5.	Respiratory conditions	4·1
6.	Other causes	22·0

(i) Represent the information given above graphically.

(ii) Which condition is the major cause of women's ill health and death world-wide?

(iii) Try to find out, with the help of your teacher, any two factors which play a major role in the cause in (ii) above being the major cause.

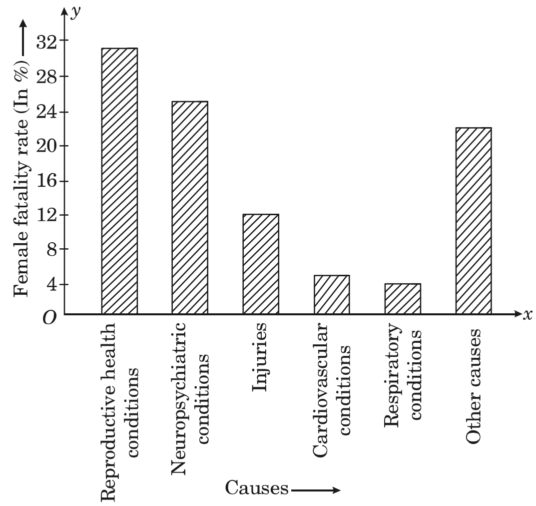
Sol. (i) We draw the bar graph of this data in the following steps : (a) We draw a horizontal and vertical line.

(b) We represent the causes of death on horizontal axis, choosing any scale. We take equal widths for all bars and maintain equal gaps in between. Let one head be represented by one unit.

(c) We represent the female fatality rate on the vertical axis. Let 1 unit = 4%.

(d) To represent first head *i.e.*, reproductive health conditions, we draw a rectangular bar with width 1 unit and height 7·95 units.

(e) Similarly, other heads are represented leaving a gap of 1 unit in between two consecutive bars.



(ii) Reproductive health conditions is the major cause of women's ill health and death world wide.

(iii) Two factors which play a major role is :

(a) There are unqualified doctors in a maternity hospitals, where the pregnant women did not get better treatment.

(b) Unwanted female child.

Q.2. The following data on the number of girls (to the nearest ten) per thousand boys in different sections of Indian society is given below:

Section	Number of girls per thousand boys
Scheduled Caste (SC)	940
Scheduled Tribe (ST)	970
Non SC/ST	920
Backward districts	950
Non-backward districts	920
Rural	930
Urban	910

(i) Represent the information above by a bar graph.

(ii) In the classroom discuss what conclusions can be arrived at from the graph.

Sol. (i) We draw the bar graph of this data in the following steps :

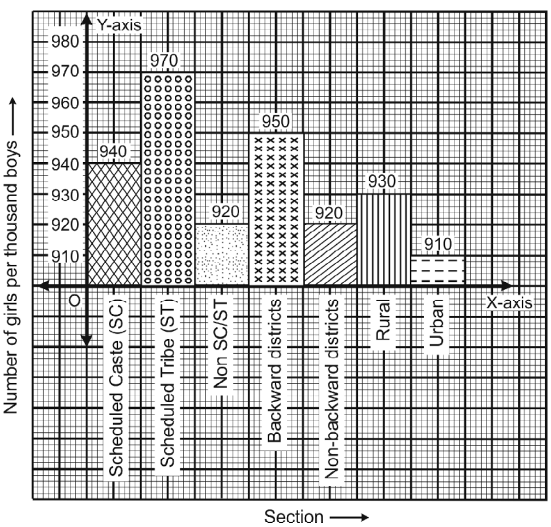
(a) We draw a horizontal and vertical line.

(b) We represent section of society on horizontal axis, choosing any scale. We take equal widths for all bars and maintain equal gaps in between. Let one head be represented by one unit.

(c) We represent the number of girls (to nearest ten) per thousand boys on the vertical axis. Let 1 unit = 10 girls.

(d) To represent the first head *i.e.*, scheduled caste (s.c.), we draw a rectangular bar with width 1 unit and height 4 units.

(e) Similarly, other heads are represented leaving a gap of 1 unit in between two consecutive bars.



(ii) From the graph, we observe that the number of girls to the nearest ten per thousand boys are maximum in scheduled tribe but minimum in urban.

Q.3. Given below are the seats won by different political parties in the polling outcome of a state assembly elections :

Political Party	A	B	C	D	E	F
Seats Won	75	55	37	29	10	37

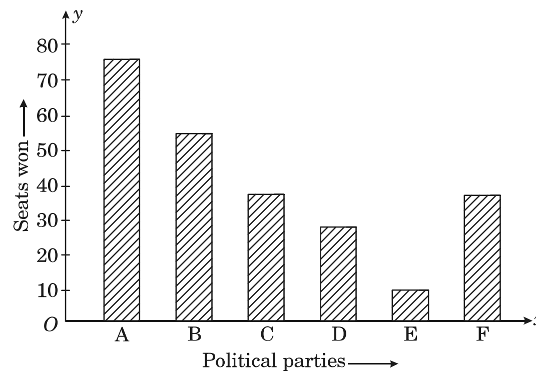
(i) Draw a bar graph to represent the polling results.

(ii) Which political party won the maximum number of seats ?

Sol. (i) We draw the bar graph of the given data in the following steps :

(a) We draw a horizontal and vertical line.

(b) We represent political party on the horizontal axis, choosing any scale. We take equal widths for all bars and maintain equal gaps in between. Let one political party represented by 1 unit.



(c) We represent the number of seats won on the vertical axis. Let 1 unit = 10 seats.

(d) To represent the political party A, we draw a rectangular bar with width 1 unit and height 7·5 units.

(e) Similarly, other political parties we represented leaving a gap of 1 unit in between two consecutive bars.

(ii) Party A won the maximum number of seats. **Ans.**

Q.4. The length of 40 leaves of a plant are measured correct to one millimetre, and the obtained data is represented in the following table:

Length (in mm)	Number of leaves
118 – 126	3
127 – 135	5
136 – 144	9
145 – 153	12
154 – 162	5
163 – 171	4
172 – 180	2

(i) Draw a histogram to represent the given data.

(ii) Is there any other suitable graphical representation for the same data ?

(iii) Is it correct to conclude that the maximum number of leaves are 153 mm long ? Why?

Sol. (i) The given frequency distribution is in inclusive form. So, first we convert it into exclusive form.

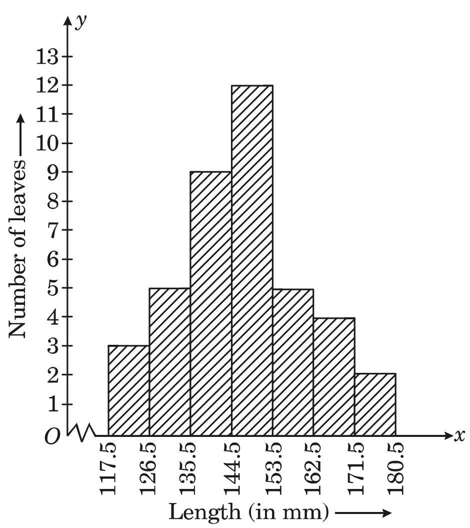
The difference between the lower limit of a class and upper limit of preceding class (h) = 1. To convert the given frequency distribution into exclusive frequency distribution, we subtract $\frac{h}{2} = \frac{1}{2} = 0{\cdot}5$ from each of lower limit and add 0·5 to each upper limit. The exclusive frequency distribution table obtained as follows :

Length (in mm)	Number of leaves
117·5 – 126·5	3
126·5 – 135·5	5
135·5 – 144·5	9
144·5 – 153·5	12
153·5 – 162·5	5
162·5 – 171·5	4
171·5 – 180·5	2

We represent the length (in mm) along x-axis on a suitable scale and the corresponding frequencies along y-axis on a suitable scale.

We construct rectangles with class intervals as bases and the corresponding frequencies as heights.

Thus, we obtain a histogram as shown below:



(ii) Frequency polygon is the another method of representing this data graphically.

(iii) No, it is not correct to conclude that maximum number of leaves are 153 mm long because number of leaves having length 153 mm or less than 153 mm = 3 + 5 + 9 + 12 = 29 and number of leaves having length more than 153 mm = 5 + 4 + 2 = 11.

But there might be no leave of length 153 mm.

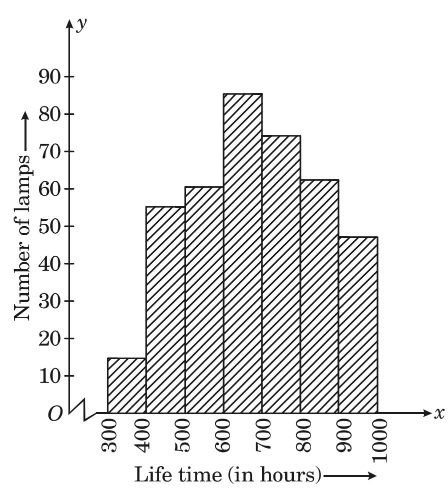
Q.5. The following table gives the life times of 400 neon lamps :

Life time (in hours)	Number of lamps
300 – 400	14
400 – 500	56
500 – 600	60
600 – 700	86
700 – 800	74
800 – 900	62
900 – 1000	48

(i) Represent the given information with the help of a histogram.

(ii) How many lamps have a life time of more than 700 hours ?

Sol. (i) The given frequency distribution is in the exclusive form. So we represent the life time (in hours) along x-axis on a suitable scale and the corresponding frequencies along y-axis on a suitable scale. We construct rectangles with class intervals as bases and corresponding frequencies as heights. Thus, we obtain a histogram as shown below :



(ii) Number of lamps having life time more than 700 hrs = 74 + 62 + 48 = 184.

Q.6. The following table gives the distribution of students of two sections according to the marks obtained by them :

Section A		Section B	
Marks	Frequency	Marks	Frequency
0 — 10	3	0 — 10	5
10 — 20	9	10 — 20	19
20 — 30	17	20 — 30	15
30 — 40	12	30 — 40	10
40 — 50	9	40 — 50	1

Represent the marks of the students of both the sections on the same graph by two frequency polygons. From the two polygons compare the performance of the two sections.

Sol. To represent the frequency polygon, we need the class marks of given classes.
For class interval (0 – 10), the upper limit is 10 and lower limit is 0.

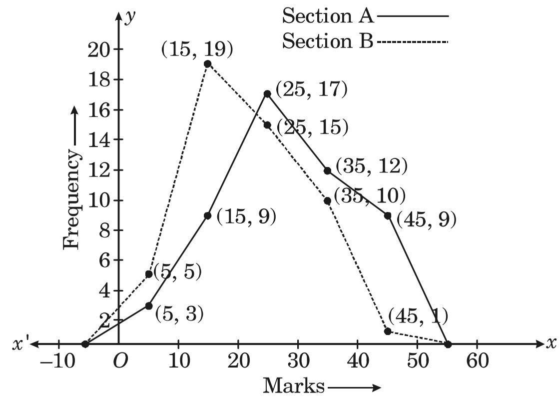
So, the class mark = $\frac{10+0}{2} = 5$

Continuing in the same manner, we find the class marks of the other classes as well. So, we obtained the following table :

Marks	Class Marks	Frequency	
		Section A	Section B
0 — 10	5	3	5
10 — 20	15	9	19
20 — 30	25	17	15
30 — 40	35	12	10
40 — 50	45	9	1

We represent class marks along the x-axis on a suitable scale and the corresponding frequency along the y-axis on a suitable scale. To obtain the frequency polygon of section A, we plot the points (5, 3), (15, 9), (25, 17), (35, 12), (45, 9) and join these points by line segments.

To obtain the frequency polygon of section B, we plot the points (5, 5), (15, 19), (25, 15), (35, 10) and (45, 1) on the same scale and join these points by dotted line segments.
The two frequency polygons are shown given figure.



Q.7. The runs scored by two teams *A* and *B* on the first 60 balls in a cricket match are given below :

Number of balls	Team ***A***	Team ***B***
1 – 6	2	5
7 – 12	1	6
13 – 18	8	2
19 – 24	9	10
25 – 30	4	5
31 – 36	5	6
37 – 42	6	3
43 – 48	10	4
49 – 54	6	8
55 – 60	2	10

Represent the data of both the teams on the same graph by frequency polygons.

Sol. Here the class intervals are discontinuous. So, we convert it into continuous class intervals. For this, we subtract $\frac{h}{2} = \frac{1}{2}$ = 0·5 from each of lower limit and add 0·5 to each upper limit. So, continuous class intervals obtained as follows:

Number of balls	Frequency	
	Team A	Team B
0·5 — 6·5	2	5
6·5 — 12·5	1	6
12·5 — 18·5	8	2
18·5 — 24·5	9	10
24·5 — 30·5	4	5
30·5 — 36·5	5	6
36·5 — 42·5	6	3
42·5 — 48·5	10	4
48·5 — 54·5	6	8
54·5 — 60·5	2	10

To represent the frequency polygon, we need the class marks of the given classes. For class interval (0·5 – 6·5), the upper limit is 6·5 and lower limit is 0·5.

So, the class mark $= \dfrac{6·5 + 0·5}{2} = 3·5$

Continuing in the same manner, we find the class marks of the other classes as well. So, we obtained the following table :

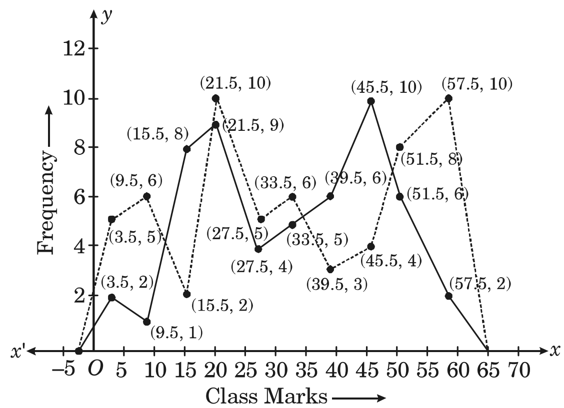
Number of balls	Class Marks	Frequency	
		Team A	Team B
0·5 — 6·5	3·5	2	5
6·5 — 12·5	9·5	1	6
12·5 — 18·5	15·5	8	2
18·5 — 24·5	21·5	9	10
24·5 — 30·5	27·5	4	5
30·5 — 36·5	33·5	5	6
36·5 — 42·5	39·5	6	3
42·5 — 48·5	45·5	10	4
48·5 — 54·5	51·5	6	8
54·5 — 60·5	57·5	2	10

We represent the class marks along x-axis on a suitable scale and the corresponding frequencies along y-axis on a suitable scale.

To obtain the frequency polygon of team A, we plot the points (3·5, 2), (9·5, 1), (15·5, 8), (21·5 – 9), (27·5, 4), (33·5, 5), (39·5, 6), (45·5, 10), (51·5, 6) and (57·5, 2) and join these points by line segments.

To obtain the frequency polygon of team B, we plot the points (3·5, 5), (9·5, 6), (15·5, 2), (21·5, 10), (27·5, 5), (33·5, 6), (39·5, 3), (45·5, 4), (51·5, 8) and (57·5, 10) on the same scale and join these points by dotted line segments.

The two frequency polygons are shown given figure.



Q.8. A random survey of the number of children of various age groups playing in a park was found as follows :

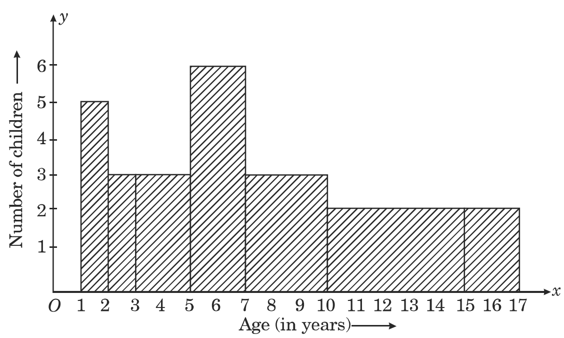
Age (in years)	Number of children
1 – 2	5
2 – 3	3
3 – 5	6
5 – 7	12
7 – 10	9
10 – 15	10
15 – 17	4

Draw a histogram to represent the data above.

Sol. Here the class intervals are not same size. Here, minimum class size is 1, then lengths of the rectangles are modified to be proportionate to the class size 1. Lengths of rectangles are computed as follows in the given table :

Age (in years)	Frequency	Width of the class	Length of the rectangle
1 – 2	5	1	$\frac{1}{1}\times 5 = 5$
2 – 3	3	1	$\frac{1}{1}\times 3 = 3$
3 – 5	6	2	$\frac{1}{2}\times 6 = 3$
5 – 7	12	2	$\frac{1}{2}\times 12 = 6$
7 – 10	9	3	$\frac{1}{3}\times 9 = 3$
10 – 15	10	5	$\frac{1}{5}\times 10 = 2$
15 – 17	4	2	$\frac{1}{2}\times 4 = 2$

We represent the ages of children along x-axis on a suitable scale and number of children along y-axis on a suitable scale. We construct rectangles with class intervals as bases and corresponding frequencies as heights.



Q.9. 100 surnames were randomly picked up from a local telephone directory and a frequency distribution of the number of letters in the English alphabet in the surnames was found as follows :

Number of letters	Number of surnames
1 – 4	6
4 – 6	30
6 – 8	44
8 – 12	16
12 – 20	4

(i) Draw a histogram to depict the given information.

(ii) Write the class interval in which the maximum number of surnames lie.

Sol. (i) Here the class intervals are not same size and minimum class size is 2, then lengths of the rectangles are modified to be proportionate to the class size 2.

Lengths of the rectangles are computed as follow in the given table :

Number of Letters	Frequency	Width of rectangle	Length of rectangle
1 – 4	6	3	$\frac{2}{3}\times 6 = 4$
4 – 6	30	2	$\frac{2}{2}\times 30 = 30$
6 – 8	44	2	$\frac{2}{2}\times 44 = 44$
8 – 12	16	4	$\frac{2}{4}\times 16 = 8$
12 – 20	4	8	$\frac{2}{8}\times 4 = 1$

We represent the number of letters along x-axis on a suitable scale and number of surnames along y-axis on a suitable scale. We construct rectangles with class intervals as bases and corresponding frequencies as heights.

APPENDIX - 1 : Proofs in Mathematics

Exercises A 1.1

Q.1. State whether the following statements are always true, false or ambiguous. Justify your answers.

(i) There are 13 months in a year.
(ii) Diwali falls on a Friday.
(iii) The temperature in Magadi is 26°C.
(iv) The earth has one moon.
(v) Dogs can fly.
(vi) February has only 28 days.

Sol. (i) Always false. Because there are 12 months in a year.
(ii) Ambiguous. ∵ In a given year, Diwali may or may not fall on Friday.
(iii) Ambiguous. ∵ At some time in the year, the temperature in Magadi, may be or may not be 26°C.
(iv) Always true. [from Scientific facts]
(v) Always false. ∵ Dogs cannot fly.
(vi) Ambiguous. ∵ In a leap year, February has 29 days and in non-leap years February has 28 days.

Q.2. State whether the following statements are true or false. Give reasons for your answers.

(i) The sum of the interior angles of a quadrilateral is 350°.
(ii) For any real number x, $x^2 = 0$.
(iii) A rhombus is a parallelogram.
(iv) The sum of two even numbers is even.
(v) The sum of two odd numbers is odd.

Sol. (i) False (ii) True (iii) True
(iv) True (v) False

Q.3. Restate the following statements with appropriate conditions, so that they become true statements.

(i) All prime numbers are odd.
(ii) Two times a real number is always even.
(iii) For any x, $3x + 1 > 4$.
(iv) For any x, $x^3 \geq 0$.
(v) In every triangle, a median is also an angle bisector.

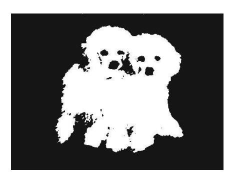
Sol. (i) All prime numbers greater than 2 are odd.
(ii) Two times a natural number is always even.
(iii) For any $x > 1$, $3x + 1 > 4$.
(iv) For any $x \geq 0$, $x^3 \geq 0$.
(v) In an equilateral triangle, a median is also an angle bisector.

Exercises A 1.2

Q.1. Use deductive reasoning to answer the following:

(i) Humans are mammals. All mammals are vertebrates. Based on these two statements, what can you conclude about humans?
(ii) Anthony is a barber. Dinesh had his hair cut. Can you conclude that Antony cut Dinesh's hair?
(iii) Martians have red tongues. Gulag is a Martian. Based on these two statements, what can you conclude about Gulag?
(iv) If it rains for more than four hours on a particular day, the gutters will have to be cleaned the next day. It has rained for 6 hours today. What can we conclude about the condition of the gutters tomorrow?
(v) What is the fallacy in the cow's reasoning in the cartoon below?

All dogs have tails. I have a tail. Therefore I am a dog.



Sol. (i) Humans are vertebrates.
(ii) No, Dinesh could have got his hair cut by anybody else.
(iii) Gulag has a red tongue.

(iv) We conclude that the gutters will have to be cleaned tomorrow.

(v) All animals have tails need not be dogs. E.g. animals such as buffaloes, monkeys, cats etc. have tails but are not dogs.

Q.2. Once again you are given four cards. Each card has a number printed on one side and a letter on the other side. Which are the only two cards you need to turn over to check whether the following rule holds?

"If a card has a consonant on one side, then it has an odd number on the other side."

$\boxed{B}$ $\boxed{3}$ $\boxed{U}$ $\boxed{8}$

Sol. You need to turn over B and 8. If B has an even number on the other side, then the rule has been broken. Similarly, if 8 has a consonant on the other side, then again the rule has been broken.

Exercises A 1.3

Q.1. Take any three consecutive even numbers and their product; for example,

$2 \times 4 \times 6 = 48$, $4 \times 6 \times 8 = 192$ and so on. Make three conjectures about these products.

Sol. Three possible conjectures are:

(i) The product of any three consecutive even numbers is even.

(ii) The product of any three consecutive even numbers is divisible by 4.

(iii) The product of any three consecutive even numbers is divisible by 6.

Q.2. Go back to Pascal's triangle?

Line 1 : $1 = 11^0$ Line 2 : $11 = 11^1$

Line 3 : $121 = 11^2$

Make a conjecture about the line 4 and line 5. Does your conjecture hold? Does your conjecture hold for line 6 too?

Sol. Line 4

$1 \quad 3 \quad 3 \quad 1 = 11^3$

Line 5

$1 \quad 4 \quad 6 \quad 4 \quad 1 = 11^4$.

$\because$ The conjecture holds for line 4 and line 5.

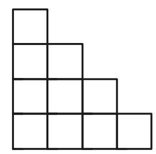
For line 6: (add two consecutive integers)

$1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1 \neq 11^5$

So, answer is 'no', this does not hold for line 6.

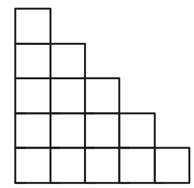
Q.3. Let us look at the triangular numbers (see figure). Add two consecutive triangular numbers (see figure). For example, $T_1 + T_2 = 4$, $T_2 + T_3 = 9$, $T_3 + T_4 = 16$. What about $T_4 + T_5$? Make a conjecture about $T_{n-1} + T_n$.

Sol.



T_4

(A)



T_5

(B)

Observe pattern

$T_1 + T_2 = 4 = 2^2$

$T_2 + T_3 = 9 = 3^2$

$T_3 + T_4 = 16 = 4^2$

Doing in same way, we get:

$T_4 + T_5 = 5^2 = 25$

& conjecture about $T_{n-1} + T_n = n^2$

Q.4. Look at the following pattern:

$1^2 = 1$ $11^2 = 121$

$111^2 = 12321$ $1111^2 = 1234321$

$11111^2 = 123454321$

Make a conjecture about each of the following:

$111111^2 =$

$1111111^2 =$

Check if your conjecture is true.

Sol. By following given pattern, we can write:

$111111^2 = 12345654321$

And $1111111^2 = 1234567654321$

Yes, conjecture is true (verified by actual multiplication).

Q.5. List five axioms (postulates) used in this book.

Sol. Euclid's postulates are:

5.1. Given two distinct points, there is a unique line that passes through them.

5.2. A terminated line can be produced indefinitely.

5.3. A circle can be drawn with any centre and any radius.

5.4. All right angles are equal to one another.

5.5. If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produced indefinitely, meet on that side in which the sum of angles is less than two right angles.

Exercises A 1.4

Q.1. Find counter-examples to disprove the following statements:

(i) If the corresponding angles in two triangles are equal, then the triangles are congruent.

(ii) A quadrilateral with all sides equal is a square.

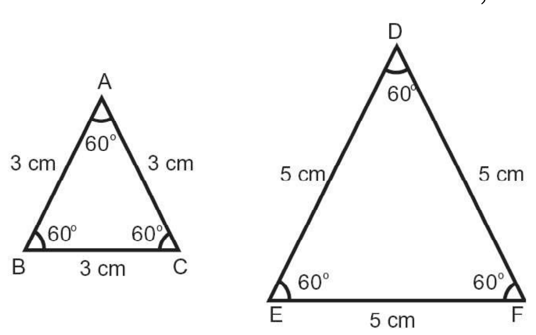
(iii) A quadrilateral with all angles equal is a square.

(iv) For integers a and b, $\sqrt{a^2+b^2} = a+b$

(v) $2n^2 + 11$ is a prime for all whole numbers n.

(vi) $n^2 - n + 41$ is a prime for all positive integers n.

Sol. (i) Counter – example:
Consider ΔABC & ΔDEF in which,



All corresponding angles are same but sides are different and ΔABC is not congruent to ΔDEF. ($\because$ both can't cover each other exactly.)

(ii) Counter example : A quadrilateral with all sides equal is a rhombus, may not be a square.

(iii) **Counter example :** A rectangle has all angels equal, but may not be a square.

(iv) **Counter example :**

Let $a = 2$ and $b = 3$

L.H.S. $\sqrt{a^2+b^2} = \sqrt{2^2+3^2}$

$= \sqrt{4+9} = \sqrt{13}$

R.H.S. $2 + 3 = 5$

$\therefore \sqrt{a^2+b^2} \neq a+b$ for $a = 2$ and $b = 3$

(v) Counter example: Let $n = 11$ (by hit and trial method)

$2n^2 + 11$

$= 2(11)^2 + 11$

$= 2 \times 121 + 11$

$= 242 + 11 = 253$

253 is divisible by 11.

Therefore, 253 is not a prime number so given statement is false for n = 11.

(vi) Counter example is : Let $n = 41$

[by Random choice (Hit and trial)]

$= n^2 - n + 41 = (41)^2 - 41 + 41$

$= (41)^2 - 1681$, that is divisible by 41.

So, 1681 is not a prime number.

So, given statement is false for $n = 41$.

Q.2. Take your favourite proof and analyse it step-by-step along the lines discussed in Section A1.5 (what is given, what has been proved, what theorems and axioms have been used, and so on).

Sol. We are taking Exterior angle theorem:

Statement: If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles.

Given: A ΔABC and one exterior angle

Step (1)

To prove : $\angle 4 = \angle 1 + \angle 2$

Proof : In ΔABC,

$\angle 1 + \angle 2 + \angle 3 = 180°$ ...(1)

(By ASP) Step (2)

Also, $\angle 3 + \angle 4 = 180°$ ...(2)

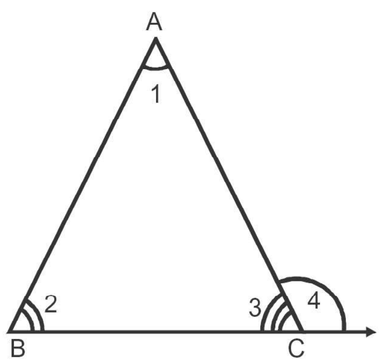
(Linear Pair) Step (3)

Comparing (1) and (2) Step (4)

$\angle 1 + \angle 2 + \angle 3 = \angle 3 + \angle 4$ Step (5)

$\angle 1 + \angle 2 = \angle 4$

Hence proved



Analysis of Proof :

Step 1: We begin with a triangle and one exterior angle.

Step 2: Here we use angle sum property of triangle that is sum of three interior angles of a triangle is 180° to show $\angle 1 + \angle 2 + \angle 3 = 180°$. (Theorem)

Step 3: Here we use linear pain axiom, which states that the angles on a straight line add up to 180° to show that $\angle 3 + \angle 4 = 180°$.

Step 4: Here we use Euclid.s axiom which states that: "Things which are equal to the same thing are equal to each other."

Step 5: Here we use Euclid.s axiom which states that: "If equals are subtracted from equals, the remainders are equal."

So notice Step 5 is the claim made in the theorem we set out to prove.

Q.3. Prove that the sum of two odd numbers is even.

Proof: Let x and y be two odd numbers.

Then $x = 2k + 1$ for some natural number k.

And $y = 2l + 1$ for some natural number l.

So, sum of two odd numbers is:

$$x + y = 2k + 1 + 2l + 1$$
$$= 2l + 2l + 2$$
$$= 2(k + l + 1)$$

(Taking 2 common)

$$= 2M \text{ where } M = k + l + 1$$

Therefore, $x + y$ is divisible by 2 and is even.

Hence proved

Q.4. Prove that the product of two odd numbers is odd.

Proof: Let x and y be two odd numbers.

Then $x = 2k + 1$ for some natural number k.

And $y = 2l + 1$ for some natural number l.

So, product of two odd numbers is:

$$xy = (2k + 1)(2l + 1)$$
$$xy = 2k(2l + 1) + 1(2l + 1)$$
$$xy = 4kl + 2k + 2l + 1$$
$$xy = 2(kl + k + l) + 1$$
$$xy = 2M + 1$$

where $2kl + k + l = M$

$\therefore$ xy is odd.

$\therefore$ Product of two odd numbers is odd.

Q.5. Prove that the sum of three consecutive even numbers is divisible by 6.

Sol. Let three consecutive even numbers be: $2n$, $2n + 2$ and $2n + 4$.

Their sum is: $2n + 2n + 2 + 2n + 4$

$= (2n + 2n + 2n) + (2 + 4)$

[Collecting like terms]

$= 6n + 6 = 6(n + 1)$ [Taking 6 common]

$= 6k$ where $k = n + 1$

So it is divisible by 6.

$\therefore$ The sum of three consecutive even numbers is divisible by 6.

Q.6. Prove that infinitely many points lie on the line whose equation is $y = 2x$.

Sol. Consider the point $(n, 2n)$ for any integer n.

Given equation is $y = 2x$ ...(1)

Put $x = n$ and $y = 2n$ in equation (1)

$$2n = 2(n)$$

$$\Rightarrow \quad 2n = 2n$$

Which is always true.

So, $(n, 2n)$ lies on (1).

Now for infinite values of n, we can obtain infinite points which lie on line whose equation is $y = 2x$. **Hence proved**

Q.7. You must have had a friend who must have told you to think of a number and do various things to it, and then without knowing your original number, telling you what number you ended up with. Here are two examples. Examine why they work.

(i) Choose a number. Double it. Add nine. Add your original number. Divide by three. Add four. Subtract your original number. Your result is seven.

(ii) Write down any three-digit number (for example, 425). Make a six-digit number by repeating these digits in the same order (425425). Your new number is divisible by 7, 11 and 13.

Sol. (i) Let your original number be n.

Now we do the following operations:

(Follow given instructions)

$n \to 2n \to 2n + 9 \to 2n + 9 + n = 3n + 9$

$3n + 9 \to \frac{3n+9}{3} = n + 3 \to n + 3 + 4$

$= n + 7$

$n + 7 \to n + 7 - n = 7$

(ii) Note that $7 \times 11 \times 13 = 1001$.

Take any three digits number say, xyz.

Then xyz × 1001 = xyzxyz

Therefore, the six digit number xyzxyz is divisible by 7, 11 and 13.

APPENDIX - 2 : Introduction to Mathematical Modelling

Exercises A 2.1

In each of the following problems, clearly state what the relevant and irrelevant factors are while going through steps 1, 2 and 3 as given in solved examples.

Q.1. Suppose a company needs a computer for some period of time. The company can either hire a computer for Rs. 2000 per month or buy one for Rs. 25,000. If the company has to use the computer for a long period, the company will pay such a high rent, that buying a computer will be cheaper. On the other hand, if the company has to use the computer for say, just one month, then hiring a computer will be cheaper. Find the number of months beyond which it will be cheaper to buy a computer.

Sol. Step 1:

Formulation: The relevant factors are the time period for hiring a computer and the two costs given to us. We assume that there is no significant change in the cost of purchasing or hiring the computer. So, we treat any such change as irrelevant. We also treat all brands and generations of computers as the same, i.e. these differences are also irrelevant. The expense of hiring the computer for x months is Rs. 2000 x. If this becomes more than the cost of purchasing a computer, we will be better off buying a computer. So, the equation is:

$$2000x = 25000 \qquad ...(1)$$

Step 2: Finding the solution:

Solving equation (1), we get

$$x = \frac{25000}{2000} = \frac{25}{2} = 12.5$$

Step 3: Interpretation: Since the cost of hiring a computer becomes more after 12.5 months, it is cheaper to buy a computer, if you have to use it for more than 12 months.

Q.2. Suppose a car starts from a place A and travels at a speed of 40 km/h towards another place B. At the same instance, another car starts from B and travels towards A at a speed of 30 km/h. If the distance between A and B is 100 km, after how much time will the cars meet?

Sol. Step 1:

Formulation: We will suppose that cars travel at a constant speed. So, any change of speed will be treated as irrelevant. If the cars meet after x hours, the first car would have travelled a distance of $40x$ km from A and second car would have travelled $30x$ km, so that it will be at a distance of $(100 - 30x)$ km from A. So, the equation will be:

$$40x = 100 - 30x.$$

$$\Rightarrow \quad 70x = 100 \quad \text{...(1)}$$

Step 2: Finding the solution:

Solving equation (1); we get:

$$x = \frac{100}{70} = \frac{10}{7} = 1.4 \text{ hours}$$

Step 3:

Interpretation: So the cars will meet after 1.4 hours.

Q.3. The moon is about 3,84,000 km from the earth, and its path around the earth is nearly circular. Find the speed at which it orbits the earth, assuming that it orbits the earth in 24 hours. (Take $\pi = 3.14$)

Sol. Step 1:

Formulation: The speed at which the moon orbits the earth is

$$= \frac{\text{Length of the orbit}}{\text{time taken}}$$

Step 2: Finding the solution:

Since the orbit is nearly circular.

$\therefore$ Its length is its circumference

i.e. $2\pi r = 2 \times \pi \times 384000$ km

$= 2 \times 3.14 \times 384000$

$= 2411520$ km

The moon takes 24 hours to complete on orbit.

$$\text{So, speed} = \frac{2411520}{24}$$

$= 100480$ km/hour

Step 3: Interpretation: The speed is 100480 km/h.

Q.4. A family pays ₹ 1000 for electricity on an average in those months in which it does not use a water heater. In the months in which it uses a water heater, the average electricity bill is ₹ 1240. The cost of using the water heater is ₹ 8.00 per hour. Find the average number of hours the water heater is used in a day.

Sol. Step 1:

Formulation: An assumption is that the difference in the bill is only because of using the water heater.

Let the average number of hours for which the water heater is used in a day = x.

Difference in bill per month due to using water heater = ₹ 1240 – ₹ 1000 = ₹ 240.

Cost of using water heater for 1 hour = ₹ 8.

So, the cost of using the water heater for 30 days = $8 \times 30 \times x$.

Also, the cost of using the water heater for 30 days = Difference in bill due to using water heater.

$$\text{So,} \quad 240x = 240 \quad \text{...(1)}$$

Step 2: Finding the solution:

From equation (1), we get;

$$x = \frac{240}{240} = 1$$

Step 3:

Interpretation: Since $x = 1$, the water heater is used for an average of 1 hour in a day.

Exercises A 2.2

Q.1. We have given the timings of the gold medalists in the 400 meter race from the time the event was included in the Olympics, in the table below. Construct a mathematical model relating the years and timings. Use it to estimate the timing in the next Olympics.

Table - A 2.6

Year	Timing (in seconds)
1964	52.01
1968	52.03
1972	51.08
1976	49.28
1980	48.88
1984	48.83
1988	48.65
1992	48.83
1996	48.25
2000	49.11
2004	49.11

Sol. Let us first convert the problem into a mathematical problem.

Step 1: Formulation: Let us take 1964 as 0^{th} year, and write 1 for 1968, 2 for 1972 and so on.

So table A2.6 will now look as table A2.7.

Table - A 2.7

Year	Timing (in seconds)
0	52.01
1	52.03
2	51.08
3	49.28
4	48.88
5	48.83
6	48.65
7	48.83
8	48.25
9	49.11
10	49.11

The difference in timing is given in the following table:

Table - A 2.8

Year	Timing (in seconds)	Difference
0	52.01	0
1	52.03	0.02
2	51.08	–0.95
3	49.28	–1.8
4	48.88	–0.4
5	48.83	–0.05
6	48.65	–0.18
7	48.83	0.18
8	48.25	–0.58
9	49.11	0.86
10	49.41	0.3
		= –2.6

Mean of differences = $-\frac{2.6}{11} = 0.24$

Let us assume that the timing steadily decreases at the rate of 0.24

Mathematical Description: We have assumed that the timing decreases steadily at the rate of 0.24 per year.

So, timing in the first year

$$= 52.01 - 0.24$$

Timing in the second year

$$= 52.01 - 0.24 - 0.24$$
$$= 52.01 - 2 \times 0.24$$

Similarly in nth year,

$$t = 52.01 - 0.24n \text{ for } n \geq 1 \quad \ldots(1)$$

Now we have to find t for n = 11.

So we have formula to find 't'? is:

$$t = 52.01 - 0.24n \ldots(2)$$

Step 2: Finding a solution:

Put $n = 11$ in (2) to get 't'

$$t = 52.01 - 0.24(11)$$
$$t = 52.01 - 2.64$$
$$= 49.37 \text{ seconds}$$

Step 3: Interpretation: In a word problem, we generally stop here. But, since we are dealing with a real life situation. We have to see to what extent this value matches with the real situation.

Step 4: Validation: Let us check if formula (2) is in agreement with the reality. Let us find the values for the years we already know, using formula (2) and compare it with known values by finding the difference. The values are given in the table A2.9.

Table - A 2.9

Year	Timing (given)	Timing (by 2 formula)	Deviation
0	52.01	52.01	0
1	52.03	51.75	0.28
2	51.08	51.49	–0.41
3	49.28	51.23	–1.95
4	48.88	50.97	–2.09
5	48.83	50.71	–1.88
6	48.65	50.45	–1.8
7	48.83	50.19	–1.36
8	48.25	49.93	–1.68
9	49.11	49.67	–0.56
10	49.41	49.41	0.00
	Total		–11.45

Suppose we decide that this error is negligible. In this case (2) is our mathematical model. And if we decide that this error is not acceptable. Then we have to go back to step 1, the formulation and change equation (2). Let us do so.

Step 1: Reformulation: We still assume that the values decrease steadily by 0.26, but we will now introduce a correction factor to reduce the error. For this, we find the mean of all the deviations. This is:

$$\text{Mean of deviations} = \frac{11.45}{11} = -1.041$$

We take the mean of the errors and correct our formula by this value.

Revised Mathematical Description: Let us now add the mean of the deviations to our formula in (2). So, our corrected formula is:

$$t = 52.01 - 0.24n - 1.041$$

$$t = 50.969 - 0.24n \qquad \text{...(3)}$$

for $n \geq 1$

Step 2: Altered solution:

Solving equation (3) for t, we get;

Put $n = 11$ in (3)

$$t = 50.969 - 0.24(11)$$

$$t = 50.969 - 2.64$$

$$= 48.395s$$

Step 3:

Interpretation: So in 11th year (in next Olympics) $t = 48.005$ seconds.

Step 4:

Validation: Once again, let us compare the values got by using formula (3) with the actual values. Table A2.10 gives the comparison.

Table - A 2.10

Year	Timing (given)	Timing (by 3 formula)	Deviation
0	52.01	52.01	0
1	52.03	50.65	1.425
2	51.08	50.345	0.735
3	49.28	50.085	–0.805
4	48.88	49.825	–0.945
5	48.83	49.565	–0.735
6	48.65	49.305	–0.655
7	48.83	49.045	–0.215
8	48.25	48.785	–0.535
9	49.11	48.525	0.585
10	49.41	48.265	1.145
	Total		0

As we can see many of the values that (3) gives are closer to the actual value than the values that (2) gives. The mean of the deviations is 0 in this case.

∴ We will stop our process here. So equation (3) is our mathematical description that gives a mathematical relationship between years and timing for race. We have constructed a 'Mathematical Model'.

"The process that we have followed in the situation above is called Mathematical Modelling".

Exercises A 2.3

Q.1. How is the solving of word problems that you come across in textbooks different form the process of mathematical modelling?

Sol. We have already mentioned that the formulation part could be very detailed in real-life situations. Also, we don.t validate the answer in word problems. Apart from this word problems have a 'correct answer'. This need not be the case in real-life situations.

Q.2. Suppose you want to minimize the waiting time of vehicles at a traffic function of four roads. Which of these factors are important and which are not?

(i) Price of Petrol.

(ii) The rate at which the vehicles arrive at the four different roads.

(iii) The proportion of slow moving vehicles like cycles and rickshaws and fast moving vehicles like cars and motorcycles.

Sol. The important factors are (ii) and (iii). Here (i) is not an important factor although it can have an effect on the number of vehicles sold.

❑❑