



CHAPTERWISE SOLUTION

Based on **NCERT** Textbook

MATHEMATICS

Class

10



Student Advisor
(Editorial Team)



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Textbook Exercises 1.1

Q.1. Express each number as a product of its prime factors :

(i) 140 (ii) 156 (iii) 3825 (iv) 5005 (v) 7429

Sol. (i) 140

$$140 = 2 \times 2 \times 5 \times 7 \\ = 2^2 \times 5 \times 7$$

$$\begin{array}{r|l} 2 & 140 \\ 2 & 70 \\ 5 & 35 \\ 7 & 7 \\ \hline & 1 \end{array}$$

(ii) 156

$$156 = 2 \times 2 \times 3 \times 13 \\ = 2^2 \times 3 \times 13$$

$$\begin{array}{r|l} 2 & 156 \\ 2 & 78 \\ 3 & 39 \\ 13 & 13 \\ \hline & 1 \end{array}$$

(iii) 3825

$$3825 = 3 \times 3 \times 5 \times 5 \times 17 \\ = 3^2 \times 5^2 \times 17$$

$$\begin{array}{r|l} 3 & 3825 \\ 3 & 1275 \\ 5 & 425 \\ 5 & 85 \\ 17 & 17 \\ \hline & 1 \end{array}$$

(iv) 5005

$$5005 = 5 \times 7 \times 11 \times 13$$

$$\begin{array}{r|l} 5 & 5005 \\ 7 & 1001 \\ 11 & 143 \\ 13 & 13 \\ \hline & 1 \end{array}$$

(v) 7429

$$7429 = 17 \times 19 \times 23$$

$$\begin{array}{r|l} 17 & 7429 \\ 19 & 437 \\ 23 & 23 \\ \hline & 1 \end{array}$$

Q.2. Find the LCM and HCF of the following pairs of integers and verify that $\text{LCM} \times \text{HCF} = \text{product of the two numbers}$:

(i) 26 and 91 (ii) 510 and 92 (iii) 336 and 54

Sol. (i) 26 and 91

$$26 = 2 \times 13 \\ 91 = 7 \times 13 \\ \text{HCF} (26, 91) = 13$$

$$\begin{array}{r|l} 2 & 26 \\ 13 & 13 \\ \hline & 1 \end{array}$$

(product of the smallest power of each common prime factor in the numbers).

$$\text{LCM} (26, 91) = 2 \times 7 \times 13 = 182$$

(product of the greatest power of each prime factor in the numbers).

$$\text{LCM} (26, 91) \times \text{HCF} (26, 91) = 182 \times 13 = 2366$$

$$\text{Product of two numbers} = 26 \times 91 = 2366$$

Hence, $\text{LCM} \times \text{HCF} = \text{product of two numbers}$

Verified

(ii) 510 and 92

$$510 = 2 \times 3 \times 5 \times 17 \\ 92 = 2 \times 2 \times 23 \\ = 2^2 \times 23$$

$$\text{HCF} (510, 92) = 2$$

$$\text{LCM} (510, 92)$$

$$= 2^2 \times 3 \times 5 \times 17 \times 23$$

$$= 23460$$

$$\text{LCM} (510, 92) \times \text{HCF} (510, 92)$$

$$= 23460 \times 2 = 46920.$$

$$\text{Product of two numbers} = 510 \times 92 = 46920$$

$$\text{Product of two numbers} = 510 \times 92 = 46920$$

Hence, $\text{LCM} \times \text{HCF}$

$$= \text{Product of two numbers}$$

Verified

(iii) 336 and 54

$$336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7 \\ = 2^4 \times 3 \times 7$$

$$54 = 2 \times 3 \times 3 \times 3 \\ = 2 \times 3^3$$

$$\text{HCF} (336, 54) = 2 \times 3 = 6$$

$$\text{LCM} (336, 54) = 2^4 \times 3^3 \times 7 = 3024$$

$$\text{LCM} (336, 54) \times \text{HCF} (336, 54)$$

$$= 3024 \times 6 = 18144$$

$$\text{Product of two numbers} = 336 \times 54 = 18144$$

Hence, $\text{LCM} \times \text{HCF} = \text{Product of the two numbers}$.

Verified

Q.3. Find the LCM and HCF of the following integers by applying the prime factorization method.

(i) 12, 15 and 21 (ii) 17, 23 and 29

(iii) 8, 9, and 25.

Sol. (i) 12, 15 and 21

$$12 = 2 \times 2 \times 3$$

$$= 2^2 \times 3$$

$$15 = 3 \times 5$$

$$21 = 3 \times 7$$

$$\begin{array}{r|l} 2 & 12 \\ 2 & 6 \\ 3 & 3 \\ \hline & 1 \end{array} \quad \begin{array}{r|l} 3 & 15 \\ 5 & 5 \\ \hline & 1 \end{array} \quad \begin{array}{r|l} 3 & 21 \\ 7 & 7 \\ \hline & 1 \end{array}$$

$$\text{LCM} (12, 15, 21) = 2^2 \times 3 \times 5 \times 7 = 420.$$

$$\text{HCF} (12, 15, 21) = 3$$

$$\therefore \text{LCM} = 420 \text{ and } \text{HCF} = 3.$$

(ii) 17, 23 and 29

$$17 = 1 \times 17 \quad 23 = 1 \times 23 \quad 29 = 1 \times 29$$

$$\text{LCM} (17, 23, 29) = 17 \times 23 \times 29 = 11339$$

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$$\text{HCF}(17, 23, 29) = 1$$

$$\therefore \text{LCM} = 11339, \text{HCF} = 1$$

(iii) 8, 9 and 25

$$\begin{array}{l} 8 = 2 \times 2 \times 2 \\ = 2^3 \\ 9 = 3 \times 3 = 3^2 \\ 25 = 5 \times 5 = 5^2 \end{array} \quad \begin{array}{c|c|c|c} 2 & 8 & 3 & 9 & 5 & 25 \\ \hline 2 & 4 & 3 & 3 & 5 & 5 \\ \hline 2 & 2 & 1 & 1 & 1 & 1 \\ \hline & & & & & \end{array}$$

$$\text{LCM}(8, 9, 25) = 2^3 \times 3^2 \times 5^2 = 1800$$

$$\text{HCF}(8, 9, 25) = 1$$

$$\therefore \text{LCM} = 1800, \text{HCF} = 1$$

Q.4. Given that $\text{HCF}(306, 657) = 9$, find $\text{LCM}(306, 657)$.

$$\text{HCF}(306, 657) = 9 \quad \text{Numbers} = 306, 657$$

$$\text{LCM}(306, 657) = ?$$

We know that,

$$\begin{aligned} \text{Sol. LCM} &= \frac{\text{Product of two numbers}}{\text{HCF}} \\ &= \frac{306 \times 657}{9} = 34 \times 657 = 22338 \end{aligned}$$

Q.5. Check whether 6^n can end with the digit 0 for any natural number n .

Sol. If the number 6^n ends with the digit 0, then it is divisible by 5, hence, it has 5 as a prime factor.

But $6^n = (2 \times 3)^n = 2^n \times 3^n$
i.e. 6^n contain prime 2 and 3 factors.

Uniqueness of fundamental theorem of arithmetic guarantees that there are no other primes in the factorization of 6^n .

$\Rightarrow 6^n$ does not contain prime 5.

Hence, 6^n cannot end with the digit 0 for any natural number n .

Q.6. Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

$$\begin{aligned} \text{Sol. Since } 7 \times 11 \times 13 + 13 \\ &= (7 \times 11 + 1) \times 13 = 78 \times 13 \\ &= 2 \times 3 \times 13 \times 13. \end{aligned}$$

It is the product of prime. By fundamental theorem of arithmetic $7 \times 11 \times 13 + 13$ is a composite number.

$$\begin{aligned} \text{Again } 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 \\ &= (7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1) \times 5 \\ &= 1009 \times 5 \end{aligned}$$

Which is the product of primes. By the fundamental theorem of arithmetic $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ is composite number.

Q.7. There is a circular path around a sports field, Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

Sol. The time they meet again at the starting point

$$= \text{LCM of 18 and 12 minutes.}$$

$$\begin{array}{l} 18 = 2 \times 3 \times 3 \\ = 2 \times 3^2 \\ 12 = 2 \times 2 \times 3 \\ = 2^2 \times 3 \end{array} \quad \begin{array}{c|c|c|c} 2 & 18 & 2 & 12 \\ \hline 3 & 9 & 2 & 6 \\ \hline 3 & 3 & 3 & 3 \\ \hline & 1 & 1 & 1 \end{array}$$

$$\text{LCM}(18, 12) = 2^2 \times 3^2 = 36.$$

Hence, they will meet again at the starting point after 36 minutes.

Textbook Exercises 1.2

Q.1. Prove that $\sqrt{5}$ is irrational. [CBSE 2020]

Sol. Let us assume, that $\sqrt{5}$ is a rational. It can be expressed in the form of $\frac{a}{b}$, where a and b are coprime positive integers and $b \neq 0$.

$$\therefore \sqrt{5} = \frac{a}{b} \quad (\text{Where } a \text{ and } b \text{ are coprime})$$

$$\therefore \text{HCF of } a \text{ and } b \text{ is } 1$$

Squaring on both sides

$$5 = \frac{a^2}{b^2} \Rightarrow 5b^2 = a^2 \quad \dots(i)$$

Therefore, 5 divides a^2 . It follows that 5 divides a

Let $a = 5c$ and put this value in equation (i)
[Where c is any positive integer]

$$5b^2 = (5c)^2$$

$$\Rightarrow \frac{25}{5} c^2 = b^2 \Rightarrow 5c^2 = b^2 \quad \dots(ii)$$

It means b^2 is divisible by 5. It follows that b is divisible by 5.

From equations (i) and (ii) we say that 5 is a common factor of both a and b . But this contradicts the fact that a and b are coprime so, they have no common factor. So, our assumption that $\sqrt{5}$ is a rational number is wrong.

Therefore, $\sqrt{5}$ is an irrational number.

Proved

Q.2. Prove that $3 + 2\sqrt{5}$ is an irrational.

Sol. Let us assume $3 + 2\sqrt{5}$ is a rational. It can be expressed in the form of $\frac{a}{b}$, where a and b are coprime positive integers and $b \neq 0$.

$$\begin{aligned}\therefore 3 + 2\sqrt{5} &= \frac{a}{b} \\ \Rightarrow \frac{a}{b} - 3 &= 2\sqrt{5} \\ \Rightarrow \frac{a - 3b}{2b} &= \sqrt{5} \quad \dots(i) \\ \frac{a - 3b}{2b} &= \text{rational} \\ (\because a \text{ and } b \text{ are positive integers})\end{aligned}$$

So, from equation (i) $\sqrt{5}$ is a rational.

But this contradicts the fact $\sqrt{5}$ is an irrational.

So, our assumption that $3 + 2\sqrt{5}$ is a rational, is wrong.

Hence, $3 + 2\sqrt{5}$ is an irrational number.

Proved

Q.3. Prove that the following are irrationals :

$$(i) \frac{1}{\sqrt{2}} \quad (ii) 7\sqrt{5} \quad (iii) 6 + \sqrt{2}.$$

Sol. (i) Let us assume, that $\frac{1}{\sqrt{2}}$ is a rational.

It can be expressed in the form of $\frac{a}{b}$, where a and b are coprime positive integer.

$$\begin{aligned}\therefore \frac{1}{\sqrt{2}} &= \frac{a}{b} \\ (\text{Where HCF of } a \text{ and } b \text{ is } 1 \text{ and } b \neq 0) \\ \frac{1}{2} &= \frac{a^2}{b^2}\end{aligned}$$

(Squaring both sides)

$$\Rightarrow b^2 = 2a^2 \quad \dots(i)$$

It means b^2 is divisible by 2. It follows that b , is divisible by 2.

Let $b = 2c$

(Where c is any positive integer).

And put $b = 2c$ in equation (i)

$$\begin{aligned}(2c)^2 &= 2a^2 \\ \Rightarrow 2c^2 &= a^2 \quad \dots(ii)\end{aligned}$$

It means a^2 is divisible by 2. It follows that a , is divisible by 2.

From (i) and (ii) we say that 2 is the common factor of a and b . But this contradicts the fact that a and b are coprime. So, our assumption

that $\frac{1}{\sqrt{2}}$ is a rational, is wrong.

Hence, $\frac{1}{\sqrt{2}}$ is an irrational number. *Proved*

(ii) Let us assume, that $7\sqrt{5}$ is a rational. It can be expressed in the form of $\frac{a}{b}$, where a and b are coprime positive integer and $b \neq 0$.

$$\therefore 7\sqrt{5} = \frac{a}{b} \quad (\text{Where HCF of } a \text{ and } b \text{ is } 1)$$

$$\Rightarrow \sqrt{5} = \frac{a}{7b}$$

$\therefore a$ and b are positive integers.

$$\therefore \frac{a}{7b} \text{ is a rational.}$$

Therefore, $\sqrt{5}$ is a rational.

But this contradicts the fact that $\sqrt{5}$ is an irrational. So, our assumption that $7\sqrt{5}$ is a rational, is wrong.

Hence, $7\sqrt{5}$ is an irrational number. *Proved*

(iii) Let us assume, that $6 + \sqrt{2}$ is a rational.

It can be expressed in the form of $\frac{a}{b}$, where a and b are coprime positive integer and $b \neq 0$.

$$\therefore 6 + \sqrt{2} = \frac{a}{b} \quad (\text{Where HCF of } a \text{ and } b \text{ is } 1)$$

$$\Rightarrow \sqrt{2} = \frac{a}{b} - 6 \Rightarrow \sqrt{2} = \frac{a - 6b}{b}$$

$\therefore a$ and b are positive integers.

$$\therefore \frac{a - 6b}{b} \text{ is a rational.}$$

Therefore, $\sqrt{2}$ is a rational.

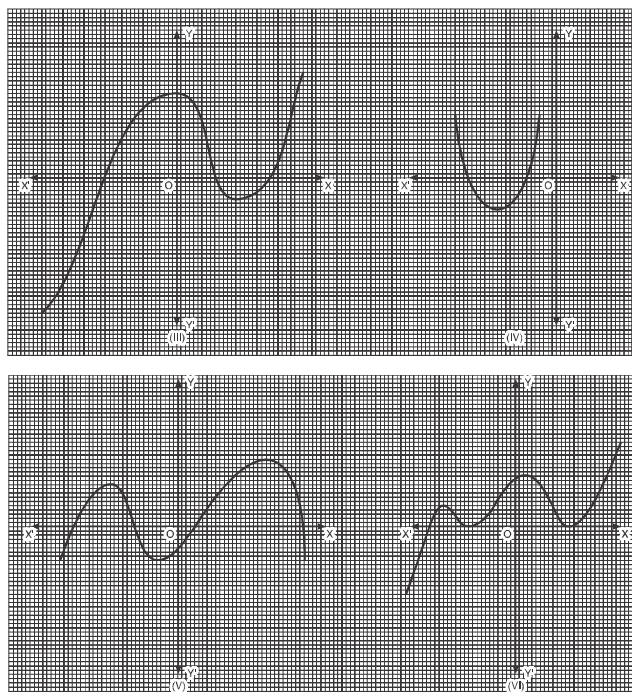
But this contradicts the fact that $\sqrt{2}$ is an irrational. So, our assumption that $6 + \sqrt{2}$ is a rational, is wrong.

Hence, $6 + \sqrt{2}$ is an irrational number.

Proved

Textbook Exercises 2.1

Q.1. The graphs of $y = p(x)$ are given in the figure (A) as shown below, for some polynomials $p(x)$. Find the number of zeroes of $p(x)$, in each case.



Sol. (i) The graph is parallel to x -axis. It does not intersect the x -axis. So the number of zeroes is 0.

(ii) The graph intersects the x -axis at one point. So, the number of zeroes is 1.

(iii) The graph intersects the x -axis at three points. So, the number of zeroes is 3.

(iv) The graph intersects the x -axis at two points. So, the number of zeroes is 2.

(v) The graph intersects the x -axis at four points. So, the number of zeroes is 4.

(vi) The graph intersects the x -axis at three points. So, the number of zeroes is 3.

(iii) $6x^2 - 3 - 7x$

(v) $t^2 - 15$

(iv) $4u^2 + 8u$

(vi) $3x^2 - x - 4$.

Sol. (i) We have,

$$\begin{aligned} f(x) &= x^2 - 2x - 8 \\ &= x^2 - 4x + 2x - 8 \\ &= (x^2 - 4x) + (2x - 8) \\ &= (x - 4)(x + 2) \end{aligned}$$

To find the zeroes of the polynomial $f(x)$, put $f(x) = 0$

$$\Rightarrow (x - 4)(x + 2) = 0 \Rightarrow x = 4, -2$$

Therefore, the zeroes of polynomial $f(x)$ are : $\alpha = 4, \beta = -2$.

Now, Sum of the zeroes

$$= \alpha + \beta = (4 - 2) = 2 = \frac{-(-2)}{1}$$

$$= \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$\text{Product of zeroes} = \alpha \times \beta = 4 \times (-2) = -\frac{8}{1}$$

Textbook Exercise 2.2

Q.1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(i) $x^2 - 2x - 8$

(ii) $4s^2 - 4s + 1$

$$= \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

Hence, zeroes of the polynomial $f(x)$ are 4, -2.

(ii) We have, $f(s) = 4s^2 - 4s + 1$

$$\begin{aligned} &= 4s^2 - 2s - 2s + 1 \\ &= (4s^2 - 2s) - (2s - 1) \\ &= 2s(2s - 1) - 1(2s - 1) \\ &= (2s - 1)(2s - 1) \end{aligned}$$

To find the zeroes of the polynomial $f(s)$,
put $f(s) = 0$.

$$\Rightarrow (2s - 1)(2s - 1) = 0 \quad \Rightarrow s = \frac{1}{2}, \frac{1}{2}$$

Therefore, zeroes of the polynomial $f(s)$

$$\text{are : } \alpha = \frac{1}{2}, \beta = \frac{1}{2}$$

$\therefore$ It has equal roots

Now, Sum of zeroes

$$\begin{aligned} &= (\alpha + \beta) = \frac{1}{2} + \frac{1}{2} = 1 = \frac{-(-4)}{4} \\ &= \frac{-\text{Coefficient of } s}{\text{Coefficient of } s^2} \end{aligned}$$

$$\begin{aligned} \text{Product of zeroes} &= \alpha \times \beta = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \\ &= \frac{\text{Constant term}}{\text{Coefficient of } s^2} \end{aligned}$$

$$\text{Hence, zeroes of polynomial } f(s) = \frac{1}{2}, \frac{1}{2}$$

(iii) We have,

$$\begin{aligned} p(x) &= 6x^2 - 3 - 7x \\ &= 6x^2 - 7x - 3 \\ &= 6x^2 - 9x + 2x - 3 \\ &= 3x(2x - 3) + 1(2x - 3) \\ &= (2x - 3)(3x + 1) \end{aligned}$$

To find the zeroes of the polynomial $p(x)$

$$\text{put } p(x) = 0$$

$$\Rightarrow (2x - 3)(3x + 1) = 0 \Rightarrow x = \frac{3}{2}, -\frac{1}{3}$$

Therefore, zeroes of the polynomial $p(x)$

$$\text{are : } \alpha = \frac{3}{2} \text{ and } \beta = -\frac{1}{3}$$

$$\text{Now, Sum of zeroes} = \alpha + \beta = \frac{3}{2} - \frac{1}{3}$$

$$= \frac{9 - 2}{6} = \frac{7}{6} = \frac{-(-7)}{6}$$

$$= \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

Product of zeroes

$$\begin{aligned} &= \alpha \times \beta = \frac{3}{2} \times \left(-\frac{1}{3}\right) = -\frac{3}{6} \\ &= \frac{\text{Constant term}}{\text{Coefficient of } x^2} \end{aligned}$$

Hence, zeroes of the polynomial

$$p(x) = \frac{3}{2}, -\frac{1}{3}$$

(iv) We have, $f(u) = 4u^2 + 8u = 4u(u + 2)$

To find the zeroes of the polynomial $f(u)$,

$$\text{put } f(u) = 0$$

$$\Rightarrow 4u(u + 2) = 0$$

$$\Rightarrow u = 0, -2.$$

Therefore, zeroes of the polynomial $f(u)$

are : $\alpha = 0$, and $\beta = -2$.

Now, Sum of zeroes

$$\begin{aligned} &= \alpha + \beta = 0 + (-2) = -2 = \frac{-8}{4} \\ &= \frac{-\text{Coefficient of } u}{\text{Coefficient of } u^2} \end{aligned}$$

Product of zeroes

$$\begin{aligned} &= \alpha \times \beta = 0 \times (-2) = \frac{0 \times (-8)}{4} = \frac{0}{4} \\ &= \frac{\text{Constant term}}{\text{Coefficient of } u^2} \end{aligned}$$

Hence, zeroes of the polynomial

$$f(u) = 0, -2.$$

(v) We have, $f(t) = t^2 - 15$

$$= (t)^2 - (\sqrt{15})^2 = (t + \sqrt{15})(t - \sqrt{15})$$

To find the zeroes of polynomial $f(t)$, put

$$f(t) = 0.$$

$$\Rightarrow (t + \sqrt{15})(t - \sqrt{15}) = 0.$$

$$\Rightarrow t = -\sqrt{15}, \sqrt{15}$$

Therefore, zeroes of polynomial $f(t)$ are:

$$\alpha = -\sqrt{15} \text{ and } \beta = \sqrt{15}$$

Now, Sum of zeroes = $\alpha + \beta$

$$= -\sqrt{15} + \sqrt{15} = 0 = -\frac{0}{1}$$

$$= \frac{-\text{Coefficient of } t}{\text{Coefficient of } t^2}$$

Product of zeroes = $\alpha \times \beta$

$$= -\sqrt{15} \times \sqrt{15} = -\frac{15}{1}$$

$$= \frac{\text{Constant term}}{\text{Coefficient of } t^2}$$

Hence, zeroes of the polynomial $f(t)$

$$= -\sqrt{15}, \sqrt{15}$$

$$\begin{aligned} \text{(vi) We have, } f(x) &= 3x^2 - x - 4 \\ &= 3x^2 - (4 - 3)x - 4 \\ &= 3x^2 - 4x + 3x - 4 \\ &= x(3x - 4) + 1(3x - 4) \\ &= (3x - 4)(x + 1) \end{aligned}$$

To find the zeroes of the polynomial $f(x)$,
put $f(x) = 0$

$$\Rightarrow (3x - 4)(x + 1) = 0 \Rightarrow x = \frac{4}{3}, -1$$

Therefore, zeroes of the polynomial $f(x)$

are : $\alpha = \frac{4}{3}$ and $\beta = -1$

Now, Sum of zeroes

$$\begin{aligned} = \alpha + \beta &= \frac{4}{3} - 1 = \frac{4-3}{3} = \frac{1}{3} = \frac{-(-1)}{3} \\ &= \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2} \end{aligned}$$

Product of zeroes = $\alpha \times \beta$

$$\begin{aligned} &= \frac{4}{3} \times (-1) = -\frac{4}{3} \\ &= \frac{\text{Constant term}}{\text{Coefficient of } x^2} \end{aligned}$$

Hence, zeroes of polynomial

$$f(x) = \frac{4}{3}, -1.$$

Q.2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

$$\text{(i) } \frac{1}{4}, -1 \quad \text{(ii) } \sqrt{2}, \frac{1}{3} \quad \text{(iii) } 0, \sqrt{5}$$

$$\text{(iv) } 1, 1 \quad \text{(v) } -\frac{1}{4}, \frac{1}{4} \quad \text{(vi) } 4, 1.$$

Sol. (i) We have, Sum of zeroes = $(\alpha + \beta) = \frac{1}{4}$

Product of zeroes = $\alpha \times \beta = -1$

So, the required quadratic polynomial is

$$\begin{aligned} f(x) &= x^2 - (\alpha + \beta)x + \alpha \times \beta \\ &= x^2 - \frac{1}{4}x - 1 \end{aligned}$$

$$= \frac{1}{4}(4x^2 - x - 4)$$

$\therefore$ Quadratic polynomial = $4x^2 - x - 4$.

(ii) We have,

Sum of zeroes = $(\alpha + \beta) = \sqrt{2}$

Product of zeroes = $\alpha \times \beta = \frac{1}{3}$

So, the required quadratic polynomial is

$$\begin{aligned} p(x) &= x^2 - (\alpha + \beta)x + \alpha \times \beta \\ &= x^2 - \sqrt{2}x + \frac{1}{3} \\ &= \frac{1}{3}(3x^2 - 3\sqrt{2}x + 1) \end{aligned}$$

$\therefore$ Quadratic polynomial

$$= 3x^2 - 3\sqrt{2}x + 1$$

(iii) We have,

Sum of zeroes = $\alpha + \beta = 0$.

Product of zeroes = $\alpha \times \beta = \sqrt{5}$

So, the required quadratic polynomial is

$$\begin{aligned} f(x) &= x^2 - (\alpha + \beta)x + \alpha \times \beta \\ &= x^2 - 0 \times x + \sqrt{5} \\ &= x^2 + \sqrt{5} \end{aligned}$$

(iv) We have,

Sum of zeroes = $\alpha + \beta = 1$

Product of zeroes = $\alpha \times \beta = 1$

So, the required quadratic polynomial is

$$\begin{aligned} f(x) &= x^2 - (\alpha + \beta)x + \alpha \times \beta \\ &= x^2 - x + 1 \end{aligned}$$

(v) We have, Sum of zeroes = $\alpha + \beta = -\frac{1}{4}$

Product of zeroes = $\alpha \times \beta = \frac{1}{4}$

So, the required quadratic polynomial is

$$\begin{aligned} f(x) &= x^2 - (\alpha + \beta)x + \alpha \times \beta \\ &= x^2 + \frac{1}{4}x + \frac{1}{4} \\ &= \frac{1}{4}(4x^2 + x + 1) \end{aligned}$$

$\therefore$ Quadratic polynomial = $4x^2 + x + 1$

(vi) We have, Sum of zeroes = $\alpha + \beta = 4$

Product of zeroes = $\alpha \times \beta = 1$

So, the required polynomial is

$$\begin{aligned} f(x) &= x^2 - (\alpha + \beta)x + \alpha \times \beta \\ &= x^2 - 4x + 1 \end{aligned}$$

Text Book Exercise 3.1

Q.1. Form the pair of linear equations in the following problems, and find their solutions graphically.

- (i) 10 students of class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.
- (ii) 5 pencils and 7 pens together cost ₹ 50, whereas 7 pencils and 5 pens together cost ₹ 46. Find the cost of one pencil and that of one pen.

Sol. (i) Let the number of boys in the class be x and number of girls be y .

According to question,

$$x + y = 10 \quad \dots(1)$$

and $y = x + 4$

$$\Rightarrow -x + y = 4 \quad \dots(2)$$

$\therefore$ Linear equations are : $x + y = 10$

$$-x + y = 4$$

Now, we draw the graphs of linear equations as follows : $x + y = 10$

$$\Rightarrow y = 10 - x$$

We put the different values of x in this equation, then we get different values of y and we prepare the table of x, y for equation $x + y = 10$

Table 1

x	2	5	0
y	8	5	10

and $-x + y = 4$

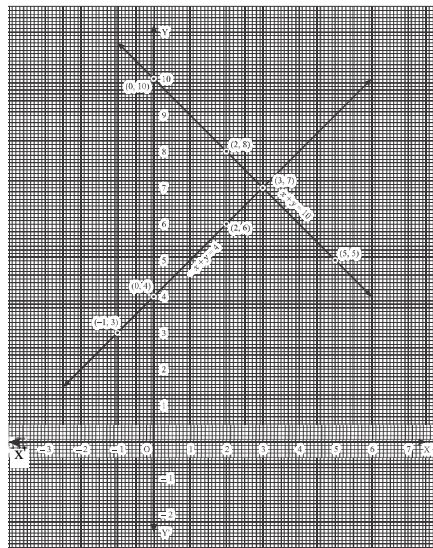
$$y = 4 + x$$

We put the different values of x in this equation, then we get different values of y and we prepare the table of x, y for equation $-x + y = 4$

Table 2

x	0	2	-1
y	4	6	3

Now, we plot the points (2, 8), (5, 5) and (0, 10) on the graph paper and we draw a graph which passes through these points.



$\therefore$ We get a graph of a linear equation $x + y = 10$.

Again, we plot the points (0, 4), (2, 6) and (-1, 3) on the graph paper and we draw a graph which passes through these points.

$\therefore$ We get a graph of a linear equation $-x + y = 4$.

We observe that the graphs of the linear equations $x + y = 10$ and $-x + y = 4$, intersect each other at the point (3, 7). Hence $x = 3$, $y = 7$ is the solution of the linear equations.

Therefore, pair of linear equations is, $x + y = 10$, $-x + y = 4$ and number of boys = 3, and number of girls = 7.

(ii) Let the cost of 1 pencil be ₹ x

and the cost of 1 pen be ₹ y

According to question,

$$5x + 7y = 50 \quad \dots(1)$$

and $7x + 5y = 46 \quad \dots(2)$

Now, we draw the graphs of linear equations as follows :

$$5x + 7y = 50 ; \Rightarrow y = \frac{50 - 5x}{7}$$

We put the different values of x in this equation, then we get different values of y and we prepare the table of x, y for equation $5x + 7y = 50$

Table 1

x	3	10	-4
y	5	0	10

and $7x + 5y = 46$

$$\Rightarrow 5y = 46 - 7x$$

$$\Rightarrow y = \frac{46 - 7x}{5}$$

We put the different values of x in this equation, then we get different values of y and we prepare the table of x, y for equation $7x + 5y = 46$

Table 2

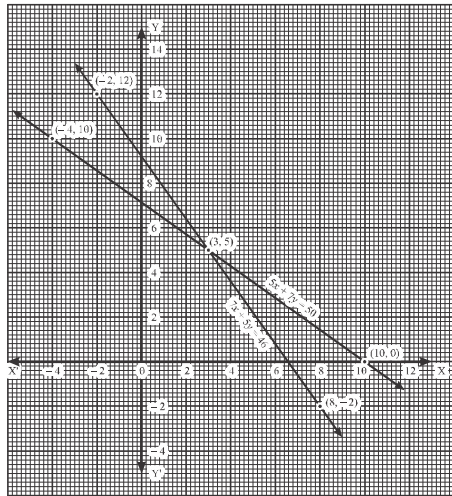
x	8	3	-2
y	-2	5	12

Now, we plot the points (3, 5), (10, 0), (-4, 10) on the graph paper and we draw a graph which passes through these points.

$\therefore$ We get a graph of linear equation $5x + 7y = 50$.

Again, we plot the points (8, -2), (3, 5) and (-2, 12) on the graph paper and we draw a graph which passes through these points.

$\therefore$ We get a graph of linear equation $7x + 5y = 46$.



We observe that the graphs of equations $5x + 7y = 50$, $7x + 5y = 46$; intersect each other at point (3, 5). Hence, $x = 3$, $y = 5$ is the solution of linear equations.

Therefore, the pair of linear equations are : $5x + 7y = 50$, $7x + 5y = 46$.

and the cost of 1 Pencil is ₹ 3 and the cost of 1 pen is ₹ 5.

Q.2. On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident :

(i) $5x - 4y + 8 = 0$; $7x + 6y - 9 = 0$

(ii) $9x + 3y + 12 = 0$; $18x + 6y + 24 = 0$

(iii) $6x - 3y + 10 = 0$; $2x - y + 9 = 0$.

Sol. (i) On comparing the linear equations $5x - 4y + 8 = 0$, $7x + 6y - 9 = 0$ with the general form of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get

$$a_1 = 5, b_1 = -4, c_1 = 8, a_2 = 7, b_2 = 6, c_2 = -9$$

$$\frac{a_1}{a_2} = \frac{5}{7}; \frac{b_1}{b_2} = \frac{-4}{6} \text{ and } \frac{c_1}{c_2} = \frac{8}{-9}$$

$$\text{Since, } \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Therefore, the given lines intersect at a point.

(ii) On comparing the linear equations $9x + 3y + 12 = 0$ and $18x + 6y + 24 = 0$ with the general form of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get

$$a_1 = 9, b_1 = 3, c_1 = 12, a_2 = 18, b_2 = 6, c_2 = 24$$

$$\frac{a_1}{a_2} = \frac{9}{18}; \frac{b_1}{b_2} = \frac{3}{6} \text{ and } \frac{c_1}{c_2} = \frac{12}{24}$$

$$\text{Since, } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Therefore, the given lines are coincident lines.

(iii) On comparing the linear equations $6x - 3y + 10 = 0$ and $2x - y + 9 = 0$ with the general form of linear equations :

$$a_1x + b_1y + c_1 = 0$$

and $a_2x + b_2y + c_2 = 0$, we get

$$a_1 = 6, b_1 = -3, c_1 = 10, a_2 = 2,$$

$$b_2 = -1, c_2 = 9$$

$$\frac{a_1}{a_2} = 3, \frac{b_1}{b_2} = 3 \text{ and } \frac{c_1}{c_2} = \frac{10}{9}$$

$$\text{Since, } \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Therefore, the given lines are parallel lines.

Q.3. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and

$\frac{c_1}{c_2}$, find out whether the following pair of linear equations are consistent, or inconsistent :

(i) $3x + 2y = 5$

$2x - 3y = 7$

(ii) $2x - 3y = 8$

$4x - 6y = 9$

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7$

$9x - 10y = 14$

(iv) $5x - 3y = 11$

$-10x + 6y = -22$

(v) $\frac{4}{3}x + 2y = 8$

$2x + 3y = 12.$

Sol. (i) We can write the given linear equations in the form of :

$$3x + 2y - 5 = 0; 2x - 3y - 7 = 0$$

On comparing these equations with the general form of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get

$$a_1 = 3, b_1 = 2, c_1 = -5, a_2 = 2,$$

$$b_2 = -3, c_2 = -7$$

$$\frac{a_1}{a_2} = \frac{3}{2}, \frac{b_1}{b_2} = \frac{-2}{3}$$

Since, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Therefore, the pair of linear equations has unique solution. So, the equations are consistent.

(ii) We can write the given linear equations in the form of :

$$2x - 3y - 8 = 0; 4x - 6y - 9 = 0$$

On comparing these equations with the general form of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get

$$a_1 = 2, b_1 = -3, c_1 = -8, a_2 = 4,$$

$$b_2 = -6, c_2 = -9$$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2}$$

and $\frac{c_1}{c_2} = \frac{-8}{-9} = \frac{8}{9}$

Since, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Therefore, the pair of linear equations are parallel lines. So, the equations are inconsistent.

(iii) We can write the given linear equations in the form of :

$$\frac{3}{2}x + \frac{5}{3}y - 7 = 0; 9x - 10y - 14 = 0$$

On comparing these equations with the general form of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get

$$a_1 = \frac{3}{2}, b_1 = \frac{5}{3}, c_1 = -7, a_2 = 9,$$

$$b_2 = -10, c_2 = -14$$

$$\frac{a_1}{a_2} = \frac{1}{6}, \frac{b_1}{b_2} = -\frac{1}{6}$$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Therefore, the pair of linear equations has a unique solution. So, the equations are consistent.

(iv) We can write the given linear equations in the form of :

$$5x - 3y - 11 = 0; -10x + 6y + 22 = 0$$

On comparing these equations with the general form of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get

$$a_1 = 5, b_1 = -3, c_1 = -11, a_2 = -10,$$

$$b_2 = 6, c_2 = 22$$

$$\frac{a_1}{a_2} = \frac{-1}{2}, \frac{b_1}{b_2} = \frac{-1}{2}$$

and $\frac{c_1}{c_2} = \frac{-1}{2}$

Since, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Therefore, the given linear equations are coincident lines. So, the equations are consistent.

(v) We can write the given linear equations in the form of :

$$\frac{4}{3}x + 2y - 8 = 0$$

$$2x + 3y - 12 = 0$$

On comparing these equations with the general form of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get

$$a_1 = \frac{4}{3}, b_1 = 2, c_1 = -8, a_2 = 2,$$

$$b_2 = 3, c_2 = -12$$

$$\frac{a_1}{a_2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{2}{3} \text{ and } \frac{c_1}{c_2} = \frac{2}{3}$$

$$\text{Since, } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Therefore, the given linear equations are coincident lines. So, the equations are consistent.

Q.4. Which of the following pairs of linear equations are consistent/inconsistent? If consistent, obtain the solution graphically:

(i) $x + y = 5; 2x + 2y = 10$

(ii) $x - y = 8; 3x - 3y = 16$

(iii) $2x + y - 6 = 0; 4x - 2y - 4 = 0$

(iv) $2x - 2y - 2 = 0; 4x - 4y - 5 = 0$

Sol. (i) We can write the given linear equations in the form of :

$$x + y - 5 = 0; 2x + 2y - 10 = 0$$

On comparing these equations with the general form of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get

$$a_1 = 1, b_1 = 1, c_1 = -5, a_2 = 2, b_2 = 2, c_2 = -10$$

$$\frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{1}{2}; \text{ and } \frac{c_1}{c_2} = \frac{1}{2},$$

$$\text{Since } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Therefore, the given linear equations are coincident lines. So, the equations are consistent.

Graphical solution : We draw the graphs of given linear equations as follows:

$$x + y - 5 = 0 \Rightarrow y = 5 - x$$

We put the different values of x in this equation then we get different values of y and we prepare the table of x, y for equation $x + y - 5 = 0$.

Table 1

x	5	2	1
y	0	3	4

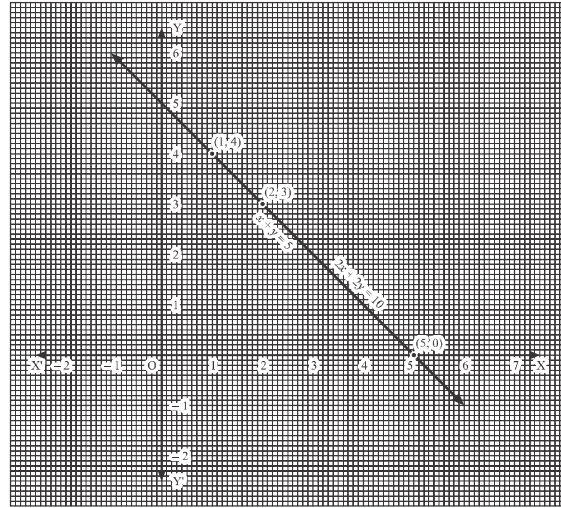
$$\text{and } 2x + 2y - 10 = 0$$

$$\Rightarrow y = \frac{10 - 2x}{2} = 5 - x$$

The table of (x, y) for equation $2x + 2y - 10 = 0$

Table 2

x	5	2	1
y	0	3	4



Now, we plot the points (5, 0), (2, 3) and (1, 4) from table 1 and 2. Then we draw the graphs of linear equations $x + y - 5 = 0$ and $2x + 2y - 10 = 0$ which passes through these points.

We observe that the graphs of linear equations are coincident lines which overlap each other so, it has infinitely many solutions. Solutions are (5, 0), (2, 3) and (1, 4),

(ii) We can write the given linear equations in the form of :

$$x - y - 8 = 0; 3x - 3y - 16 = 0$$

On comparing these equations with the general form of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get

$$a_1 = 1, b_1 = -1, c_1 = -8, a_2 = 3,$$

$$b_2 = -3, c_2 = -16$$

$$\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{1}{3}, \text{ and } \frac{c_1}{c_2} = \frac{1}{2}$$

$$\text{Since, } \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Therefore, the given linear equations are parallel lines. So, the equations are inconsistent.

(iii) On comparing the linear equations $2x + y - 6 = 0$ and $4x - 2y - 4 = 0$ with the general

form of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get

$$a_1 = 2, b_1 = 1, c_1 = -6, a_2 = 4,$$

$$b_2 = -2, c_2 = -4$$

$$\frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-1}{2} \text{ and } \frac{c_1}{c_2} = \frac{3}{2}$$

$$\text{Since, } \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Therefore, the given linear equations intersect each other at a point. So, the equations are consistent.

Graphical solution : We draw the graphs of linear equations as follows :

$$2x + y - 6 = 0; y = 6 - 2x$$

We put the different values of x in this equation then we get different values of y and we prepare the table of x, y .

Table 1

x	0	1	3
y	6	4	0

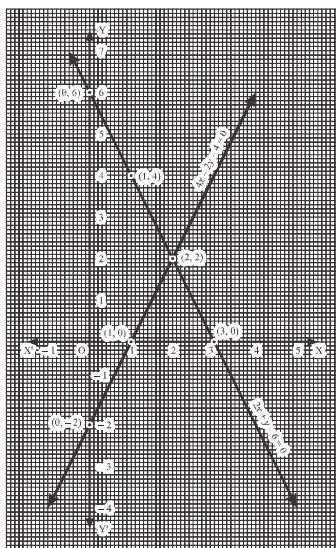
$$\text{and } 4x - 2y - 4 = 0$$

$$\Rightarrow 2y = 4x - 4 = 2x - 2$$

We put the different values of x in this equation, then we get different values of y and we prepare the table of x, y .

Table 2

x	0	1	2
y	-2	0	2



Now, we plot the points (0, 6), (1, 4) and (3, 0) on the graph paper and we draw a graph which passes through these points.

$\therefore$ We get a graph of linear equation $2x + y - 6 = 0$.

Again, we plot the points (0, -2), (1, 0) and (2, 2) on the graph paper and we draw a graph which passes through these points.

$\therefore$ We get a graph of linear equation $4x - 2y - 4 = 0$.

We observe that the graphs of linear equations $2x + y - 6 = 0$ and $4x - 2y - 4 = 0$ intersect each other at point (2, 2). Hence, $x = 2, y = 2$ is the solution of the given linear equations.

(iv) On comparing the linear equations $2x - 2y - 2 = 0$ and $4x - 4y - 5 = 0$, with the general form of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get

$$a_1 = 2, b_1 = -2, c_1 = -2, a_2 = 4,$$

$$b_2 = -4 \text{ and } c_2 = -5$$

$$\frac{a_1}{a_2} = \frac{1}{2}; \frac{b_1}{b_2} = \frac{1}{2} \text{ and } \frac{c_1}{c_2} = \frac{2}{5}$$

$$\text{Since, } \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Therefore, the given linear equations are parallel lines. So, the equations are inconsistent.

Q.5. Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.

Sol. Let the length of rectangular garden be x m and width be y m.

According to question,

$$\frac{2(x + y)}{2} = 36$$

$$x + y = 36 \quad \dots(1)$$

$$\text{and } x = y + 4 \Rightarrow x - y = 4 \quad \dots(2)$$

We draw the graphs of these equations as follows : $x + y = 36 \Rightarrow y = 36 - x$

We put the different values of x in this equation then we get different values of y and we prepare the table of x, y for equation $x + y = 36$

Table 1

x	10	20	12
y	26	16	24

and $x - y = 4$
 $\Rightarrow y = x - 4$

We put the different values of x in this equation then we get different values of y and we prepare the table of x, y for equation $x - y = 4$

Table 2

x	20	2	4
y	16	-2	0

Now, we plot the points (10, 26), (20, 16) and (12, 24) on the graph paper and we draw a graph which passes through these points.

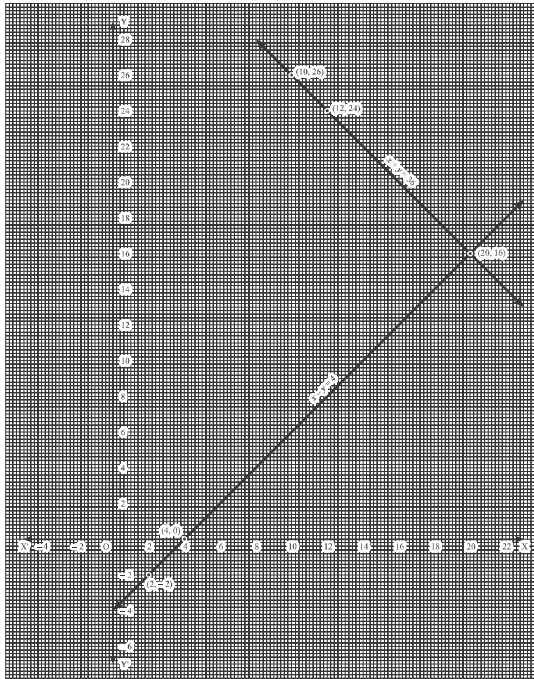
$\therefore$ We get a graph of linear equation

$$x + y = 36.$$

Again, we plot the points (20, 16), (2, -2) and (4, 0) on the graph paper and we draw a graph which passes through these points.

$\therefore$ We get a graph of linear equation

$$x - y = 4.$$



We observe that the graphs of linear equations $x + y = 36$ and $x - y = 4$ intersect each other at point (20, 16). Hence, $x = 20, y = 16$ is the solution of given linear equations.

Therefore, Length of garden = 20 m

Width of garden = 16 m.

Q.6. Given the linear equation $2x + 3y - 8 = 0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is :

- (i) intersecting lines (ii) parallel lines
 (iii) coincident lines.

Sol. On comparing the given linear equation $2x + 3y - 8 = 0$ with the general form of linear equation $a_1x + b_1y + c_1 = 0$, we get

$$a_1 = 2, b_1 = 3, c_1 = -8.$$

(i) When a pair of linear equations are intersecting lines, then condition is :

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{2}{a_2} \neq \frac{3}{b_2} \Rightarrow \frac{a_2}{b_2} \neq \frac{2}{3}$$

Hence, required possible another linear equation is $3x + 2y - 10 = 0$.

(ii) When a pair of linear equations are parallel lines, then condition is :

$$\begin{aligned} \frac{a_1}{a_2} &= \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \\ \Rightarrow \frac{2}{a_2} &= \frac{3}{b_2} \neq \frac{-8}{c_2} \\ \Rightarrow \frac{2}{a_2} &= \frac{3}{b_2} \Rightarrow \frac{a_2}{b_2} = \frac{2}{3} \\ \text{and } \frac{3}{b_2} &\neq \frac{-8}{c_2} \Rightarrow \frac{b_2}{c_2} \neq \frac{3}{-8} \\ \Rightarrow \frac{b_2}{c_2} &\neq \frac{-3}{8} \end{aligned}$$

Hence, required possible another linear equation is $2x + 3y - 12 = 0$.

(iii) When the pair of linear equations are coincident lines, then the condition is :

$$\begin{aligned} \frac{a_1}{a_2} &= \frac{b_1}{b_2} = \frac{c_1}{c_2} \\ \Rightarrow \frac{2}{a_2} &= \frac{3}{b_2} = \frac{-8}{c_2} \end{aligned}$$

$$\Rightarrow \frac{a_2}{2} = \frac{b_2}{3} = \frac{c_2}{-8}$$

$$\text{Let } \frac{a_2}{2} = \frac{b_2}{3} = \frac{c_2}{-8} = k$$

[Where k is constant]

$$a_2 = 2k; b_2 = 3k \text{ and } c_2 = -8k.$$

Hence, the required linear equation is :
 $2kx + 3ky - 8k = 0$ [Where k is constant].

Therefore, the required possible linear equation is $4x + 6y - 16 = 0$.

Q.7. Draw the graphs of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and the x -axis and shade the triangular region.

Sol. We draw the graph of given linear equations as follows :

$$x - y + 1 = 0 \Rightarrow y = x + 1$$

We put the different values of x in this equation, then we get different values of y and we prepare the table of x, y for equation $x - y + 1 = 0$

Table 1

x	2	4	5
y	3	5	6

and $3x + 2y - 12 = 0 \Rightarrow 2y = 12 - 3x$

$$\Rightarrow y = \frac{12 - 3x}{2}$$

We put the different values of x in this equation then we get different values of y and we prepare the table of x, y for equation $3x + 2y - 12 = 0$

Table 2

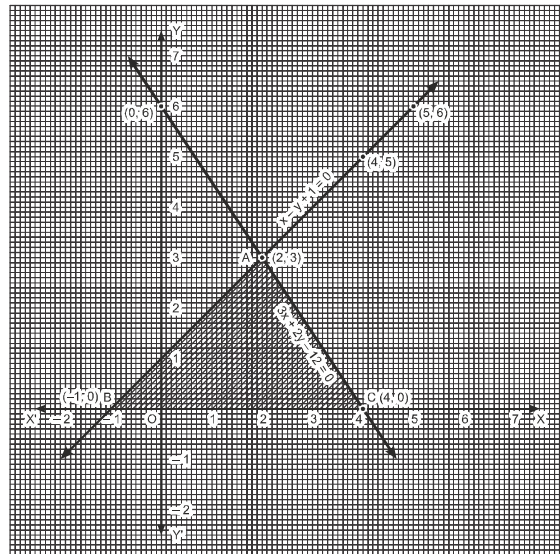
x	0	2	4
y	6	3	0

Now, we plot the points (2, 3), (4, 5) and (5, 6) on the graph paper and we draw a graph which passes through these points.

$\therefore$ We get a graph of linear equation $x - y + 1 = 0$.

Again, we plot the points (0, 6), (2, 3) and (4, 0) on the graph paper and we draw a graph, which passes through these points.

$\therefore$ We get a graph of linear equation $3x + 2y - 12 = 0$.



We observe that the graphs of linear equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$ intersect at point (2, 3). The graphs of linear equations meet x -axis at points B and C. So, the coordinates of $\triangle ABC$ formed are A(2, 3), B(-1, 0) and C(4, 0).

Textbook Exercise 3.2

Q.1. Solve the following pair of linear equations by the substitution method :

(i) $x + y = 14; x - y = 4$

(ii) $s - t = 3; \frac{s}{3} + \frac{t}{2} = 6$

(iii) $3x - y = 3; 9x - 3y = 9$

(iv) $0.2x + 0.3y = 1.3; 0.4x + 0.5y = 2.3$

(v) $\sqrt{2}x + \sqrt{3}y = 0; \sqrt{3}x - \sqrt{8}y = 0$

(vi) $\frac{3x}{2} - \frac{5y}{3} = -2; \frac{x}{3} + \frac{y}{2} = \frac{13}{6}$

Sol. (i) The given equations are :

$$x + y = 14 \quad \dots(1)$$

$$x - y = 4 \quad \dots(2)$$

From equation (2), we get

$$x = 4 + y$$

Substituting the value of x in equation (1), we get

$$\begin{aligned} 4 + y + y &= 14 \\ \Rightarrow 2y &= 14 - 4 \\ \Rightarrow 2y &= 10 \end{aligned}$$

$$\therefore y = \frac{10}{2} = 5$$

Putting the value of y in equation (2), we get

$$\begin{aligned} x - 5 &= 4 \\ \Rightarrow x &= 4 + 5 = 9 \end{aligned}$$

Hence, $x = 9$, $y = 5$ is the required solution.

(ii) The given equations are :

$$s - t = 3 \quad \dots(1)$$

$$\frac{s}{3} + \frac{t}{2} = 6$$

$$\Rightarrow 2s + 3t = 36 \quad \dots(2)$$

From equation (1), we get

$$s = 3 + t$$

Substituting the value of s in equation (2), we get

$$\begin{aligned} 2(3 + t) + 3t &= 36 \\ \Rightarrow 6 + 5t &= 36 \\ \Rightarrow 5t &= 30 \\ \therefore t &= 6 \end{aligned}$$

Putting the value of t in equation (1), we get

$$s - 6 = 3; \Rightarrow s = 3 + 6 = 9$$

Hence, $s = 9$, $t = 6$ is the required solution.

(iii) The given equations are :

$$3x - y = 3 \quad \dots(1)$$

$$9x - 3y = 9 \quad \dots(2)$$

From equation (1), we get $y = 3x - 3$

Substituting the value of y in equation (2), we get $9x - 3(3x - 3) = 9 \Rightarrow 9 = 9$

It is a true statement. Hence, we do not get specific values of x and y as a solution. So, equations (1) and (2) have infinitely many solutions.

Putting the different values of x in $y = 3x - 3$, we get different values of y .

(iv) The given equations are :

$$0.2x + 0.3y = 1.3$$

$$0.4x + 0.5y = 2.3$$

Multiplying both equations by 10, we get

$$2x + 3y = 13 \quad \dots(1)$$

$$4x + 5y = 23 \quad \dots(2)$$

From equation (1), we get

$$3y = 13 - 2x$$

$$\Rightarrow y = \frac{13 - 2x}{3}$$

Substituting the value of y in equation (2), we get

$$4x + \frac{5(13 - 2x)}{3} = 23$$

$$\Rightarrow \frac{12x + 65 - 10x}{3} = 23$$

$$\Rightarrow 2x + 65 = 69$$

$$\Rightarrow x = 2$$

Putting the value of x in equation (1), we get

$$2 \times 2 + 3y = 13$$

$$\Rightarrow 3y = 13 - 4 = 9$$

$$\therefore y = 3$$

Hence, $x = 2$, $y = 3$ is the required solution.

(v) The given equations are

$$\sqrt{2}x + \sqrt{3}y = 0 \quad \dots(1)$$

$$\sqrt{3}x - \sqrt{8}y = 0 \quad \dots(2)$$

From equation (1), we get

$$\sqrt{2}x = -\sqrt{3}y \Rightarrow x = -\frac{\sqrt{3}}{\sqrt{2}}y$$

Substituting the value of x in equation (2), we get

$$\sqrt{3} \times \left(-\frac{\sqrt{3}}{\sqrt{2}}y\right) - \sqrt{8}y = 0$$

$$\Rightarrow -\frac{3}{\sqrt{2}}y - \sqrt{8}y = 0 \Rightarrow y = 0$$

Putting the value of y in equation (1), we get $\sqrt{2}x + \sqrt{3} \times 0 = 0$

$$\Rightarrow \sqrt{2}x = 0 \Rightarrow x = 0$$

Hence, $x = 0$, $y = 0$ is the required solution.

(vi) The given equations are :

$$\frac{3x}{2} - \frac{5y}{3} = -2$$

$$\text{or } 9x - 10y = -12 \quad \dots(1)$$

$$\text{and } \frac{x}{3} + \frac{y}{2} = \frac{13}{6} \text{ or } 2x + 3y = \frac{13}{6} \times 6$$

$$\text{or } 2x + 3y = 13 \quad \dots(2)$$

From equation (2), we get

$$2x = 13 - 3y$$

$$\Rightarrow x = \frac{13 - 3y}{2}$$

Substituting the value of x in equation (1), we get

$$\begin{aligned} 9 \times \left(\frac{13 - 3y}{2} \right) - 10y &= -12 \\ \Rightarrow \frac{117 - 27y - 20y}{2} &= -12 \\ \Rightarrow 117 - 47y &= -24 \\ \Rightarrow -47y &= -24 - 117 \\ \Rightarrow y &= \frac{-141}{-47} = 3 \end{aligned}$$

Putting the value of y in equation (2), we get

$$\begin{aligned} 2x + 3 \times 3 &= 13 \\ \Rightarrow 2x &= 13 - 9 = 4 \\ \Rightarrow x &= 2 \end{aligned}$$

Hence, $x = 2$, $y = 3$ is the required solution.

Q.2. Solve $2x + 3y = 11$ and $2x - 4y = -24$ and hence find the value of ' m ' for which $y = mx + 3$.

Sol. The given equations are :

$$2x + 3y = 11 \quad \dots(1)$$

$$2x - 4y = -24 \quad \dots(2)$$

From equation (2), we get

$$\begin{aligned} 2x &= -24 + 4y \\ \Rightarrow x &= \frac{-24 + 4y}{2} \end{aligned}$$

$$\Rightarrow x = -12 + 2y$$

Substituting the value of x in equation (1), we get

$$\begin{aligned} 2 \times (-12 + 2y) + 3y &= 11 \\ \Rightarrow -24 + 7y &= 11 \\ \Rightarrow 7y &= 11 + 24 = 35 \\ \therefore y &= 5 \end{aligned}$$

Putting the value of y in equation (2), we get

$$\begin{aligned} 2x - 4 \times 5 &= -24 \\ \Rightarrow 2x &= -24 + 20 = -4 \\ \Rightarrow x &= -2 \end{aligned}$$

Putting the $x = -2$, $y = 5$ in the equation $y = mx + 3$, we get $5 = m \times (-2) + 3$

$$\begin{aligned} \Rightarrow 2m &= 3 - 5 \\ \Rightarrow 2m &= -2 \\ \Rightarrow m &= -1 \end{aligned}$$

Hence, $x = -2$, $y = 5$ and $m = -1$.

Q.3. Form the pair of linear equations for the following problems and find their solution by substitution method :

- (i) The difference between two numbers is 26 and one number is three times the other. Find them.

- (ii) The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them. **(CBSE 2019)**

- (iii) The coach of a cricket team buys 7 bats and 6 balls for ₹ 3800. Later, she buys 3 bats and 5 balls for ₹ 1750. Find the cost of each bat and each ball.

- (iv) The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is ₹ 105 and for a journey of 15 km, the charge paid is ₹ 155. What are the fixed charges and the charge per km? How much does a person have to pay for travelling a distance of 25 km?

- (v) A fraction becomes $\frac{9}{11}$, if 2 is added to both the numerator and the denominator. If, 3 is added to both the numerator and the denominator it becomes $\frac{5}{6}$. Find the fraction.

- (vi) Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages?

Sol. (i) Let one number be x and other number be y

According to question,

$$x - y = 26 \quad \dots(1)$$

and $x = 3y$

$$\Rightarrow x - 3y = 0 \quad \dots(2)$$

From equation (2), we get

$$x = 3y$$

Substituting the value of x in equation (1), we get

$$3y - y = 26$$

$$\Rightarrow y = 13$$

Putting the value of y in equation (2), we get

$$\begin{aligned} x - 3 \times 13 &= 0 \\ x &= 39 \end{aligned}$$

Hence, the required equations are

$$x - y = 26, x - 3y = 0$$

and $x = 39$, $y = 13$ is the required solution.

Therefore, two numbers are 39 and 13.

- (ii) Let the larger supplementary angle be and the smaller supplementary angle be y°

$$x^\circ + y^\circ = 180^\circ$$

[$\because$ Sum of supplementary angles is 180°]

...(1)

and According to question,

$$x^\circ - y^\circ = 18^\circ \quad \dots(2)$$

From equation (2), we get

$$x^\circ = 18^\circ + y^\circ$$

Substituting the value of x in equation (1), we get

$$\begin{aligned} 18^\circ + y^\circ + y^\circ &= 180^\circ \\ \Rightarrow 2y^\circ &= 180^\circ - 18^\circ \\ \Rightarrow 2y^\circ &= 162^\circ \\ \Rightarrow y^\circ &= \frac{162^\circ}{2} = 81^\circ \end{aligned}$$

Putting the value of y in equation (2), we get

$$\begin{aligned} x^\circ - 81^\circ &= 18^\circ \\ \Rightarrow x^\circ &= 18^\circ + 81^\circ \\ \Rightarrow x^\circ &= 99^\circ \end{aligned}$$

Hence, the required equations are $x^\circ + y^\circ = 180^\circ$, $x^\circ - y^\circ = 18^\circ$

and $x^\circ = 99^\circ$, $y^\circ = 81^\circ$ is the required solution.

Therefore, supplementary angles are 99° and 81° .

(iii) Let cost of 1 bat be ₹ x and cost of 1 ball be ₹ y .

According to question,

$$7x + 6y = 3800 \quad \dots(1)$$

$$\text{and } 3x + 5y = 1750 \quad \dots(2)$$

From equation (2), we get

$$\begin{aligned} 5y &= 1750 - 3x \\ \Rightarrow y &= \frac{1750 - 3x}{5} \end{aligned}$$

Substituting the value of y in equation (1), we get

$$7x + 6 \times \left(\frac{1750 - 3x}{5} \right) = 3800$$

$$\Rightarrow \frac{35x + 10500 - 18x}{5} = 3800$$

$$\Rightarrow \frac{17x + 10500}{5} = 3800$$

$$\Rightarrow 17x = 19000 - 10500 = 8500$$

$$\Rightarrow x = 500$$

Putting the value of x in equation (2), we get

$$\begin{aligned} 3 \times 500 + 5y &= 1750 \\ \Rightarrow 5y &= 1750 - 1500 = 250 \\ \Rightarrow y &= 50 \end{aligned}$$

Hence, the required equations are $7x + 6y = 3800$, $3x + 5y = 1750$

and $x = 500$, $y = 50$ is the required solution.

Therefore, cost of 1 bat = ₹ 500 and cost of 1 ball = ₹ 50

(iv) Let the fixed charge of a taxi be ₹ x and charge per km be ₹ y .

According to question,

$$x + 10y = 105 \quad \dots(1)$$

$$\text{and } x + 15y = 155 \quad \dots(2)$$

From equation (1), we get

$$x = 105 - 10y$$

Substituting the value of x in equation (2), we get

$$\begin{aligned} 105 - 10y + 15y &= 155 \\ \Rightarrow y &= 10 \end{aligned}$$

Putting the value of y in equation (1), we get

$$\begin{aligned} x + 10 \times 10 &= 105 \\ \Rightarrow x &= 105 - 100 = 5 \end{aligned}$$

Amount paid for 25 km = $x + 25y = 5 + 25 \times 10 = 5 + 250 = ₹ 255$

Hence, the equations required are, $x + 10y = 105$, $x + 15y = 155$

and $x = 5$, $y = 10$ is the required solution.

Therefore, Fixed charge = ₹ 5

Charge per km = ₹ 10

Charge for 25 km = ₹ 255

(v) Let the numerator be x and denominator be y .

$$\text{then fraction} = \frac{x}{y}$$

Now, according to question,

$$\frac{x+2}{y+2} = \frac{9}{11}$$

$$\Rightarrow 11x + 22 = 9y + 18 \quad \dots(1)$$

$$\text{and } \frac{x+3}{y+3} = \frac{5}{6}$$

$$\Rightarrow 6x + 18 = 5y + 15 \quad \dots(2)$$

From equation (2), we get

$$6x = -3 + 5y$$

$$\Rightarrow x = \frac{-3 + 5y}{6}$$

Substituting the value of x in equation (1), we get

$$11 \times \left(\frac{-3 + 5y}{6} \right) - 9y = -4$$

$$\Rightarrow \frac{-33 + 55y}{6} - 9y = -4$$

$$\Rightarrow \frac{-33 + y}{6} = -4$$

$$\Rightarrow y = -24 + 33 = 9$$

Putting the value of y in equation (2), we get

$$\begin{aligned} 6x - 5 \times 9 &= -3 \\ \Rightarrow 6x &= -3 + 45 = 42 \\ \Rightarrow x &= 7 \end{aligned}$$

Hence, the equations are $11x - 9y = -4$, $6x - 5y = -3$ and $x = 7, y = 9$ is the required solution.

Therefore, fraction = $\frac{7}{9}$.

(vi) Let the present age of Jacob be x years and present age of his son be y years

After 5 years Jacob's age = $(x + 5)$ years

After 5 years son's age = $(y + 5)$ years

According to question,

$$\begin{aligned} (x + 5) &= 3(y + 5) \\ \Rightarrow x + 5 &= 3y + 15 \\ \Rightarrow x - 3y &= 15 - 5 = 10 \quad \dots(1) \end{aligned}$$

Five years ago Jacob's age = $(x - 5)$ years

Five years ago son's age = $(y - 5)$ years

According to question,

$$\begin{aligned} (x - 5) &= 7(y - 5) \\ \Rightarrow x - 7y &= -35 + 5 = -30 \quad \dots(2) \end{aligned}$$

From equation (1), we get

$$x = 10 + 3y$$

Substituting the value of x in equation (2), we get

$$\begin{aligned} 10 + 3y - 7y &= -30 \\ \Rightarrow y &= 10 \end{aligned}$$

Putting the value of y in equation (1), we get

$$\begin{aligned} x - 3 \times 10 &= 10 \\ \Rightarrow x - 30 &= 10 \\ \Rightarrow x &= 10 + 30 = 40 \end{aligned}$$

Hence, the equations are $x - 3y = 10$, $x - 7y = -30$

and $x = 40, y = 10$ is the required solution.

Therefore, present age of Jacob = 40 years and present age of his son = 10 years.

Text Book Exercise 3.3

Q.1. Solve the following pair of linear equations by the elimination method and the substitution method :

(i) $x + y = 5$ and $2x - 3y = 4$

(ii) $3x + 4y = 10$ and $2x - 2y = 2$

(iii) $3x - 5y - 4 = 0$ and $9x = 2y + 7$

(iv) $\frac{x}{2} + \frac{2y}{3} = -1$ and $x - \frac{y}{3} = 3$.

Sol. (i) By Elimination Method :

The given equations are :

$$x + y = 5 \quad \dots(1)$$

$$2x - 3y = 4 \quad \dots(2)$$

Multiplying equation (1) by 3, we get

$$3x + 3y = 15 \quad \dots(3)$$

Adding equation (3) and equation (2), we get

$$\begin{array}{r} 3x + 3y = 15 \\ 2x - 3y = 4 \\ \hline 5x = 19 \end{array}$$

$$\Rightarrow x = \frac{19}{5}$$

Substituting the value of x in equation (2), we get

$$2 \times \frac{19}{5} - 3y = 4$$

$$\Rightarrow -3y = \frac{4}{1} - \frac{38}{5}$$

$$\Rightarrow -3y = \frac{20 - 38}{5} = -\frac{18}{5}$$

$$\Rightarrow y = \frac{6}{5}$$

Hence, $x = \frac{19}{5}, y = \frac{6}{5}$ is the required solution.

By Substitution Method :

The given equations are :

$$x + y = 5 \quad \dots(1)$$

$$2x - 3y = 4 \quad \dots(2)$$

From equation (1), we get

$$y = 5 - x$$

Substituting the value of y in equation (2), we get

$$\begin{aligned} 2x - 3(5 - x) &= 4 \\ \Rightarrow 5x &= 4 + 15 \end{aligned}$$

$$\Rightarrow x = \frac{19}{5}$$

Putting the value of x in equation (1), we get

$$\frac{19}{5} + y = 5$$

$$\Rightarrow y = 5 - \frac{19}{5} \quad y = \frac{6}{5}$$

Hence, $x = \frac{19}{5}, y = \frac{6}{5}$ is the required solution.

(ii) By Elimination Method :

The given equations are :

$$3x + 4y = 10 \quad \dots(1)$$

$$\begin{aligned}
 2x - 2y &= 2 \\
 \text{or } x - y &= 1 \quad \dots(2) \\
 \text{Multiplying the equation (2) by 3, we get} \\
 3x - 3y &= 3 \quad \dots(3) \\
 \text{Subtracting equation (3) from equation (1), we get} \\
 \begin{array}{r}
 3x + 4y = 10 \\
 3x - 3y = 3 \\
 \hline
 - \quad + \quad - \\
 7y = 7 \\
 \Rightarrow y = 1
 \end{array}
 \end{aligned}$$

Substituting the value of y in equation (2), we get

$$\begin{aligned}
 x - 1 &= 1 \\
 x &= 1 + 1 = 2
 \end{aligned}$$

Hence, $x = 2$, $y = 1$ is the required solution. **Ans.**

By Substitution Method :

The given equations are :

$$3x + 4y = 10 \quad \dots(1)$$

$$2x - 2y = 2$$

$$\text{or } x - y = 1 \quad \dots(2)$$

From equation (2), we get

$$x = 1 + y$$

Substituting the value of x in equation (1), we get

$$\begin{aligned}
 3(1 + y) + 4y &= 10 \\
 \Rightarrow 7y &= 10 - 3 \Rightarrow y = 1
 \end{aligned}$$

Putting the value of y in equation (2), we get

$$x - 1 = 1 \Rightarrow x = 1 + 1 = 2$$

Hence, $x = 2$, $y = 1$ is the required solution.

(iii) By Elimination Method :

The given equations are :

$$3x - 5y - 4 = 0$$

$$\Rightarrow 3x - 5y = 4 \quad \dots(1)$$

$$\text{and } 9x = 2y + 7$$

$$\Rightarrow 9x - 2y = 7 \quad \dots(2)$$

Multiplying equation (1) by 3, we get

$$9x - 15y = 12 \quad \dots(3)$$

Subtracting equation (2) from equation (3), we get

$$\begin{aligned}
 9x - 15y &= 12 \\
 9x - 2y &= 7 \\
 \hline
 - \quad + \quad - \\
 -13y &= 5 \\
 \Rightarrow y &= -\frac{5}{13}
 \end{aligned}$$

Substituting the value of y in equation (1), we get

$$3x - 5 \times \left(-\frac{5}{13}\right) = 4$$

$$\Rightarrow 3x = \frac{4}{1} - \frac{25}{13} \Rightarrow 3x = \frac{27}{13}$$

$$\Rightarrow x = \frac{9}{13}$$

Hence, $x = \frac{9}{13}$, $y = -\frac{5}{13}$ is the required solution.

By Substitution Method :

The given equations are :

$$3x - 5y - 4 = 0$$

$$\Rightarrow 3x - 5y = 4 \quad \dots(1)$$

$$\text{and } 9x = 2y + 7$$

$$\Rightarrow 9x - 2y = 7 \quad \dots(2)$$

From equation (1), we get

$$3x = 4 + 5y$$

$$\Rightarrow x = \frac{4 + 5y}{3}$$

Substituting the value of x in equation (2), we get

$$\begin{aligned}
 \frac{9(4 + 5y)}{3} - 2y &= 7 \\
 \Rightarrow 3(4 + 5y) - 2y &= 7 \\
 \Rightarrow 13y &= 7 - 12 \Rightarrow 13y = -5 \\
 \Rightarrow y &= -\frac{5}{13}
 \end{aligned}$$

Putting the value of y in equation (1), we

$$3x - 5 \times \left(-\frac{5}{13}\right) = 4 \Rightarrow 3x = \frac{4}{1} - \frac{25}{13}$$

$$\Rightarrow 3x = \frac{27}{13} \Rightarrow x = \frac{9}{13}$$

Hence, $x = \frac{9}{13}$, $y = -\frac{5}{13}$ is the required solution.

(iv) By Elimination Method :

The given equations are :

$$\frac{x}{2} + \frac{2y}{3} = -1$$

$$\Rightarrow 3x + 4y = -6 \quad \dots(1)$$

$$\text{and } x - \frac{y}{3} = 3 \Rightarrow \frac{3x - y}{3} = 3$$

$$\Rightarrow 3x - y = 9 \quad \dots(2)$$

Subtracting the equation (2) from equation (1), we get

$$\begin{array}{r} 3x + 4y = -6 \\ - \quad + \quad - \\ \hline 5y = -15 \end{array}$$

$$\Rightarrow y = -3$$

Substituting the value of y in equation (2), we get

$$3x - (-3) = 9$$

$$\Rightarrow 3x + 3 = 9 \Rightarrow 3x = 9 - 3 = 6$$

$$\Rightarrow x = \frac{6}{3} = 2$$

Hence, $x = 2$, $y = -3$ is the required solution.

By Substitution Method :

The given equations are :

$$\frac{x}{2} + \frac{2y}{3} = -1 \Rightarrow \frac{3x + 4y}{6} = -1$$

$$\Rightarrow 3x + 4y = -6 \quad \dots(1)$$

$$\text{and } x - \frac{y}{3} = 3 \Rightarrow \frac{3x - y}{3} = 3$$

$$\Rightarrow 3x - y = 9 \quad \dots(2)$$

From equation (2), we get

$$3x = 9 + y \Rightarrow x = \frac{9 + y}{3}$$

Substituting the value of x in equation (1), we get

$$3 \times \left(\frac{9 + y}{3}\right) + 4y = -6$$

$$\Rightarrow 5y = -6 - 9 = -15$$

$$\Rightarrow y = -3$$

Putting the value of y in equation (2), we get

$$3x - (-3) = 9$$

$$\Rightarrow 3x + 3 = 9 \Rightarrow x = 2$$

Hence, $x = 2$, $y = -3$ is the required solution.

Q.2. Form the pair of linear equations in the following problems and find their solutions (if they exist) by the elimination method.

(i) If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces

to 1. It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction?

(ii) Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?

(iii) The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

(iv) Meena went to a bank to withdraw ₹ 2000. She asked the cashier to give her ₹ 50 and ₹ 100 notes only. Meena got 25 notes in all. Find how many notes of ₹ 50 and ₹ 100 she received?

(v) A lending library has a fixed charge for the first three days and an additional charge for each day thereafter, Sarita paid ₹ 27 for a book kept for seven days, while Susy paid ₹ 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

Sol. (i) Let the numerator be x and denominator be y , then

$$\text{The fraction} = \frac{x}{y}$$

According to question,

$$\frac{x+1}{y-1} = 1$$

$$\Rightarrow x + 1 = y - 1$$

$$\Rightarrow x - y = -2 \quad \dots(1)$$

$$\text{and } \frac{x}{y+1} = \frac{1}{2}$$

$$\Rightarrow 2x = y + 1$$

$$\Rightarrow 2x - y = 1 \quad \dots(2)$$

Subtracting equation (2) from equation (1), we get

$$x - y = -2$$

$$2x - y = 1$$

$$\begin{array}{r} - \quad + \quad - \\ \hline -x = -3 \end{array}$$

$$\Rightarrow x = 3$$

Substituting the value of x in equation (1), we get

$$3 - y = -2$$

$$\Rightarrow -y = -2 - 3$$

$$\Rightarrow y = 5$$

Hence, the equations are $x - y = -2$, $2x - y = 1$ and $x = 3$, $y = 5$ is the required solution.

$$\text{Therefore, fraction} = \frac{3}{5}.$$

(ii) Let the present age of Nuri be x years and present age of Sonu be y years.

Five years ago age of Nuri = $(x - 5)$ years

Five years ago age of Sonu = $(y - 5)$ years

According to question,

$$(x - 5) = 3(y - 5)$$

$$\Rightarrow x - 3y = -15 + 5$$

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$\Rightarrow x - 3y = -10 \quad \dots(1)$
 Ten years later age of Nuri = $(x + 10)$ years

Ten years later age of Sonu = $(y + 10)$ years

According to question,

$$x + 10 = 2(y + 10)$$

$$\Rightarrow x - 2y = 20 - 10$$

$$\Rightarrow x - 2y = 10 \quad \dots(2)$$

Subtracting equation (2) from equation (1), we get

$$x - 3y = -10$$

$$x - 2y = 10$$

$$\begin{array}{r} - \quad + \quad - \\ - \quad y = -20 \end{array}$$

$$\Rightarrow y = 20$$

Substituting the value of y in equation (2), we get

$$x - 2 \times 20 = 10$$

$$\Rightarrow x = 10 + 40 = 50$$

Hence, the equations are :

$$x - 3y = -10, x - 2y = 10$$

and $x = 50, y = 20$ is the required solution.

Therefore, present age of Nuri = 50 years and present age of Sonu = 20 years.

(iii) Let the digit of unit's place be x and digit of ten's place be y

Then required number = $10y + x$

After reversing the digits, digit of unit's place be y and ten's place be x .

The required number after interchange of digits = $10x + y$

According to question,

$$x + y = 9 \quad \dots(1)$$

$$\text{and } 9(10y + x) = 2(10x + y)$$

$$\Rightarrow 90y + 9x = 20x + 2y$$

$$\Rightarrow 11x - 88y = 0$$

$$\Rightarrow x - 8y = 0 \quad \dots(2)$$

Subtracting equation (2) from equation (1), we get

$$x + y = 9$$

$$x - 8y = 0$$

$$\begin{array}{r} - \quad + \quad - \\ 9y = 9 \end{array}$$

$$9y = 9$$

$$y = \frac{9}{9} = 1$$

Substituting the value of y in equation (1), we get

$$x + 1 = 9$$

$$\Rightarrow x = 8.$$

Hence, the equations are $x + y = 9, x - 8y = 0$ and $x = 8, y = 1$ is the required solution.

Therefore, the required number = $10 \times 1 + 8 = 18$.

(iv) Let the number of ₹ 50 notes be x and number of ₹ 100 notes be y .

Total Amount of ₹ 50 notes = $50x$ and total amount of ₹ 100 notes = $100y$

According to question,

$$50x + 100y = 2000$$

$$\Rightarrow x + 2y = 40 \quad \dots(1)$$

$$\text{and } x + y = 25 \quad \dots(2)$$

Subtracting equation (2) from equation (1), we get

$$x + 2y = 40$$

$$x + y = 25$$

$$\begin{array}{r} - \quad - \quad - \\ y = 15 \end{array}$$

$$y = 15$$

Substituting the value of y in equation (2), we get

$$x + 15 = 25$$

$$\Rightarrow x = 10$$

Hence, the equations are $x + 2y = 40, x + y = 25$ and $x = 10, y = 15$ is the required solution.

Therefore, ₹ 50 notes = 10 and ₹ 100 notes = 15.

(v) Let the fixed charge be ₹ x

and charge for each extra day be ₹ y

According to question,

$$x + 4y = 27 \quad \dots(1)$$

$$\text{and } x + 2y = 21 \quad \dots(2)$$

Subtracting equation (2) from equation (1), we get

$$x + 4y = 27$$

$$x + 2y = 21$$

$$\begin{array}{r} - \quad - \quad - \\ 2y = 6 \end{array}$$

$$2y = 6$$

$$\Rightarrow y = 3$$

Substituting the value of y in equation (1), we get

$$x + 4 \times 3 = 27$$

$$\Rightarrow x + 12 = 27$$

$$\Rightarrow x = 15$$

Hence, the equations are $x + 4y = 27, x + 2y = 21$ and $x = 15, y = 3$ is the required solution.

Therefore, fixed charge = ₹ 15

and charge for each extra day = ₹ 3

Textbook Exercise 4.1

Q.1. Check whether the following are quadratic equations :

- (i) $(x + 1)^2 = 2(x - 3)$
 (ii) $x^2 - 2x = (-2)(3 - x)$
 (iii) $(x - 2)(x + 1) = (x - 1)(x + 3)$
 (iv) $(x - 3)(2x + 1) = x(x + 5)$
 (v) $(2x - 1)(x - 3) = (x + 5)(x - 1)$
 (vi) $x^2 + 3x + 1 = (x - 2)^2$
 (vii) $(x + 2)^3 = 2x(x^2 - 1)$
 (viii) $x^3 - 4x^2 - x + 1 = (x - 2)^3$.

Sol. (i) The given equation is

$$\begin{aligned}(x + 1)^2 &= 2(x - 3) \\ \Rightarrow x^2 + 2x + 1 &= 2x - 6 \Rightarrow x^2 + 7 = 0 \\ \Rightarrow 1 \cdot x^2 + 0 \cdot x + 7 &= 0\end{aligned}$$

It is of the form $ax^2 + bx + c = 0$, where $a \neq 0$.

Hence, the given equation is a quadratic equation.

(ii) The given equation is :

$$\begin{aligned}x^2 - 2x &= (-2)(3 - x) \\ \Rightarrow x^2 - 2x &= -6 + 2x \\ \Rightarrow x^2 - 4x + 6 &= 0 \\ \Rightarrow 1 \cdot x^2 + (-4) \cdot x + 6 &= 0\end{aligned}$$

It is of the form $ax^2 + bx + c = 0$, where $a \neq 0$.

Hence, the given equation is a quadratic equation.

(iii) The given equation is :

$$\begin{aligned}(x - 2)(x + 1) &= (x - 1)(x + 3) \\ \Rightarrow x^2 + x - 2x - 2 &= x^2 + 3x - x - 3 \\ \Rightarrow -3x + 1 &= 0 \\ \Rightarrow 0 \cdot x^2 + (-3) \cdot x + 1 &= 0 \\ \therefore a &= 0\end{aligned}$$

$\therefore$ It is not of the form $ax^2 + bx + c = 0$, where $a \neq 0$.

Hence, the given equation is not a quadratic equation.

(iv) The given equation is :

$$\begin{aligned}(x - 3)(2x + 1) &= x(x + 5) \\ \Rightarrow 2x^2 + x - 6x - 3 &= x^2 + 5x \\ \Rightarrow 2x^2 - 5x - 3 - x^2 - 5x &= 0 \\ \Rightarrow x^2 - 10x - 3 &= 0 \\ \Rightarrow 1 \cdot x^2 + (-10) \cdot x + (-3) &= 0\end{aligned}$$

It is of the form $ax^2 + bx + c = 0$, where $a \neq 0$.

Hence, the given equation is a quadratic equation.

(v) The given equation is :

$$\begin{aligned}(2x - 1)(x - 3) &= (x + 5)(x - 1) \\ \Rightarrow 2x^2 - 6x - x + 3 &= x^2 - x + 5x - 5 \\ \Rightarrow 2x^2 - 7x + 3 - x^2 - 4x + 5 &= 0 \\ \Rightarrow x^2 - 11x + 8 &= 0 \\ \Rightarrow 1 \cdot x^2 + (-11) \cdot x + 8 &= 0\end{aligned}$$

It is of the form $ax^2 + bx + c = 0$, where $a \neq 0$.

Hence, the given equation is a quadratic equation.

(vi) The given equation is :

$$\begin{aligned}x^2 + 3x + 1 &= (x - 2)^2 \\ \Rightarrow x^2 + 3x + 1 &= x^2 - 4x + 4 \\ \Rightarrow 7x - 3 &= 0 \Rightarrow 0 \cdot x^2 + 7 \cdot x + (-3) = 0 \\ \therefore a &= 0\end{aligned}$$

$\therefore$ It is not of the form

$$ax^2 + bx + c = 0,$$

Hence, the given equation is not a quadratic equation.

(vii) The given equation is :

$$\begin{aligned}(x + 2)^3 &= 2x(x^2 - 1) \\ \Rightarrow x^3 + 6x^2 + 12x + 8 &= 2x^3 - 2x \\ \Rightarrow -x^3 + 6x^2 + 14x + 8 &= 0\end{aligned}$$

It is not of the form $ax^2 + bx + c = 0$.

Hence, the given equation is not a quadratic equation.

(viii) The given equation is :

$$\begin{aligned}x^3 - 4x^2 - x + 1 &= (x - 2)^3 \\ \Rightarrow x^3 - 4x^2 - x + 1 &= x^3 - 6x^2 + 12x - 8 \\ \Rightarrow 2x^2 - 13x + 9 &= 0 \\ \Rightarrow 2 \cdot x^2 + (-13) \cdot x + 9 &= 0\end{aligned}$$

It is of the form $ax^2 + bx + c = 0$, where $a \neq 0$.

Hence, the given equation is a quadratic equation.

Q.2. Represent the following situations in the form of quadratic equations :

- (i) The area of a rectangular plot is 528 m². The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

- (ii) The product of two consecutive positive integers is 306. We need to find the integers.
- (iii) Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.
- (iv) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

(CBSE 2020)

Sol. (i) Let the breadth of plot be x m.
Then length = $(2x + 1)$ m.

According to question,

$$\begin{aligned} x(2x + 1) &= 528 \\ \Rightarrow 2x^2 + x - 528 &= 0 \end{aligned}$$

Hence, the quadratic equation is $2x^2 + x - 528 = 0$.

(ii) Let the two consecutive integers be x and $(x + 1)$

According to question,

$$\begin{aligned} x(x + 1) &= 306 \Rightarrow x^2 + x = 306 \\ \Rightarrow x^2 + x - 306 &= 0 \end{aligned}$$

Hence, the quadratic equation is $x^2 + x - 306 = 0$.

(iii) Let the Rohan's present age be x years.
Then, his mother's present age be $(x + 26)$ years.

After three years, age of Rohan
= $(x + 3)$ years

After three years, age of his mother
= $x + 26 + 3 = (x + 29)$ years

According to question, $(x + 3)(x + 29) = 360$
 $\Rightarrow x^2 + 29x + 3x + 87 = 360$

$\Rightarrow x^2 + 32x - 273 = 0$
Hence, the quadratic equation is $x^2 + 32x - 273 = 0$.

(iv) Let the speed of the train be x km/h

Time taken to travel 480 km = $\frac{480}{x}$ hours

If speed is 8 km/h less then new speed = $(x - 8)$ km/h

Time taken to travel 480 km = $\frac{480}{x-8}$ hours

According to question, $\frac{480}{x-8} - \frac{480}{x} = 8$

$$\Rightarrow \frac{480x - 480(x-8)}{x(x-8)} = 8$$

$$\begin{aligned} \Rightarrow 480x - 480x + 480 \times 8 &= 3x(x - 8) \\ \Rightarrow 3840 &= 3x^2 - 24x \end{aligned}$$

$$\Rightarrow x^2 - 8x - 1280 = 0$$

Hence, the quadratic equation is

$$x^2 - 8x - 1280 = 0.$$

Textbook Exercise 4.2

Q.1. Find the roots of the following quadratic equations by factorisation :

(i) $x^2 - 3x - 10 = 0$ (ii) $2x^2 + x - 6 = 0$

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$ (iv) $2x^2 - x + \frac{1}{8} = 0$

(v) $100x^2 - 20x + 1 = 0$.

Sol. (i) The given equation is :

$$\begin{aligned} x^2 - 3x - 10 &= 0 \\ \Rightarrow x^2 - 5x + 2x - 10 &= 0 \\ \Rightarrow x(x - 5) + 2(x - 5) &= 0 \\ \Rightarrow (x - 5)(x + 2) &= 0 \\ \Rightarrow x - 5 = 0 \text{ or } x + 2 &= 0 \\ \Rightarrow x = 5 \text{ or } x = -2 \end{aligned}$$

Hence, $x = 5$ and $x = -2$ are the roots of the given quadratic equation.

(ii) The given equation is : $2x^2 + x - 6 = 0$

$$\begin{aligned} \Rightarrow 2x^2 + (4 - 3)x - 6 &= 0 \\ \Rightarrow 2x(x + 2) - 3(x + 2) &= 0 \\ \Rightarrow (x + 2)(2x - 3) &= 0 \\ \Rightarrow x + 2 = 0 \text{ or } 2x - 3 &= 0 \\ \Rightarrow x = -2 \text{ or } x = \frac{3}{2} \end{aligned}$$

Hence, $x = -2$ and $x = \frac{3}{2}$ are the roots of the given quadratic equation.

(iii) The given equation is :

$$\begin{aligned} \sqrt{2}x^2 + 7x + 5\sqrt{2} &= 0 \\ \Rightarrow \sqrt{2}x^2 + (2 + 5)x + 5\sqrt{2} &= 0 \\ \Rightarrow \sqrt{2}x(x + \sqrt{2}) + 5(x + \sqrt{2}) &= 0 \\ \Rightarrow (x + \sqrt{2})(\sqrt{2}x + 5) &= 0 \\ \Rightarrow x + \sqrt{2} = 0 \text{ or } \sqrt{2}x + 5 &= 0 \\ \Rightarrow x = -\sqrt{2} \text{ or } x = -\frac{5}{\sqrt{2}} \end{aligned}$$

Hence, $x = -\sqrt{2}$ and $x = -\frac{5}{\sqrt{2}}$ are the roots of the given quadratic equation.

(iv) The given equation is :

$$2x^2 - x + \frac{1}{8} = 0$$

$$\begin{aligned}
&\Rightarrow 16x^2 - 8x + 1 = 0 \\
&\Rightarrow 16x^2 - 4x - 4x + 1 = 0 \\
&\Rightarrow 4x(4x - 1) - 1(4x - 1) = 0 \\
&\Rightarrow (4x - 1)(4x - 1) = 0 \\
&\Rightarrow 4x - 1 = 0 \text{ or } 4x - 1 = 0 \\
&\Rightarrow x = \frac{1}{4} \text{ or } x = \frac{1}{4}
\end{aligned}$$

Hence, $x = \frac{1}{4}$ and $x = \frac{1}{4}$ are the roots of the given quadratic equation.

(v) The given equation is :

$$\begin{aligned}
&100x^2 - 20x + 1 = 0 \\
&\Rightarrow 100x^2 - (10 + 10)x + 1 = 0 \\
&\Rightarrow 10x(10x - 1) - 1(10x - 1) = 0 \\
&\Rightarrow (10x - 1)(10x - 1) = 0 \\
&\Rightarrow 10x - 1 = 0 \text{ or } 10x - 1 = 0 \\
&\Rightarrow x = \frac{1}{10} \text{ or } x = \frac{1}{10}
\end{aligned}$$

Hence, $x = \frac{1}{10}$ and $x = \frac{1}{10}$ are the roots of the given quadratic equation.

Q.2. Solve the following problems :

(i) John and Jivanti together have 45 marbles. Both of them lost 5 marbles each and the product of the number of marbles they now have is 124. We would like to find out how many marbles they had to start with.

(ii) A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in ₹) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was ₹ 750. We would like to find out the number of toys produced on that day.

Sol. (i) Let the number of marbles John has be x .

Then, the number of marbles Jivanti has = $45 - x$.

The number of marbles left with John, when he lost 5 marbles = $x - 5$

and the number of marbles left with Jivanti, when she lost 5 marbles

$$= 45 - x - 5 = 40 - x$$

According to question,

$$\begin{aligned}
&(x - 5)(40 - x) = 124 \\
&\Rightarrow 40x - x^2 - 200 + 5x = 124 \\
&\Rightarrow -x^2 + 45x - 324 = 0 \\
&\Rightarrow x^2 - (36 + 9)x + 324 = 0
\end{aligned}$$

$$\begin{aligned}
&\Rightarrow x^2 - 36x - 9x + 324 = 0 \\
&\Rightarrow x(x - 36) - 9(x - 36) = 0 \\
&\Rightarrow (x - 36)(x - 9) = 0 \\
&\Rightarrow x - 36 = 0 \text{ or } x - 9 = 0 \\
&\Rightarrow x = 36 \text{ or } x = 9
\end{aligned}$$

Hence, either John had 36 marbles and Jivanti had 9 marbles or vice-versa.

(ii) Let the number of toys produced on that day be x .

Therefore, the cost of production of each toy = ₹ $(55 - x)$

So, the total cost of production on that day = ₹ $x(55 - x)$

According to question,

$$\begin{aligned}
&x(55 - x) = 750 \Rightarrow 55x - x^2 = 750 \\
&\Rightarrow -x^2 + 55x - 750 = 0 \\
&\Rightarrow x^2 - 25x - 30x + 750 = 0 \\
&\Rightarrow x(x - 25) - 30(x - 25) = 0 \\
&\Rightarrow (x - 25)(x - 30) = 0 \\
&\Rightarrow x - 25 = 0 \text{ or } x - 30 = 0 \\
&\Rightarrow x = 25 \text{ or } x = 30
\end{aligned}$$

Hence, the number of toys produced on that day = 25 or 30.

Q.3. Find two numbers whose sum is 27 and product is 182.

Sol. Let 1st number be x , then 2nd number = $27 - x$

According to question, $x(27 - x) = 182$

$$\begin{aligned}
&\Rightarrow 27x - x^2 = 182 \\
&\Rightarrow x^2 - 27x + 182 = 0 \\
&\Rightarrow x^2 - 14x - 13x + 182 = 0 \\
&\Rightarrow x(x - 14) - 13(x - 14) = 0 \\
&\Rightarrow (x - 14)(x - 13) = 0 \\
&\Rightarrow x - 14 = 0 \text{ or } x - 13 = 0 \\
&\Rightarrow x = 14 \text{ or } x = 13
\end{aligned}$$

Hence, the numbers are 14, 13 or 13, 14.

Q.4. Find two consecutive positive integers, sum of whose squares is 365.

Sol. Let the consecutive positive integers be x and $x + 1$.

According to question, $x^2 + (x + 1)^2 = 365$

$$\begin{aligned}
&\Rightarrow x^2 + x^2 + 2x + 1 = 365 \\
&\Rightarrow 2x^2 + 2x - 364 = 0 \\
&\Rightarrow x^2 + x - 182 = 0 \\
&\Rightarrow x^2 + 14x - 13x - 182 = 0 \\
&\Rightarrow x(x + 14) - 13(x + 14) = 0 \\
&\Rightarrow (x + 14)(x - 13) = 0
\end{aligned}$$

$$\Rightarrow x + 14 = 0 \text{ or } x - 13 = 0$$

$$\Rightarrow x = -14 \text{ or } x = 13$$

Since, x is a positive integer, it cannot be negative.

Therefore, $x = 13$.

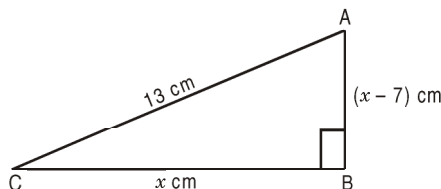
Hence, the consecutive positive integers are 13, 14.

Q.5. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

Sol. Let base of a right triangle be x cm.

then altitude of the right triangle = $(x - 7)$ cm and hypotenuse = 13 cm (given)

In a right $\triangle ABC$,



By Pythagoras theorem, we get

$$AC^2 = BC^2 + AB^2$$

$$\Rightarrow 13^2 = x^2 + (x - 7)^2$$

$$\Rightarrow 169 = x^2 + x^2 - 14x + 49$$

$$\Rightarrow 2x^2 - 14x - 120 = 0$$

$$\Rightarrow x^2 - 7x - 60 = 0$$

$$\Rightarrow x^2 - (12 - 5)x - 60 = 0$$

$$\Rightarrow x(x - 12) + 5(x - 12) = 0$$

$$\Rightarrow (x - 12)(x + 5) = 0$$

$$\Rightarrow x - 12 = 0 \text{ or } x + 5 = 0$$

$$\Rightarrow x = 12 \text{ or } x = -5$$

Since, x is length of base, it cannot be negative. Therefore, $x = 12$.

Hence, the base of triangle = 12 cm and altitude of triangle = $12 - 7 = 5$ cm.

Q.6. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in Rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was ₹ 90. Find the number of articles produced and the cost of each article.

Sol. Let the number of articles produced in a day be x ,

then cost of one article = ₹ $(2x + 3)$

According to question,

$$x(2x + 3) = 90 \Rightarrow 2x^2 + 3x = 90$$

$$\Rightarrow 2x^2 + 3x - 90 = 0$$

$$\Rightarrow 2x^2 + 15x - 12x - 90 = 0$$

$$\Rightarrow x(2x + 15) - 6(2x + 15) = 0$$

$$\Rightarrow (2x + 15)(x - 6) = 0$$

$$\Rightarrow 2x + 15 = 0 \text{ or } x - 6 = 0$$

$$\Rightarrow x = -\frac{15}{2} \text{ or } x = 6$$

Since, x is number of articles. It cannot be negative. Therefore, $x = 6$

Hence, the number of articles = 6

Cost of each article = ₹ $2 \times 6 + 3 = ₹ 15$

Textbook Exercise 4.3

Q.1. Find the nature of the roots of the following quadratic equations. If the real roots exist, find them :

(i) $2x^2 - 3x + 5 = 0$

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

(iii) $2x^2 - 6x + 3 = 0$

Sol. (i) The given equation is :

$$2x^2 - 3x + 5 = 0$$

Comparing the given equation with the general form of quadratic equation, $ax^2 + bx + c = 0$, we get

$$a = 2, b = -3, c = 5$$

$$\therefore D = b^2 - 4ac \Rightarrow D = (-3)^2 - 4 \times 2 \times 5$$

$$\Rightarrow D = 9 - 40 = -31.$$

$$\therefore D < 0$$

$\therefore$ The real roots do not exist.

(ii) The given equation is :

$$3x^2 - 4\sqrt{3}x + 4 = 0$$

Comparing the given equation with the general form of quadratic equation, $ax^2 + bx + c = 0$, we get

$$a = 3, b = -4\sqrt{3}, c = 4$$

$$\therefore D = b^2 - 4ac$$

$$\Rightarrow D = (-4\sqrt{3})^2 - 4 \times 3 \times 4 \Rightarrow D = 48 - 48 = 0$$

$\therefore$ The roots of the equation are real and equal.

$$\therefore x = -\frac{b}{2a} \text{ and } -\frac{b}{2a}$$

$$\Rightarrow x = \frac{4\sqrt{3}}{2 \times 3} \text{ and } \frac{4\sqrt{3}}{2 \times 3}$$

$$\Rightarrow x = \frac{2}{\sqrt{3}} \text{ and } \frac{2}{\sqrt{3}}$$

Hence, the roots of the equation are $\frac{2}{\sqrt{3}}$ and $\frac{2}{\sqrt{3}}$.

(iii) The given equation is :

$$2x^2 - 6x + 3 = 0$$

Comparing the given equation with the general form of quadratic equation $ax^2 + bx + c = 0$, we get

$$a = 2, b = -6, c = 3$$

$$\therefore D = b^2 - 4ac$$

$$\Rightarrow D = (-6)^2 - 4 \times 2 \times 3$$

$$\Rightarrow D = 36 - 24 = 12$$

$$\therefore D > 0$$

The roots of the equation are real and distinct.

$$\therefore x = \frac{-b \pm \sqrt{D}}{2a} \Rightarrow x = \frac{6 \pm \sqrt{12}}{2 \times 2}$$

$$\Rightarrow x = \frac{6 \pm 2\sqrt{3}}{2 \times 2} \Rightarrow x = \frac{3 \pm \sqrt{3}}{2}$$

$$\Rightarrow x = \frac{3 + \sqrt{3}}{2} \quad \text{or} \quad x = \frac{3 - \sqrt{3}}{2}$$

Hence, the roots of the equation are $\frac{3 + \sqrt{3}}{2}$ and $\frac{3 - \sqrt{3}}{2}$.

Q.2. Find the values of k for each of the following quadratic equations, so that they have two equal roots :

(i) $2x^2 + kx + 3 = 0$

(ii) $kx(x - 2) + 6 = 0$.

Sol. (i) The given equation is :

$$2x^2 + kx + 3 = 0$$

Comparing the given equation with general form of the quadratic equation $ax^2 + bx + c = 0$, we get $a = 2, b = k, c = 3$

$$\therefore D = b^2 - 4ac$$

$$\Rightarrow D = k^2 - 4 \times 2 \times 3 \Rightarrow D = k^2 - 24$$

Since, the roots of the given equation are equal.

Therefore, $D = 0$

$$\Rightarrow k^2 - 24 = 0 \Rightarrow k^2 = 24$$

$$\Rightarrow k = \pm\sqrt{24} = \pm 2\sqrt{6}$$

Hence, $k = \pm 2\sqrt{6}$.

(ii) The given equation is :

$$kx(x - 2) + 6 = 0$$

$$\Rightarrow kx^2 - 2kx + 6 = 0$$

Comparing the given equation with the general form of quadratic equation $ax^2 + bx + c = 0$, we get $a = k, b = -2k, c = 6$

$$\therefore D = b^2 - 4ac$$

$$\Rightarrow D = (-2k)^2 - 4 \times k \times 6 \Rightarrow D = 4k^2 - 24k$$

Since the roots of the given equation are equal.

Therefore, $D = 0$

$$\Rightarrow 4k^2 - 24k = 0 \Rightarrow 4k(k - 6) = 0$$

$$\Rightarrow 4k = 0 \text{ or } k - 6 = 0 \Rightarrow k = 0 \text{ or } k = 6$$

But

$$k = 0 \quad [\text{Not possible}]$$

Hence, $k = 6$.

Q.3. Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 m^2 ? If so, find its length and breadth.

Sol. Let the breadth of the rectangular grove be $x \text{ m}$.

Then length of rectangular grove = $2x \text{ m}$

According to question, $2x \times x = 800$

[$\because$ Area of rectangle = Length $\times$ Breadth]

$$\Rightarrow 2x^2 = 800 \Rightarrow x^2 = \frac{800}{2}$$

$$\Rightarrow x^2 = 400 \Rightarrow x = \sqrt{400}$$

$$\Rightarrow x = \pm 20 \quad \text{Reject } x = -20$$

[$\because$ Breadth cannot be negative.]

$$\therefore x = 20$$

Hence, the rectangular grove is possible and its breadth = 20 m and length $2 \times 20 = 40 \text{ m}$.

Q.4. Is the following situation possible? If so, determine their present ages.

The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48 .

Sol. Let the present age of first friend be x years.

Then present age of second friend = $(20 - x)$ years

Four years ago age of first friend = $(x - 4)$ years

Four years ago age of second friend = $(20 - x - 4)$ years = $(16 - x)$ years

According to question, $(x - 4)(16 - x) = 48$

$$\Rightarrow 16x - x^2 - 64 + 4x = 48$$

$$\Rightarrow -x^2 + 20x - 112 = 0$$

$$\Rightarrow x^2 - 20x + 112 = 0$$

Here, $a = 1, b = -20, c = 112$

$$\therefore D = b^2 - 4ac$$

$$\Rightarrow D = (-20)^2 - 4 \times 1 \times 112 \Rightarrow D = 400 - 448$$

$$\Rightarrow D = -48 \quad \therefore D < 0$$

$\therefore$ The equation has no real value of x .

Hence, the situation is not possible. **Ans.**

Q.5. Is it possible to design a rectangular park of perimeter 80 m and area 400 m^2 ? If so, find its length and breadth.

Sol. Let the breadth of rectangular park be x m.

Perimeter of rectangular park
 $= 2(\text{length} + \text{breadth})$

$$\Rightarrow 80 = 2(\text{length} + x)$$

$$\Rightarrow \frac{80}{2} = \text{length} + x$$

$$\Rightarrow 40 = \text{length} + x$$

$$\Rightarrow \text{length} = (40 - x) \text{ m}$$

Now, according to question,

Area of rectangular park

$$= \text{Length} \times \text{Breadth}$$

$$400 = (40 - x) \times x$$

$$\Rightarrow 400 = 40x - x^2$$

$$\Rightarrow x^2 - 40x + 400 = 0$$

Here, $a = 1$, $b = -40$, $c = 400$

$$\therefore D = b^2 - 4ac$$

$$\Rightarrow D = (-40)^2 - 4 \times 1 \times 400$$

$$\Rightarrow D = 1600 - 1600$$

$$\Rightarrow D = 0$$

The equation has real and equal roots.

$$\therefore x = -\frac{b}{2a} \text{ and } -\frac{b}{2a}$$

$$\Rightarrow x = \frac{40}{2 \times 1} \text{ and } \frac{40}{2 \times 1}$$

$$\Rightarrow x = 20 \text{ and } 20$$

Hence, the rectangular park is possible and its length = 20 m and breadth = 20 m.

5

Arithmetic Progressions

Textbook Exercise 5.1

Q.1. In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?

(i) The taxi fare after each km when the fare is ₹ 15 for the first km and ₹ 8 for each additional km.

(ii) The amount of air present in a cylinder

when a vacuum pump removes $\frac{1}{4}$ of the air remaining in the cylinder at a time.

(iii) The cost of digging a well after every metre of digging, when it costs ₹ 150 for the first metre and rises by ₹ 50 for each subsequent metre.

(iv) The amount of money in the account every year, when ₹ 10000 is deposited at compound interest at 8% per annum.

Sol. (i) We have,

Taxi fare for 1 km = ₹ 15

Taxi fare for each additional km = ₹ 8

Let $a_1, a_2, a_3, a_4 \dots$ be the fares for 1 km, 2 km, 3 km, 4 km ... respectively.

$$\therefore a_1 = ₹ 15$$

$$a_2 = 15 + 8 = ₹ 23$$

$$a_3 = 15 + 2 \times 8 = ₹ 31$$

$$a_4 = 15 + 3 \times 8 = ₹ 39 \text{ and so on.}$$

The sequence is 15, 23, 31, 39, ...

$$\text{Now, } a_2 - a_1 = 23 - 15 = 8$$

$$a_3 - a_2 = 31 - 23 = 8$$

$$a_4 - a_3 = 39 - 31 = 8$$

$$\therefore a_2 - a_1 = a_3 - a_2 = a_4 - a_3$$

$\therefore$ The sequence 15, 23, 31, 39, ... forms an AP.

(ii) Let the amount of air present in the cylinder be x units and let the amount of air remained after each pump be $a_2, a_3, a_4, \dots$ respectively.

$$\therefore a_1 = x \text{ units.}$$

$$a_2 = x - \frac{1}{4} \text{ of } x = x - \frac{1}{4}x = \frac{3x}{4}$$

$$a_3 = \frac{3x}{4} - \frac{1}{4} \text{ of } \frac{3x}{4}$$

$$= \frac{3x}{4} - \frac{3x}{16} = \frac{12x - 3x}{16} = \frac{9x}{16}$$

$$a_4 = \frac{9x}{16} - \frac{1}{4} \text{ of } \frac{9x}{16}$$

$$= \frac{9x}{16} - \frac{9x}{64} = \frac{27x}{64} \text{ and so on.}$$

The sequence is $x, \frac{3x}{4}, \frac{9x}{16}, \frac{27x}{64}, \dots$

$$\text{Now, } a_2 - a_1 = \frac{3x}{4} - x = -\frac{x}{4}$$

$$a_3 - a_2 = \frac{9x}{16} - \frac{3x}{4} = \frac{9x - 12x}{16} = -\frac{3x}{4}$$

$$\therefore a_3 - a_2 \neq a_2 - a_1$$

Hence, the sequence $x, \frac{3x}{4}, \frac{9x}{16}, \frac{27x}{64}, \dots$ does not form an AP.

(iii) Let the cost of 1 m, 2 m, 3 m, ... be ₹ a_1 , ₹ a_2 , ₹ a_3 , ... respectively.

$$\begin{aligned}\therefore a_1 &= ₹ 150 \\ a_2 &= 150 + 50 = ₹ 200 \\ a_3 &= 150 + 2 \times 50 \\ &= ₹ 250 \text{ and so on.}\end{aligned}$$

The sequence is 150, 200, 250, ...

$$\text{Now, } a_2 - a_1 = 200 - 150 = ₹ 50$$

$$a_3 - a_2 = 250 - 200 = ₹ 50$$

$$\therefore a_2 - a_1 = a_3 - a_2$$

Hence, the sequence 150, 200, 250, ... forms an A.P.

(iv) Amount deposited in the bank = ₹ 10000

Rate of interest = 8% per annum.

Let the amounts in bank after 1 year, 2 year, 3 years, ... are ₹ a_1 , ₹ a_2 , ₹ a_3 , ... respectively.

$$\therefore a_1 = 10000 \left[1 + \frac{8}{100} \right] \left[\because A = P \left(1 + \frac{r}{100} \right)^n \right]$$

$$= 10000 \times \frac{108}{100} = ₹ 10800$$

$$a_2 = 10000 \left[1 + \frac{8}{100} \right]^2 = 10000 \left[1 + \frac{2}{25} \right]^2$$

$$= 10000 \times \frac{27}{25} \times \frac{27}{25}$$

$$= 16 \times 27 \times 27 = ₹ 11664$$

$$a_3 = 10000 \left[1 + \frac{8}{100} \right]^3 = 10000 \left[1 + \frac{2}{25} \right]^3$$

$$= 10000 \times \frac{27}{25} \times \frac{27}{25} \times \frac{27}{25}$$

$$= \frac{16 \times 27 \times 27 \times 27}{25}$$

$$= ₹ 12597.12 \text{ and so on.}$$

The sequence is 10800, 11664, 12597.12, ...

$$\text{Now, } a_2 - a_1 = 11664 - 10800 = 864$$

$$\begin{aligned}a_3 - a_2 &= 12597.12 - 11664 \\ &= 933.12\end{aligned}$$

$$\therefore a_2 - a_1 \neq a_3 - a_2$$

Hence, the sequence 10800, 11664, 12597.12, ... does not form an AP.

Q.2. Write first four terms of the AP, when the first term a and the common difference d are given as follows :

$$(i) \ a = 10, d = 10 \quad (ii) \ a = -2, d = 0$$

$$(iii) \ a = 4, d = -3 \quad (iv) \ a = -1, d = \frac{1}{2}$$

$$(v) \ a = -1.25, d = -0.25.$$

Sol. (i) We have, $a = 10, d = 10$

First four terms of required AP are

$$\begin{aligned}a_1 &= a = 10 \\ a_2 &= a + d = 10 + 10 = 20 \\ a_3 &= a_2 + d = 20 + 10 = 30 \\ a_4 &= a_3 + d = 30 + 10 = 40\end{aligned}$$

Hence, the first four terms of AP are 10, 20, 30, 40.

(ii) We have, $a = -2, d = 0$

First four terms of required AP are

$$\begin{aligned}a_1 &= a = -2 \\ a_2 &= a_1 + d = -2 + 0 = -2 \\ a_3 &= a_2 + d = -2 + 0 = -2 \\ a_4 &= a_3 + d = -2 + 0 = -2\end{aligned}$$

Hence, the first four terms of AP are $-2, -2, -2, -2$.

(iii) We have, $a = 4, d = -3$

First four terms of required AP are

$$\begin{aligned}a_1 &= a = 4 \\ a_2 &= a_1 + d = 4 - 3 = 1 \\ a_3 &= a_2 + d = 1 - 3 = -2 \\ a_4 &= a_3 + d = -2 - 3 = -5\end{aligned}$$

Hence, the first four terms of AP are 4, 1, $-2, -5$.

$$(iv) \text{ We have, } a = -1, d = \frac{1}{2}$$

First four terms of required AP are

$$\begin{aligned}a_1 &= a = -1 \\ a_2 &= a_1 + d = -1 + \frac{1}{2} = -\frac{1}{2} \\ a_3 &= a_2 + d = -\frac{1}{2} + \frac{1}{2} = 0 \\ a_4 &= a_3 + d = 0 + \frac{1}{2} = \frac{1}{2}\end{aligned}$$

Hence, the first four terms of AP are

$$-1, -\frac{1}{2}, 0, \frac{1}{2}.$$

(v) We have, $a = -1.25, d = -0.25$

First four terms of required AP are

$$\begin{aligned}a_1 &= a = -1.25 \\ a_2 &= a_1 + d \\ &= -1.25 - 0.25 = -1.50 \\ a_3 &= a_2 + d \\ &= -1.50 - 0.25 = -1.75 \\ a_4 &= a_3 + d \\ &= -1.75 - 0.25 = -2.00\end{aligned}$$

Hence, the first four terms of AP are $-1.25, -1.50, -1.75, -2.00$.

Q.3. For the following APs, write the first term and the common difference :

(i) 3, 1, -1, -3, ... (ii) -5, -1, 3, 7, ...

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, \dots$ (iv) 0.6, 1.7, 2.8, 3.9, ...

Sol. (i) The given sequence is 3, 1, -1, -3, ...

Here, $a = 3, d = 1 - 3 = -2$

Hence, $a = 3, d = -2$.

(ii) The given equation is -5, -1, 3, 7, ...

Here, $a = -5$

$$d = -1 - (-5) = -1 + 5 = 4$$

Hence, $a = -5, d = 4$.

(iii) The given sequence is $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, \dots$

Here, $a = \frac{1}{3}$ and $d = \frac{5}{3} - \frac{1}{3} = \frac{4}{3}$

Hence, $a = \frac{1}{3}, d = \frac{4}{3}$.

(iv) The given sequence is 0.6, 1.7, 2.8, 3.9, ...

Here, $a = 0.6$ $d = 1.7 - 0.6 = 1.1$

Hence, $a = 0.6, d = 1.1$.

Q.4. Which of the following are APs ? If they form an AP, find the common difference d and write next three more terms :

(i) 2, 4, 8, 16, ...

(ii) $2, \frac{5}{2}, 3, \frac{7}{2}, \dots$

(iii) -1.2, -3.2, -5.2, -7.2, ...

(iv) -10, -6, -2, 2, ...

(v) $3, 3 + \sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2}, \dots$

(vi) 0.2, 0.22, 0.222, 0.2222, ...

(vii) 0, -4, -8, -12, ...

(viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \dots$

(ix) 1, 3, 9, 27, ...

(x) $a, 2a, 3a, 4a, \dots$

(xi) $a, a^2, a^3, a^4, \dots$

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \dots$

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \dots$

(xiv) $1^2, 3^2, 5^2, 7^2, \dots$

(xv) $1^2, 5^2, 7^2, 73, \dots$

Sol. (i) The given sequence is 2, 4, 8, 16, ...

$$a_2 - a_1 = 4 - 2 = 2$$

$$a_3 - a_2 = 8 - 4 = 4$$

$$\therefore a_2 - a_1 \neq a_3 - a_2$$

$\therefore$ The given sequence is not an AP.

(ii) The given sequence is $2, \frac{5}{2}, 3, \frac{7}{2}, \dots$

$$a_2 - a_1 = \frac{5}{2} - 2 = \frac{5-4}{2} = \frac{1}{2}$$

$$a_3 - a_2 = 3 - \frac{5}{2} = \frac{6-5}{2} = \frac{1}{2}$$

$$a_4 - a_3 = \frac{7}{2} - 3 = \frac{7-6}{2} = \frac{1}{2}$$

$$\therefore a_2 - a_1 = a_3 - a_2 = a_4 - a_3$$

$\therefore$ The given sequence is an AP and

$$d = \frac{5}{2} - 2 = \frac{5-4}{2} = \frac{1}{2}$$

$$\therefore a_5 = a_4 + d = \frac{7}{2} + \frac{1}{2} = \frac{8}{2} = 4$$

$$\text{and } a_6 = a_5 + d = 4 + \frac{1}{2} = \frac{8+1}{2} = \frac{9}{2}$$

$$\text{and } a_7 = a_6 + d = \frac{9}{2} + \frac{1}{2} = \frac{10}{2} = 5$$

Hence, $d = \frac{1}{2}$ and the next three terms are

$4, \frac{9}{2}, 5$.

(iii) The given sequence is -1.2, -3.2, -5.2, -7.2, ...

$$a_2 - a_1 = -3.2 - (-1.2) = -3.2 + 1.2 = -2$$

$$a_3 - a_2 = -5.2 - (-3.2) = -5.2 + 3.2 = -2$$

$$a_4 - a_3 = -7.2 - (-5.2) = -7.2 + 5.2 = -2$$

$$\therefore a_2 - a_1 = a_3 - a_2 = a_4 - a_3$$

$\therefore$ The sequence is an AP and $d = a_2 - a_1$

$$= -3.2 - (-1.2)$$

$$= -3.2 + 1.2 = -2$$

$$\therefore a_5 = a_4 + d$$

$$a_5 = -7.2 - 2 = -9.2$$

$$\text{and } a_6 = a_5 + d$$

$$= -9.2 - 2 = -11.2$$

$$\text{and } a_7 = a_6 + d$$

$$= -11.2 - 2 = -13.2$$

Hence, $d = -2$, the next three terms are -9.2, -11.2, -13.2.

(iv) The given sequence is -10, -6, -2, 2, ...

$$a_2 - a_1 = -6 - (-10) = -6 + 10 = 4$$

$$a_3 - a_2 = -2 - (-6) = -2 + 6 = 4$$

$$a_4 - a_3 = 2 - (-2) = 2 + 2 = 4$$

$$\therefore a_2 - a_1 = a_3 - a_2 = a_4 - a_3$$

∴ The given sequence is an AP.

$$\text{and } d = a_2 - a_1 = -6 - (-10) \\ = -6 + 10 = 4$$

$$\therefore a_5 = a_4 + d = 2 + 4 = 6$$

$$\text{and } a_6 = a_5 + d = 6 + 4 = 10$$

$$\therefore a_7 = a_6 + d = 10 + 4 = 14$$

Hence, $d = 4$, the next three terms are 6, 10, 14.

(v) The given sequence is

$$3, 3 + \sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2}, \dots$$

$$a_2 - a_1 = 3 + \sqrt{2} - 3 = \sqrt{2}$$

$$a_3 - a_2 = 3 + 2\sqrt{2} - 3 - \sqrt{2} = \sqrt{2}$$

$$a_4 - a_3 = 3 + 3\sqrt{2} - 3 - 2\sqrt{2} = \sqrt{2}$$

$$\therefore a_2 - a_1 = a_3 - a_2 = a_4 - a_3$$

∴ The given sequence is an AP and

$$d = a_2 - a_1 = 3 + \sqrt{2} - 3 = \sqrt{2}$$

$$\therefore a_5 = a_4 + d \\ = 3 + 3\sqrt{2} + \sqrt{2} = 3 + 4\sqrt{2}$$

$$\text{and } a_6 = a_5 + d \\ = 3 + 4\sqrt{2} + \sqrt{2} = 3 + 5\sqrt{2}$$

$$\text{and } a_7 = a_6 + d \\ = 3 + 5\sqrt{2} + \sqrt{2} = 3 + 6\sqrt{2}$$

Hence, $d = \sqrt{2}$, the next three terms are $3 + 4\sqrt{2}, 3 + 5\sqrt{2}, 3 + 6\sqrt{2}$.

(vi) The given sequence is 0.2, 0.22, 0.222, 0.2222, ...

$$a_2 - a_1 = 0.22 - 0.2 = 0.02$$

$$a_3 - a_2 = 0.222 - 0.22 = 0.002$$

$$\therefore a_2 - a_1 \neq a_3 - a_2$$

∴ The given sequence is not an AP.

(vii) The given sequence is 0, -4, -8, -12, ...

$$a_2 - a_1 = -4 - 0 = -4$$

$$a_3 - a_2 = -8 - (-4) \\ = -8 + 4 = -4$$

$$a_4 - a_3 = -12 - (-8) \\ = -12 + 8 = -4$$

$$a_2 - a_1 = a_3 - a_2 = a_4 - a_3$$

∴ The given sequence is an AP and

$$d = a_2 - a_1 \\ = -4 - 0 = -4$$

$$\therefore a_5 = a_4 + d \\ = -12 - 4 = -16$$

$$\text{and } a_6 = a_5 + d \\ = -16 - 4 = -20$$

and

$$a_7 = a_6 + d \\ = -20 - 4 = -24$$

Hence, $d = -4$, the next three terms are -16, -20, -24.

(viii) The given sequence is $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \dots$

$$a_2 - a_1 = -\frac{1}{2} - \left(-\frac{1}{2}\right) = -\frac{1}{2} + \frac{1}{2} = 0$$

$$a_3 - a_2 = -\frac{1}{2} - \left(-\frac{1}{2}\right) = -\frac{1}{2} + \frac{1}{2} = 0$$

$$a_4 - a_3 = -\frac{1}{2} - \left(-\frac{1}{2}\right) = -\frac{1}{2} + \frac{1}{2} = 0$$

$$\therefore a_2 - a_1 = a_3 - a_2 = a_4 - a_3$$

∴ The given sequence is an AP and

$$d = a_2 - a_1 = -\frac{1}{2} - \left(-\frac{1}{2}\right) = -\frac{1}{2} + \frac{1}{2} = 0$$

$$\therefore a_5 = a_4 + d = -\frac{1}{2} + 0 = -\frac{1}{2}$$

$$\text{and } a_6 = a_5 + d = -\frac{1}{2} + 0 = -\frac{1}{2}$$

$$\text{and } a_7 = a_6 + d = -\frac{1}{2} + 0 = -\frac{1}{2}$$

Hence, $d = 0$, the next three terms are $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}$.

(ix) The given sequence is 1, 3, 9, 27, ...

$$a_2 - a_1 = 3 - 1 = 2$$

$$a_3 - a_2 = 9 - 3 = 6$$

$$\therefore a_2 - a_1 \neq a_3 - a_2$$

∴ The given sequence is not an AP.

(x) The given sequence is

$$a, 2a, 3a, 4a, \dots$$

$$a_2 - a_1 = 2a - a = a$$

$$a_3 - a_2 = 3a - 2a = a$$

$$a_4 - a_3 = 4a - 3a = a$$

$$\therefore a_2 - a_1 = a_3 - a_2 = a_4 - a_3$$

∴ The given sequence is an AP and $d = a_2 - a_1 = 2a - a = a$

$$\therefore a_5 = a_4 + d = 4a + a = 5a$$

$$\text{and } a_6 = a_5 + d = 5a + a = 6a$$

$$\text{and } a_7 = a_6 + d = 6a + a = 7a$$

Hence, $d = a$, the next three terms are $5a, 6a, 7a$.

(xi) The given sequence is

$$a, a^2, a^3, a^4, \dots$$

$$a_2 - a_1 = a^2 - a$$

$$a_3 - a_2 = a^3 - a^2$$

$$\therefore a_2 - a_1 \neq a_3 - a_2$$

∴ The given sequence is not an AP.

(xii) The given sequence is $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \dots$

$$\begin{aligned} a_2 - a_1 &= \sqrt{8} - \sqrt{2} = 2\sqrt{2} - \sqrt{2} = \sqrt{2} \\ a_3 - a_2 &= \sqrt{18} - \sqrt{8} = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2} \\ a_4 - a_3 &= \sqrt{32} - \sqrt{18} = 4\sqrt{2} - 3\sqrt{2} = \sqrt{2} \\ \therefore a_2 - a_1 &= a_3 - a_2 = a_4 - a_3 \\ \therefore \text{The given sequence is an AP and } d &= a_2 - a_1 \\ &= \sqrt{8} - \sqrt{2} \end{aligned}$$

$$\begin{aligned} &= 2\sqrt{2} - \sqrt{2} = \sqrt{2} \\ \therefore a_5 &= a_4 + d \\ &= \sqrt{32} + \sqrt{2} = 4\sqrt{2} + \sqrt{2} = 5\sqrt{2} = \sqrt{50} \\ \text{and } a_6 &= a_5 + d = \sqrt{50} + \sqrt{2} = 5\sqrt{2} + \sqrt{2} \\ &= 6\sqrt{2} = \sqrt{72} \\ \text{and } a_7 &= a_6 + d = \sqrt{72} + \sqrt{2} = 6\sqrt{2} + \sqrt{2} \\ &= 7\sqrt{2} = \sqrt{98} \end{aligned}$$

Hence, $d = \sqrt{2}$, the next three terms are $\sqrt{50}, \sqrt{72}, \sqrt{98}$.

(xiii) The given sequence is $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \dots$

$$\begin{aligned} a_2 - a_1 &= \sqrt{6} - \sqrt{3} \\ a_3 - a_2 &= \sqrt{9} - \sqrt{6} = 3 - \sqrt{6} \\ \therefore a_2 - a_1 &\neq a_3 - a_2 \\ \therefore \text{The given sequence is not an AP.} \end{aligned}$$

(xiv) The given sequence is $1^2, 3^2, 5^2, 7^2, \dots$

$$\begin{aligned} a_2 - a_1 &= 3^2 - 1^2 = 9 - 1 = 8 \\ a_3 - a_2 &= 5^2 - 3^2 = 25 - 9 = 16 \\ \therefore a_2 - a_1 &\neq a_3 - a_2 \end{aligned}$$

$\therefore$ The given sequence is not an AP.

(xv) The given sequence is $1^2, 5^2, 7^2, 73, \dots$

$$\begin{aligned} a_2 - a_1 &= 5^2 - 1^2 = 25 - 1 = 24 \\ a_3 - a_2 &= 7^2 - 5^2 = 49 - 25 = 24 \\ a_4 - a_3 &= 73 - 7^2 = 73 - 49 = 24 \\ \therefore a_2 - a_1 &= a_3 - a_2 = a_4 - a_3 \end{aligned}$$

$\therefore$ The given sequence is an AP and

$$\begin{aligned} d &= a_2 - a_1 \\ &= 5^2 - 1^2 = 25 - 1 = 24 \end{aligned}$$

$$a_5 = a_4 + d = 73 + 24 = 97$$

$$\text{and } a_6 = a_5 + d = 97 + 24 = 121$$

$$\text{and } a_7 = a_6 + d = 121 + 24 = 145$$

Hence, $d = 24$, the next three terms are 97, 121, 145.

Textbook Exercise 5.2

Q.1. Fill in the blanks in the following table, given that a is the first term, d the common difference and a_n the n^{th} term of the AP :

	a	d	n	a_n
i.	7	3	8	...
ii.	-18	...	10	0
ii.	...	-3	18	-5
iv.	-18.9	2.5	...	3.6
v.	3.5	0	105	...

Sol. (i) We have, $a = 7$, $d = 3$, $n = 8$,
 $a_n = ?$

We know that n^{th} term of an AP is :

$$a_n = a + (n - 1)d$$

$$\Rightarrow a_8 = 7 + (8 - 1) \times 3 \Rightarrow a_8 = 7 + 21 = 28$$

Hence, $a_8 = 28$.

(ii) We have, $a = -18$, $n = 10$, $a_n = 0$,
 $d = ?$

We know that n^{th} term of an AP is

$$a_n = a + (n - 1)d$$

$$\Rightarrow 0 = -18 + (10 - 1) \times d \Rightarrow 9d = 18$$

$$\Rightarrow d = \frac{18}{9} = 2$$

Hence, $d = 2$.

(iii) We have, $d = -3$, $n = 18$, $a_n = -5$,
 $a = ?$

We know that n^{th} term of an AP is

$$a_n = a + (n - 1)d$$

$$\Rightarrow -5 = a + (18 - 1) \times (-3)$$

$$\Rightarrow -5 = a - 51 \Rightarrow a = 51 - 5 = 46$$

Hence, $a = 46$.

(iv) We have, $a = -18.9$, $d = 2.5$,

$n = ?$, $a_n = 3.6$.

We know that n^{th} term of an AP is

$$a_n = a + (n - 1)d$$

$$\Rightarrow 3.6 = -18.9 + (n - 1) \times 2.5$$

$$\Rightarrow 3.6 = -18.9 + 2.5n - 2.5$$

$$\Rightarrow 3.6 = -21.4 + 2.5n$$

$$\Rightarrow 2.5n = 25 \Rightarrow n = \frac{25}{2.5} = 10$$

Hence, $n = 10$.

(v) We have, $a = 3.5$, $d = 0$, $n = 105$,
 $a_n = ?$

We know that n^{th} term of an AP is

$$a_n = a + (n - 1)d$$

$$\begin{aligned}\Rightarrow a_{105} &= 3.5 + (105 - 1) \times 0 \\ \Rightarrow a_{105} &= 3.5 + 0 \\ \Rightarrow a_{105} &= 3.5 \\ \text{Hence, } a_{105} &= 3.5.\end{aligned}$$

Q.2. Choose the correct choice in the following and justify :

(i) 30th term of the AP : 10, 7, 4, ... is
(A) 97 (B) 77 (C) -77 (D) -87

(ii) 11th term of the AP : -3, $-\frac{1}{2}$, 2, ... is

(A) 28 (B) 22 (C) -38 (D) $-48\frac{1}{2}$

Sol. (i) The given sequence of AP is : 10, 7, 4, ...

Here $a = 10$, $d = 7 - 10 = -3$, $n = 30$

We know that n^{th} term of an AP is

$$\begin{aligned}a_n &= a + (n - 1)d \\ \Rightarrow a_{30} &= 10 + (30 - 1) \times (-3) \\ \Rightarrow a_{30} &= 10 - 87 \\ \Rightarrow a_{30} &= -77.\end{aligned}$$

Hence, the correct option is (C).

(ii) The given sequence is : -3, $-\frac{1}{2}$, 2, ...

Here $a = -3$, $d = -\frac{1}{2} - (-3) = -\frac{1}{2} + 3$

$$= \frac{-1 + 6}{2} = \frac{5}{2} \text{ and } n = 11$$

We know that n^{th} term of an AP is

$$\begin{aligned}a_n &= a + (n - 1)d \\ \Rightarrow a_{11} &= -3 + (11 - 1) \times \frac{5}{2} \\ \Rightarrow a_{11} &= -3 + 25 \\ \Rightarrow a_{11} &= 22.\end{aligned}$$

Hence, the correct option is (B).

Q.3. In the following APs, find the missing terms in the boxes.

(i) 2, $\square$, 26 (ii) $\square$, 13, $\square$, 3

(iii) 5, $\square$, $\square$, $9\frac{1}{2}$ (iv) -4, $\square$, $\square$, $\square$, $\square$, 6

(v) $\square$, 38, $\square$, $\square$, $\square$, -22.

Sol. (i) We have, $a = 2$, $a_3 = 26$

$$\begin{aligned}\text{Now, } a_3 &= 26 \\ \Rightarrow a + (3 - 1)d &= 26 \\ \Rightarrow 2 + 2d &= 26 \quad [\because a = 2] \\ \Rightarrow 2d &= 26 - 2 \\ \Rightarrow d &= \frac{24}{2} = 12.\end{aligned}$$

$$\begin{aligned}\text{Then missing term} &= a_2 = a + d \\ &= 2 + 12 = 14\end{aligned}$$

Hence, the missing term = 14.

$$\begin{aligned}\text{(ii) We have, } a_2 &= 13 \\ \Rightarrow a + d &= 13 \quad \dots(1)\end{aligned}$$

$$\begin{aligned}\text{and } a_4 &= 3 \\ \Rightarrow a + (4 - 1)d &= 3 \\ \Rightarrow a + 3d &= 3 \quad \dots(2)\end{aligned}$$

Subtracting equation (2) from equation (1), we get

$$\begin{array}{r} a + d = 13 \\ a + 3d = 3 \\ \hline -2d = 10 \end{array}$$

$$\Rightarrow d = \frac{10}{-2} = -5$$

Putting the value of d in equation (1), we get

$$\begin{aligned}a - 5 &= 13 \Rightarrow a = 18 \\ \Rightarrow 3^{\text{rd}} \text{ missing term } (a_3) &= a + 2d \\ \Rightarrow a_3 &= 18 + 2 \times (-5) \Rightarrow a_3 = 8.\end{aligned}$$

Hence, 1st and 3rd missing terms are 18, 8.

$$\begin{aligned}\text{(iii) We have, } a &= 5 \\ a_4 &= 9\frac{1}{2} \\ \Rightarrow a + (4 - 1)d &= \frac{19}{2} \\ \Rightarrow a + 3d &= \frac{19}{2} \quad \dots(1) \\ \Rightarrow 5 + 3d &= \frac{19}{2} \quad [\because a = 5]\end{aligned}$$

$$\Rightarrow 3d = \frac{19 - 10}{2} \Rightarrow d = \frac{9}{2 \times 3} = \frac{3}{2}$$

So, 2nd missing term (a_2) = $a + d$

$$\Rightarrow a_2 = 5 + \frac{3}{2} \Rightarrow a_2 = \frac{13}{2} = 6\frac{1}{2}$$

and 3rd missing term (a_3) = $a + 2d$

$$\Rightarrow a_3 = 5 + 2 \times \frac{3}{2} \Rightarrow a_3 = 5 + 3 = 8$$

Hence, 2nd and 3rd missing terms are $6\frac{1}{2}$, 8.

$$\begin{aligned}\text{(iv) We have } a &= -4 \text{ and } a_6 = 6 \\ \Rightarrow a + (6 - 1)d &= 6 \\ \Rightarrow a + 5d &= 6 \\ \Rightarrow -4 + 5d &= 6 \quad [\because a = -4] \\ \Rightarrow 5d &= 10 = \frac{10}{5} = 2 \\ \text{So, } 2^{\text{nd}} \text{ missing term } (a_2) &= a + d\end{aligned}$$

$$\begin{aligned}
 &\Rightarrow a_2 = -4 + 2 \\
 &\Rightarrow a_2 = -2 \\
 &\quad 3^{\text{rd}} \text{ missing term } (a_3) = a + 2d \\
 &\Rightarrow a_3 = -4 + 2 \times 2 \\
 &\Rightarrow a_3 = -4 + 4 \\
 &\Rightarrow a_3 = 0 \\
 &\quad 4^{\text{th}} \text{ missing term } (a_4) = a + 3d \\
 &\Rightarrow a_4 = -4 + 3 \times 2 \\
 &\Rightarrow a_4 = -4 + 6 \\
 &\Rightarrow a_4 = 2 \\
 &\quad 5^{\text{th}} \text{ missing term } (a_5) = a + 4d \\
 &\Rightarrow a_5 = -4 + 4 \times 2 \\
 &\Rightarrow a_5 = -4 + 8 \\
 &\Rightarrow a_5 = 4
 \end{aligned}$$

Hence, 2nd, 3rd, 4th, 5th missing terms are -2, 0, 2, 4.

$$\begin{aligned}
 \text{(v) We have,} \quad &a_2 = 38 \\
 &\Rightarrow a + d = 38 \quad \dots(1) \\
 \text{and} \quad &a_6 = -22 \\
 &\Rightarrow a + (6-1)d = -22 \\
 &\Rightarrow a + 5d = -22 \quad \dots(2)
 \end{aligned}$$

Subtracting equation (2) from (1), we get
 $a + d = 38$

$$\begin{array}{r}
 a + 5d = -22 \\
 - \quad - \quad + \\
 \hline
 -4d = 60
 \end{array}$$

$$\Rightarrow d = \frac{60}{-4} = -15$$

Substituting the value of d in equation (1), we get
 $a - 15 = 38$

$$\Rightarrow a = 38 + 15 \Rightarrow a = 53$$

So, 1st missing term = 53

$$\begin{aligned}
 &\quad 3^{\text{rd}} \text{ missing term } (a_3) = a + 2d \\
 &\Rightarrow a_3 = 53 + 2 \times (-15) \Rightarrow a_3 = 53 - 30 \\
 &\Rightarrow a_3 = 23
 \end{aligned}$$

$$\begin{aligned}
 &\quad 4^{\text{th}} \text{ missing term } (a_4) = a + 3d \\
 &\Rightarrow a_4 = 53 + 3 \times (-15) \Rightarrow a_4 = 53 - 45 \\
 &\Rightarrow a_4 = 8
 \end{aligned}$$

$$\begin{aligned}
 &\quad 5^{\text{th}} \text{ missing term } (a_5) = a + 4d \\
 &\Rightarrow a_5 = 53 + 4 \times (-15) \Rightarrow a_5 = 53 - 60 \\
 &\Rightarrow a_5 = -7
 \end{aligned}$$

Hence, 1st, 3rd, 4th, 5th missing terms are : 53, 23, 8, -7.

Q.4. Which term of the AP : 3, 8, 13, 18, ... is 78?

Sol. The given sequence of AP is : 3, 8, 13, 18, ...

Here, $a = 3$, $d = a_2 - a_1 = 8 - 3 = 5$, $a_n = 78$
 We know that n^{th} term of AP is

$$\begin{aligned}
 &a_n = a + (n-1)d. \\
 &\Rightarrow 78 = 3 + (n-1) \times 5 \Rightarrow 78 = 3 + 5n - 5 \\
 &\Rightarrow 78 + 2 = 5n \Rightarrow 5n = 80 \\
 &\Rightarrow n = \frac{80}{5} = 16
 \end{aligned}$$

Hence, 16th term of the given AP is 78.

Q.5. Find the number of terms in each of the following APs :

(i) 7, 13, 19, ..., 205

(ii) $18, 15\frac{1}{2}, 13, \dots, -47$

Sol. (i) The given sequence of AP is : 7, 13, 19, ..., 205

Here, $a = 7$, $d = a_2 - a_1 = 13 - 7 = 6$,
 $a_n = 205$

We know that n^{th} term of AP is

$$\begin{aligned}
 &a_n = a + (n-1)d \\
 &\Rightarrow 205 = 7 + (n-1) \times 6 \\
 &\Rightarrow 205 = 7 + 6n - 6 \\
 &\Rightarrow 6n = 205 - 1 \\
 &\Rightarrow 6n = 204 \Rightarrow n = \frac{204}{6} = 34
 \end{aligned}$$

Hence, the number of terms = 34.

(ii) The given sequence of AP is

$$\begin{aligned}
 &18, 15\frac{1}{2}, 13, \dots, -47 \\
 \text{Here,} \quad &a = 18, d = a_2 - a_1 \\
 &= 15\frac{1}{2} - 18 = \frac{31}{2} - 18 \\
 &= \frac{31 - 36}{2} = -\frac{5}{2} \\
 &\Rightarrow d = -\frac{5}{2} \text{ and } a_n = -47
 \end{aligned}$$

We know that n^{th} term of AP is

$$\begin{aligned}
 &a_n = a + (n-1)d \\
 &\Rightarrow -47 = 18 + (n-1) \times \left(-\frac{5}{2}\right) \\
 &\Rightarrow -47 = 18 - \frac{5}{2}n + \frac{5}{2} \\
 &\Rightarrow -47 = \frac{36 + 5}{2} - \frac{5}{2}n \\
 &\Rightarrow -47 - \frac{41}{2} = -\frac{5}{2}n \\
 &\Rightarrow \frac{-94 - 41}{2} = -\frac{5}{2}n
 \end{aligned}$$

$$\Rightarrow -\frac{135}{2} = -\frac{5}{2}n$$

$$\Rightarrow n = 27$$

Hence, the number of terms = 27.

Q.6. Check whether -150 is a term of the AP : 11, 8, 5, 2, ...

Sol. The given sequence of AP is : 11, 8, 5, 2, ...

Here, $a = 11$

$$d = a_2 - a_1 = 8 - 11 = -3$$

and $a_n = -150$

We know that n^{th} term of AP is

$$a_n = a + (n - 1)d$$

$$\Rightarrow -150 = 11 + (n - 1) \times (-3)$$

$$\Rightarrow -150 = 11 - 3n + 3$$

$$\Rightarrow -150 - 14 = -3n$$

$$\Rightarrow -3n = -164$$

$$\Rightarrow n = \frac{-164}{-3} = 54\frac{2}{3}$$

$\therefore n$ is a natural number, so it is not possible.

Hence, -150 is not a term of given AP.

Q.7. Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.

Sol. We have,

$$a_{11} = 38$$

$$\Rightarrow a + (11 - 1)d = 38$$

$$\Rightarrow a + 10d = 38 \quad \dots(1)$$

$$\text{and } a_{16} = 73$$

$$\Rightarrow a + (16 - 1)d = 73$$

$$\Rightarrow a + 15d = 73 \quad \dots(2)$$

Subtracting equation (2) from equation (1), we get

$$\begin{array}{r} a + 10d = 38 \\ a + 15d = 73 \\ \hline -5d = -35 \end{array}$$

$$\Rightarrow d = \frac{-35}{-5} \Rightarrow d = 7$$

Putting the value of d in equation (1), we get

$$a + 10 \times 7 = 38 \Rightarrow a + 70 = 38$$

$$\Rightarrow a = 38 - 70 = -32$$

We know that n^{th} term of AP is :

$$a_n = a + (n - 1)d$$

$$\Rightarrow a_{31} = -32 + (31 - 1) \times 7$$

$$\Rightarrow a_{31} = -32 + 210$$

$$\Rightarrow a_{31} = 178$$

Hence, 31st term of AP = 178.

Q.8. An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.

Sol. We have,

$$n = 50$$

$$a_3 = 12$$

$$\Rightarrow a + (3 - 1)d = 12$$

$$\Rightarrow a + 2d = 12 \quad \dots(1)$$

$$\text{and last term } (a_{50}) = 106$$

$$\Rightarrow a + (50 - 1)d = 106$$

$$\Rightarrow a + 49d = 106 \quad \dots(2)$$

Subtracting equation (2) from equation (1), we get

$$a + 2d = 12$$

$$a + 49d = 106$$

$$\hline -47d = -94$$

$$\Rightarrow d = \frac{-94}{-47} = 2$$

Putting the value of d in equation (1), we get

$$a + 2 \times 2 = 12 \Rightarrow a + 4 = 12$$

$$\Rightarrow a = 12 - 4 \Rightarrow a = 8$$

We know that n^{th} term of AP is

$$a_n = a + (n - 1)d$$

$$\Rightarrow a_{29} = 8 + (29 - 1) \times 2$$

$$\Rightarrow a_{29} = 8 + 56$$

$$\Rightarrow a_{29} = 64.$$

Hence, 29th term of AP = 64.

Q.9. If the 3rd term and 9th term of an AP are 4 and -8 respectively, which term of this AP is zero?

Sol. We have, $a_3 = 4$

$$\Rightarrow a + (3 - 1)d = 4$$

$$\Rightarrow a + 2d = 4 \quad \dots(1)$$

$$\text{and } a_9 = -8$$

$$\Rightarrow a + (9 - 1)d = -8$$

$$\Rightarrow a + 8d = -8 \quad \dots(2)$$

Subtracting equation (2) from equation (1), we get

$$a + 2d = 4$$

$$a + 8d = -8$$

$$\hline -6d = 12$$

$$\Rightarrow d = \frac{12}{-6}$$

$$\Rightarrow d = -2$$

Putting the value of d in equation (1), we get

$$\begin{aligned} a + 2 \times (-2) &= 4 \\ \Rightarrow a - 4 &= 4 \\ \Rightarrow a &= 8 \end{aligned}$$

We know that n^{th} term of AP is :

$$\begin{aligned} a_n &= a + (n - 1)d \\ \Rightarrow 0 &= 8 + (n - 1) \times (-2) \\ [\because \text{given that } a_n &= 0] \\ \Rightarrow 0 &= 8 - 2n + 2 \\ \Rightarrow 0 &= 10 - 2n \\ \Rightarrow 2n &= 10 \\ \Rightarrow n &= \frac{10}{2} = 5. \end{aligned}$$

Hence, 5th term of AP is zero.

Q.10. The 17th term of an AP exceeds its 10th term by 7. Find the common difference.

$$\begin{aligned} \text{Sol. We have, } a_{17} - a_{10} &= 7 \\ \Rightarrow a + (17 - 1)d - [a + (10 - 1)d] &= 7 \\ \Rightarrow a + 16d - a - 9d &= 7 \\ \Rightarrow d &= \frac{7}{7} = 1 \end{aligned}$$

Hence, common difference = 1.

Q.11. Which term of the AP : 3, 15, 27, 39, ... will be 132 more than its 54th term?

Sol. The given sequence of AP is : 3, 15, 27, 39, ...

$$\begin{aligned} \text{Here, } a &= 3 \\ d &= a_2 - a_1 = 15 - 3 = 12 \end{aligned}$$

Let n^{th} term of given AP be 132 more than its 54th term

$$\begin{aligned} \text{i.e. } a_n &= a_{54} + 132 \\ \Rightarrow a + (n - 1)d &= 3 + (54 - 1) \times 12 + 132 \\ \Rightarrow 3 + (n - 1) \times 12 &= 3 + 636 + 132 \\ \Rightarrow 3 + 12n - 12 &= 771 \\ \Rightarrow 12n - 9 &= 771 \\ \Rightarrow 12n &= 771 + 9 \\ \Rightarrow n &= \frac{780}{12} \\ \Rightarrow n &= 65 \end{aligned}$$

Hence 65th term is 132 more than 54th term.

Q.12. Two AP's have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms?

Sol. Let the two APs, with same common difference d be

$$\begin{aligned} a, a + d, a + 2d, a + 3d, \dots \\ \text{and } b, b + d, b + 2d, b + 3d, \dots \end{aligned}$$

According to question,

$$\begin{aligned} a_{100} - b_{100} &= 100 \\ \Rightarrow a + (100 - 1)d - [b + (100 - 1)d] &= 100 \\ \Rightarrow a + 99d - b - 99d &= 100 \\ \Rightarrow a - b &= 100 \quad \dots(1) \\ \text{Now, } a_{1000} - b_{1000} &= a + (1000 - 1)d \\ &\quad - [b + (1000 - 1)d] \\ \Rightarrow a_{1000} - b_{1000} &= a + 999d - b - 999d \\ a_{1000} - b_{1000} &= a - b \\ \Rightarrow a_{1000} - b_{1000} &= 100 \\ [\text{From equation (1) } a - b &= 100] \end{aligned}$$

Hence, the difference between 1000th terms = 100.

Q.13. How many three-digit numbers are divisible by 7?

Sol. The list of three digit numbers which are divisible by 7 is :

$$105, 112, 119, 126, \dots, 994.$$

Let n numbers be divisible by 7.

It is an AP with $a = 105$, $d = 7$, $a_n = 994$.

We know that n^{th} term of AP is

$$\begin{aligned} a_n &= a + (n - 1)d \\ \Rightarrow 994 &= 105 + (n - 1) \times 7 \\ \Rightarrow 994 &= 105 + 7n - 7 \\ \Rightarrow 7n &= 994 - 98 = 896 \\ \Rightarrow n &= \frac{896}{7} = 128 \end{aligned}$$

Hence, 128 numbers are divisible by 7.

Q.14. How many multiples of 4 lie between 10 and 250?

Sol. Let the multiples of 4 lying between 10 and 250 are : 12, 16, 20, ..., 248.

Let the multiples of 4 between 10 and 250 be n .

It is an AP with $a = 12$, $d = 4$, $a_n = 248$.

We know that n^{th} term of AP is :

$$\begin{aligned} a_n &= a + (n - 1)d \\ \Rightarrow 248 &= 12 + (n - 1) \times 4 \\ \Rightarrow 248 &= 12 + 4n - 4 \\ \Rightarrow 248 &= 8 + 4n \end{aligned}$$

$$\Rightarrow 248 - 8 = 4n \Rightarrow 4n = 240$$

$$\Rightarrow n = \frac{240}{4} = 60$$

Hence, 60 multiples of 4 lie between 10 and 250.

15. For what value of n , are the n^{th} terms of two APs : 63, 65, 67, ... and 3, 10, 17, ... equal?

Sol. The first given AP is : 63, 65, 67, ...

$$\begin{aligned} \text{Here, } a &= 63 & d &= a_2 - a_1 \\ & & &= 65 - 63 = 2 \end{aligned}$$

Let n^{th} terms of given two APs are equal

$$\begin{aligned} a_n &= a + (n - 1)d \\ \Rightarrow a_n &= 63 + (n - 1) \times 2 \\ \Rightarrow a_n &= 61 + 2n \quad \dots(1) \end{aligned}$$

The second given AP is : 3, 10, 17, ...

$$\begin{aligned} \text{Here } a &= 3 \\ d &= a_2 - a_1 = 10 - 3 = 7 \\ a_n &= a + (n - 1)d \\ \Rightarrow a_n &= 3 + 7n - 7 \\ \Rightarrow a_n &= -4 + 7n \quad \dots(2) \end{aligned}$$

Since, n^{th} terms of given two APs are equal, from equations (1) and (2), we get

$$\begin{aligned} 61 + 2n &= -4 + 7n \\ \Rightarrow 7n - 2n &= 61 + 4 \Rightarrow 5n = 65 \\ \Rightarrow n &= \frac{65}{5} = 13 \end{aligned}$$

Hence, 13th terms of given two APs are equal.

Q.16. Determine the AP whose third term is 16 and the 7th term exceeds the 5th term by 12.

$$\begin{aligned} \text{Sol. We have, } a_3 &= 16 \\ a + (3 - 1)d &= 16 \\ \Rightarrow a + 2d &= 16 \quad \dots(1) \\ \text{and given, } 7^{\text{th}} \text{ term} - 5^{\text{th}} \text{ term} &= 12 \\ \Rightarrow a_7 - a_5 &= 12 \\ \Rightarrow a + (7 - 1)d - [a + (5 - 1)d] &= 12 \\ \Rightarrow a + 6d - a - 4d &= 12 \Rightarrow 2d = 12 \\ \Rightarrow d &= \frac{12}{2} = 6 \end{aligned}$$

Putting the value of d in equation (1), we get $a + 2 \times 6 = 16$

$$\Rightarrow a + 12 = 16 \Rightarrow a = 16 - 12 = 4$$

So, the AP is : $a, a + d, a + 2d, a + 3d, \dots$

i.e. 4, 4 + 6, 4 + 2 × 6, 4 + 3 × 6, ...

i.e. 4, 10, 16, 22, ...

Hence, AP is 4, 10, 16, 22, ...

Q.17. Find the 20th term from the last term of the AP : 3, 8, 13, ..., 253.

Sol. For finding the 20th term from the last term of AP. We write the given AP in reverse order as follows : 253, ... 13, 8, 3.

$$\text{Here, } a = 253 \quad d = 3 - 8 = -5$$

We know that n^{th} term of AP is :

$$\begin{aligned} a_n &= a + (n - 1)d \\ \Rightarrow a_{20} &= 253 + (20 - 1) \times (-5) \\ \Rightarrow a_{20} &= 253 + 19 \times (-5) \Rightarrow a_{20} = 253 - 95 \\ \Rightarrow a_{20} &= 158 \end{aligned}$$

Hence, 20th term from the last term of given AP = 158.

Q.18. The sum of the 4th and 8th terms of an AP is 24 and the sum of 6th and 10th terms is 44. Find the first three terms of the AP.

$$\begin{aligned} \text{Sol. We have, } a_4 + a_8 &= 24 \\ \Rightarrow a + (4 - 1)d + a + (8 - 1)d &= 24 \\ \Rightarrow a + 3d + a + 7d &= 24 \\ \Rightarrow 2a + 10d &= 24 \quad \dots(1) \\ \Rightarrow a + 5d &= 12 \text{ and } a_6 + a_{10} = 44 \\ \Rightarrow a + (6 - 1)d + a + (10 - 1)d &= 44 \\ \Rightarrow a + 5d + a + 9d &= 44 \Rightarrow 2a + 14d = 44 \\ \Rightarrow a + 7d &= 22 \quad \dots(2) \end{aligned}$$

Subtracting equation (2) from equation (1), we get

$$\begin{array}{r} a + 5d = 12 \\ a + 7d = 22 \\ \hline -2d = -10 \\ d = \frac{-10}{-2} = 5 \end{array}$$

Putting the value of d in equation (1), we get $a + 5 \times 5 = 12$

$$\Rightarrow a + 25 = 12 \Rightarrow a = 12 - 25 = -13$$

So, the first three terms of AP are : $a, a + d, a + 2d$.

i.e. -13, -13 + 5, -13 + 2 × 5

i.e. -13, -8, -3.

Hence, the first three terms of AP are -13, -8, -3.

Q.19. Subba Rao started work in 1995 at an annual salary of ₹ 5000 and received an increment of ₹ 200 each year. In which year did his income reach ₹ 7000?

Sol. We have,

Increment of each year = ₹ 200

Income in 1995 = ₹ 5000

Income in 1996 = 5000 + 200
= ₹ 5200

Income in 1997 = 5200 + 200
= ₹ 5400 and so on.

Sequence of salary of Subba Rao is : 5000, 5200, 5400, ..., 7000

$$a_2 - a_1 = 5200 - 5000 = 200$$

$$a_3 - a_2 = 5400 - 5200 = 200$$

$$\therefore a_2 - a_1 = a_3 - a_2$$

$\therefore$ The given sequence of salary forms an AP
Here

$$a = 5000$$

$$d = 5200 - 5000 = 200$$

and

$$a_n = 7000$$

We know that n^{th} term of AP is

$$a_n = a + (n - 1)d$$

$$\Rightarrow 7000 = 5000 + (n - 1) \times 200$$

$$\Rightarrow 7000 = 5000 + 200n - 200$$

$$\Rightarrow 200n = 7000 - 4800$$

$$\Rightarrow 200n = 2200$$

$$\Rightarrow n = \frac{2200}{200} \Rightarrow n = 11$$

Hence, in 11th year Subba Rao's income reached ₹ 7000.

Q.20. Ramkali saved ₹ 5 in the first week of a year and then increased her weekly savings by ₹ 1.75. If in the n^{th} week, her weekly savings become ₹ 20.75, Find n .

Sol. We have weekly increment = ₹ 1.75

Saving in first week = ₹ 5.00

Saving in second week

$$= ₹ 5.00 + ₹ 1.75 = ₹ 6.75$$

Saving in third week = 6.75 + 1.75

$$= ₹ 8.50... \text{ and so on.}$$

Sequence of savings of Ramkali is : 5.00, 6.75, 8.50, ..., 20.75.

$$a_2 - a_1 = 6.75 - 5.00 = 1.75$$

$$a_3 - a_2 = 8.50 - 6.75 = 1.75$$

$$\therefore a_2 - a_1 = a_3 - a_2$$

$\therefore$ The sequence of savings of Ramkali forms an AP

Here, $a = 5.00$ $d = 6.75 - 5.00 = 1.75$

$$a_n = 20.75$$

We know that n^{th} term of AP is

$$a_n = a + (n - 1)d$$

$$\Rightarrow 20.75 = 5 + (n - 1) \times 1.75$$

$$\Rightarrow 20.75 = 5 + 1.75n - 1.75$$

$$\Rightarrow 20.75 = 3.25 + 1.75n$$

$$\Rightarrow 1.75n = 20.75 - 3.25$$

$$\Rightarrow 1.75n = 17.50$$

$$\Rightarrow n = \frac{17.50}{1.75} \Rightarrow n = \frac{1750}{175} = 10$$

Hence, in 10th week Ramkali's savings become ₹ 20.75.

Textbook Exercise 5.3

Q.1. Find the sum of the following APs :

(i) 2, 7, 12, ..., to 10 terms.

(ii) -37, -33, -29, ..., to 12 terms.

(iii) 0.6, 1.7, 2.8, ..., to 100 terms.

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \dots$, to 11 terms.

Sol. (i) The given sequence of AP is : 2, 7, 12, ..., to 10 terms

Here, $a = 2$

$$d = a_2 - a_1 = 7 - 2 = 5$$

and $n = 10$

We know that $S_n = \frac{n}{2}[2a + (n - 1)d]$

$$\Rightarrow S_{10} = \frac{10}{2}[2 \times 2 + (10 - 1) \times 5]$$

$$\Rightarrow S_{10} = 5[4 + 45]$$

$$\Rightarrow S_{10} = 5 \times 49 = 245$$

Hence, the sum of 10 terms of given AP = 245.

(ii) The given sequence of AP is : -37, -33, -29, ..., to 12 terms

Here, $a = -37$

$$d = a_2 - a_1 \\ = -33 - (-37)$$

and $n = 12$

We know that $S_n = \frac{n}{2}[2a + (n - 1)d]$

$$\Rightarrow S_{12} = \frac{12}{2}[2 \times (-37) + (12 - 1) \times 4]$$

$$\Rightarrow S_{12} = 6[-74 + 11 \times 4]$$

$$\Rightarrow S_{12} = 6 \times (-30) = -180$$

Hence, the sum of 12 terms of given AP = -180.

(iii) The given sequence of AP is : 0.6, 1.7, 2.8, ..., to 100 terms

Here, $a = 0.6$
 $d = a_2 - a_1$
 $= 1.7 - 0.6 = 1.1$ and $n = 100$

We know that $S_n = \frac{n}{2} [2a + (n - 1)d]$
 $\Rightarrow S_{100} = \frac{100}{2} [2 \times 0.6 + (100 - 1) \times 1.1]$
 $\Rightarrow S_{100} = 50[1.2 + 108.9]$
 $\Rightarrow S_{100} = 50 \times 110.1 = 5505$
Hence, the sum of 100 terms of given AP = 5505.

(iv) The given sequence of AP is :

$\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \dots$, to 11 terms

Here, $a = \frac{1}{15}$
 $d = a_2 - a_1$
 $= \left(\frac{1}{12} - \frac{1}{15} \right)$
 $= \left(\frac{5 - 4}{60} \right) = \frac{1}{60}$
and $n = 11$

We know that $S_n = \frac{n}{2} [2a + (n - 1)d]$
 $\Rightarrow S_{11} = \frac{11}{2} \left[2 \times \frac{1}{15} + (11 - 1) \times \frac{1}{60} \right]$
 $\Rightarrow S_{11} = \frac{11}{2} \left[\frac{2}{15} + 10 \times \frac{1}{60} \right]$
 $\Rightarrow S_{11} = \frac{11}{2} \left[\frac{2}{15} + \frac{1}{6} \right] = \frac{11}{2} \left[\frac{4 + 5}{30} \right]$
 $\Rightarrow S_{11} = \frac{11}{2} \times \frac{9}{30} = \frac{33}{20}$

Hence, the sum of 11 terms of given AP = $\frac{33}{20}$.

Q.2. Find the sum of the following AP's given below:

(i) $7 + 10\frac{1}{2} + 14 + \dots + 84$ [CBSE 2022]

(ii) $34 + 32 + 30 + \dots + 10$

(iii) $-5 + (-8) + (-11) + \dots + (-230)$

Sol. (i) We have, $7 + 10\frac{1}{2} + 14 + \dots + 84$

It is an AP.

Here, $a = 7$ $d = a_2 - a_1$
 $\Rightarrow d = 10\frac{1}{2} - 7 \Rightarrow d = \frac{21 - 14}{2} = \frac{7}{2}$

and $l = a_n = 84 \Rightarrow a + (n - 1)d = 84$
 $\Rightarrow 7 + (n - 1) \times \frac{7}{2} = 84 \Rightarrow \frac{7}{2}n - \frac{7}{2} = 84 - 7$
 $\Rightarrow \frac{7}{2}n - \frac{7}{2} = 77 \Rightarrow \frac{7}{2}n = 77 + \frac{7}{2}$
 $\Rightarrow \frac{7}{2}n = \frac{154 + 7}{2} \Rightarrow n = \frac{161}{7} = 23$

We know that $S_n = \frac{n}{2} [a + l]$

$\Rightarrow S_{23} = \frac{23}{2} [7 + 84] = \frac{23}{2} \times 91$

$\Rightarrow S_{23} = 1046\frac{1}{2}$

Hence, $S_{23} = 1046\frac{1}{2}$.

(ii) We have, $34 + 32 + 30 + \dots + 10$
It is an AP.

Here, $a = 34$ $d = a_2 - a_1$
 $= 32 - 34 = -2$

and $l = a_n = 10$

$\Rightarrow a + (n - 1)d = 10$
 $\Rightarrow 34 + (n - 1) \times (-2) = 10$
 $\Rightarrow -2n + 2 = 10 - 34$
 $\Rightarrow -2n + 2 = -24$
 $\Rightarrow -2n = -24 - 2$
 $\Rightarrow -2n = -26$
 $\Rightarrow n = \frac{-26}{-2} = 13$

We know that $S_n = \frac{n}{2} [a + l]$

$S_{13} = \frac{13}{2} [34 + 10] = \frac{13}{2} \times 44$

$S_{13} = 286$

Hence, $S_{13} = 286$.

(iii) We have,

$-5 + (-8) + (-11) + \dots + (-230)$

It is an AP.

Here, $a = -5$
 $d = a_2 - a_1 = (-8) - (-5)$
 $= -8 + 5 = -3$

and $l = a_n = -230$

$\Rightarrow a + (n - 1)d = -230$
 $\Rightarrow -5 + (n - 1) \times (-3) = -230$
 $\Rightarrow -3n + 3 = -230 + 5$
 $\Rightarrow -3n + 3 = -225$

$$\Rightarrow -3n = -228$$

$$\Rightarrow n = \frac{-228}{-3} = 76$$

We know that $S_n = \frac{n}{2}[a + l]$

$$\Rightarrow S_{76} = \frac{76}{2}[-5 - 230] = 38 \times (-235)$$

$$\Rightarrow S_{76} = -8930$$

Hence, $S_{76} = -8930$.

Q.3. In an AP :

- (i) given $a = 5$, $d = 3$, $a_n = 50$, find n and S_n .
- (ii) given $a = 7$, $a_{13} = 35$, find d and S_{13} .
- (iii) given $a_{12} = 37$, $d = 3$, find a and S_{12} .
- (iv) given $a_3 = 15$, $S_{10} = 125$, find d and a_{10} .
- (v) given $d = 5$, $S_9 = 75$, find a and a_9 .
- (vi) given $a = 2$, $d = 8$, $S_n = 90$, find n and a_n .
- (vii) given $a = 8$, $a_n = 62$, $S_n = 210$, find n and d .
- (viii) given $a_n = 4$, $d = 2$, $S_n = -14$, find n and a .
- (ix) given $a = 3$, $n = 8$, $S_n = 192$, find d .
- (x) given $l = 28$, $S_n = 144$, and there are total 9 terms. Find a .

Sol. (i) We have, $a = 5$, $d = 3$, $a_n = 50$

We know that $a + (n - 1)d = a_n$

$$\Rightarrow 5 + (n - 1) \times 3 = 50$$

$$\Rightarrow 5 + 3n - 3 = 50$$

$$\Rightarrow 3n = 50 - 2 = 48$$

$$\Rightarrow n = \frac{48}{3} = 16$$

We know that $S_n = \frac{n}{2}[2a + (n - 1)d]$

$$\Rightarrow S_{16} = \frac{16}{2}[2 \times 5 + (16 - 1) \times 3]$$

$$\Rightarrow S_{16} = 8[10 + 15 \times 3]$$

$$= 8[10 + 45]$$

$$\Rightarrow S_{16} = 8 \times 55 = 440$$

Hence, $n = 16$, $S_{16} = 440$.

(ii) We have, $a = 7$, $a_{13} = 35$

We know that $a + (n - 1)d = a_n$

$$\Rightarrow 7 + (13 - 1)d = 35 \Rightarrow 7 + 12d = 35$$

$$\Rightarrow 12d = 35 - 7 = 28 \Rightarrow d = \frac{28}{12} = \frac{7}{3}$$

We know that $S_n = \frac{n}{2}[a + l]$

$$[\because l = a_{13} = 35]$$

$$\Rightarrow S_{13} = \frac{13}{2}[7 + 35] = \frac{13}{2} \times 42 = 273$$

Hence, $d = \frac{7}{3}$, $S_{13} = 273$.

(iii) We have, $d = 3$, $a_{12} = 37$

We know that $a + (n - 1)d = a_n$

$$\Rightarrow a + (12 - 1) \times 3 = 37$$

$$\Rightarrow a + 11 \times 3 = 37 \Rightarrow a + 33 = 37$$

$$\Rightarrow a = 37 - 33 = 4$$

We know that $S_n = \frac{n}{2}[a + l]$

$$\Rightarrow S_{12} = \frac{12}{2}[4 + 37] = 6 \times 41$$

$$[\because l = a_{12} = 37]$$

$$S_{12} = 246.$$

Hence, $a = 4$, $S_{12} = 246$.

(iv) We have, $a_3 = 15$, $S_{10} = 125$

Now, $a_3 = 15$

$$\Rightarrow a + (3 - 1)d = 15$$

$$[\because a_n = a + (n - 1)d]$$

$$\Rightarrow a + 2d = 15 \quad \dots(1)$$

and $S_{10} = 125$

$$\Rightarrow \frac{10}{2}[2a + (10 - 1)d] = 125$$

$$\left[\because S_n = \frac{n}{2}[2a + (n - 1)d] \right]$$

$$\Rightarrow 5[2a + 9d] = 125 \Rightarrow 2a + 9d = \frac{125}{5}$$

$$\Rightarrow 2a + 9d = 25 \quad \dots(2)$$

Multiplying equation (1) by 2 and subtracting (2) from it, we get

$$\begin{array}{r} 2a + 4d = 30 \\ 2a + 9d = 25 \\ \hline -5d = 5 \\ \Rightarrow d = \frac{5}{-5} = -1 \end{array}$$

Putting the value of d in equation (1), we get

$$a + 2 \times (-1) = 15 \Rightarrow a - 2 = 15$$

$$\Rightarrow a = 15 + 2 \Rightarrow a = 17$$

Now, $a_{10} = 17 + (10 - 1) \times (-1)$

$$\Rightarrow a_{10} = 17 + 9 \times (-1) \Rightarrow a_{10} = 17 - 9$$

$$\Rightarrow a_{10} = 8$$

Hence, $d = -1$ and $a_{10} = 8$.

(v) We have, $d = 5$, $S_9 = 75$

$$\Rightarrow \frac{9}{2}[2a + (9 - 1) \times 5] = 75$$

$$\left[\because S_n = \frac{n}{2}[2a + (n - 1)d] \right]$$

$$\Rightarrow \frac{9}{2} [2a + 8 \times 5] = 75 \Rightarrow 2a + 40 = \frac{75 \times 2}{9}$$

$$\Rightarrow 2a = \frac{150}{9} - 40 \Rightarrow 2a = \frac{150 - 360}{9}$$

$$\Rightarrow 2a = \frac{-210}{9} \Rightarrow a = \frac{-210}{9 \times 2} = \frac{-70}{6} = \frac{-35}{3}$$

Now, $a_9 = \frac{-35}{3} + (9 - 1) \times 5$
 $[\because a_n = a + (n - 1)d]$

$$\Rightarrow a_9 = \frac{-35}{3} + (8) \times 5 \Rightarrow a_9 = \frac{-35}{3} + 40$$

$$\Rightarrow a_9 = \frac{-35 + 120}{3} = \frac{85}{3} \Rightarrow a = \frac{-35}{3}$$

and $a_9 = \frac{85}{3}$.

(vi) We have, $a = 2$, $d = 8$, $S_n = 90$

$$\Rightarrow \frac{n}{2} [2a + (n - 1)d] = 90$$

$$\Rightarrow \frac{n}{2} [2 \times 2 + (n - 1) \times 8] = 90$$

$$\Rightarrow \frac{n}{2} [4 + 8n - 8] = 90$$

$$\Rightarrow n[8n - 4] = 90 \times 2$$

$$\Rightarrow 8n^2 - 4n = 180 \Rightarrow 2n^2 - n = 45$$

$$\Rightarrow 2n^2 - n - 45 = 0$$

$$\Rightarrow 2n^2 - (10 - 9)n - 45 = 0$$

$$\Rightarrow 2n^2 - 10n + 9n - 45 = 0$$

$$\Rightarrow 2n(n - 5) + 9(n - 5) = 0$$

$$\Rightarrow (n - 5)(2n + 9) = 0$$

$$\Rightarrow n - 5 = 0 \text{ or } 2n + 9 = 0$$

$$\Rightarrow n = 5 \text{ or } n = -\frac{9}{2}$$

Reject $n = -\frac{9}{2}$ $[\because n \neq -\frac{9}{2}]$

So, $n = 5$

Now, $a_n = a + (n - 1)d$

$$\Rightarrow a_5 = 2 + (5 - 1) \times 8 \Rightarrow a_5 = 2 + 4 \times 8$$

$$\Rightarrow a_5 = 2 + 32 \Rightarrow a_5 = 34$$

Hence, $n = 5$ and $a_5 = 34$.

(vii) We have, $a = 8$, $a_n = 62$, $S_n = 210$

Now $S_n = 210$

$$\Rightarrow \frac{n}{2} [a + l] = 210$$

$$\Rightarrow \frac{n}{2} [8 + 62] = 210 \quad [\because l = a_n = 62]$$

$$\Rightarrow \frac{n}{2} \times 70 = 210$$

$$\Rightarrow 35n = 210$$

$$\Rightarrow n = \frac{210}{35} = 6$$

Now, $a_n = 62$

$$\Rightarrow 8 + (6 - 1)d = 62$$

$$[\because a_n = a + (n - 1)d]$$

$$\Rightarrow 8 + 5d = 62 \Rightarrow 5d = 62 - 8$$

$$\Rightarrow 5d = 54 \Rightarrow d = \frac{54}{5}$$

Hence, $n = 6$, $d = \frac{54}{5}$.

(viii) We have, $d = 2$, $S_n = -14$,

$$a_n = 4$$

$$\Rightarrow a + (n - 1) \times 2 = 4$$

$$[\because a_n = a + (n - 1)d]$$

$$\Rightarrow a + 2n - 2 = 4 \Rightarrow a + 2n = 4 + 2$$

$$\Rightarrow a + 2n = 6 \Rightarrow a = 6 - 2n \quad \dots(i)$$

Now, $S_n = -14$

$$\Rightarrow \frac{n}{2} [a + l] = -14$$

$$\Rightarrow n[6 - 2n + 4] = -14 \times 2$$

$$[\text{From (1) Put } a = 6 - 2n]$$

$$\Rightarrow n[10 - 2n] = -28$$

$$[\because l = a_n = 4]$$

$$\Rightarrow 10n - 2n^2 = -28$$

$$\Rightarrow 2n^2 - 10n - 28 = 0$$

$$\Rightarrow n^2 - 5n - 14 = 0$$

$$\Rightarrow n^2 - (7 - 2)n - 14 = 0$$

$$\Rightarrow n^2 - 7n + 2n - 14 = 0$$

$$\Rightarrow n(n - 7) + 2(n - 7) = 0$$

$$\Rightarrow (n - 7)(n + 2) = 0$$

$$\Rightarrow n - 7 = 0 \text{ or } n + 2 = 0$$

$$\Rightarrow n = 7 \text{ or } n = -2$$

Reject $n = -2$ $[\because n \neq -2]$

So, $n = 7$

Substituting the value of n in equation (1), we

get $a = 6 - 2 \times 7$

$$\Rightarrow a = 6 - 14 \Rightarrow a = -8.$$

Hence, $n = 7$, $a = -8$.

(ix) We have, $a = 3$, $n = 8$, $S_n = 192$

i.e. $S_8 = 192$

$$\Rightarrow \frac{8}{2} [2 \times 3 + (8 - 1)d] = 192$$

$$\Rightarrow 4[6 + 7d] = 192 \Rightarrow 6 + 7d = \frac{192}{4}$$

$$\Rightarrow 6 + 7d = 48 \Rightarrow 7d = 48 - 6$$

$$\Rightarrow 7d = 42 \Rightarrow d = \frac{42}{7} = 6$$

Hence, $d = 6$.

(x) We have, $n = 9$, $l = 28$, $S_n = 144$

$$\Rightarrow \frac{n}{2}[a + l] = 144 \Rightarrow \frac{9}{2}[a + 28] = 144$$

$$\Rightarrow a + 28 = \frac{144 \times 2}{9} \Rightarrow a + 28 = 32$$

$$\Rightarrow a = 32 - 28 = 4$$

Hence, $a = 4$.

Q.4. How many terms of the AP : 9, 17, 25, ... must be taken to give a sum of 636?

Sol. The given sequence of AP is : 9, 17, 25, ...

Here, $a = 9$

$$d = a_2 - a_1 = 17 - 9 = 8$$

and $S_n = 636$ (Given)

$$\Rightarrow \frac{n}{2}[2a + (n - 1)d] = 636$$

$$\Rightarrow \frac{n}{2}[2 \times 9 + (n - 1) \times 8] = 636$$

$$\Rightarrow n[18 + 8n - 8] = 636 \times 2$$

$$\Rightarrow n[10 + 8n] = 1272$$

$$\Rightarrow 10n + 8n^2 = 1272$$

$$\Rightarrow 8n^2 + 10n - 1272 = 0$$

$$\Rightarrow 4n^2 + 5n - 636 = 0$$

$$\Rightarrow 4n^2 + (53 - 48)n - 636 = 0$$

$$\Rightarrow 4n^2 + 53n - 48n - 636 = 0$$

$$\Rightarrow n(4n + 53) - 12(4n + 53) = 0$$

$$\Rightarrow (4n + 53)(n - 12) = 0$$

$$\Rightarrow 4n + 53 = 0 \text{ or } n - 12 = 0$$

$$\Rightarrow n = -\frac{53}{4} \text{ or } n = 12$$

Reject, $n = -\frac{53}{4}$

$[\because n \text{ cannot be fraction}]$

So, $n = 12$.

Hence, 12 terms of the given AP must be taken to give a sum of 636.

Q.5. The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Sol. We have, $a = 5$, $l = a_n = 45$, $S_n = 400$

$$\Rightarrow \frac{n}{2}[a + l] = 400 \Rightarrow \frac{n}{2}[5 + 45] = 400$$

$$\Rightarrow n \times 50 = 400 \times 2 \Rightarrow 50n = 800$$

$$\Rightarrow n = \frac{800}{50} = 16$$

Now, $a_n = 45$

$$\Rightarrow a + (n - 1)d = 45 \Rightarrow 5 + (16 - 1)d = 45$$

$$\Rightarrow 5 + 15d = 45 \Rightarrow 15d = 45 - 5$$

$$\Rightarrow 15d = 40 \Rightarrow d = \frac{40}{15} = \frac{8}{3}$$

$$\text{Hence, } n = 16, d = \frac{8}{3}.$$

Q.6. The first and last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Sol. We have, $a = 17$, $d = 9$, $l = a_n = 350$

$$\Rightarrow a + (n - 1)d = 350$$

$$\Rightarrow 17 + (n - 1)9 = 350$$

$$\Rightarrow 17 + 9n - 9 = 350 \Rightarrow 8 + 9n = 350$$

$$\Rightarrow 9n = 350 - 8 \Rightarrow 9n = 342$$

$$\Rightarrow n = \frac{342}{9} = 38$$

$$\text{Now, } S_n = \frac{n}{2}[a + l] \Rightarrow S_{38} = \frac{38}{2}[17 + 350]$$

$$\Rightarrow S_{38} = 19 \times 367 = 6973$$

Hence, the number of terms

= 38 and their sum = 6973.

Q.7. Find the sum of first 22 terms of an AP in which $d = 7$ and 22nd term is 149.

Sol. We have, $d = 7$, $n = 22$, $a_{22} = 149$

$$\Rightarrow a + (22 - 1) \times 7 = 149$$

$$[\because a_n = a + (n - 1)d]$$

$$\Rightarrow a + 21 \times 7 = 149 \Rightarrow a + 147 = 149$$

$$\Rightarrow a = 149 - 147 \Rightarrow a = 2$$

$$\text{Now, } S_n = \frac{n}{2}[a + l] \Rightarrow S_{22} = \frac{22}{2}[2 + 149]$$

$$[\because l = a_n = 149]$$

$$\Rightarrow S_{22} = 11 \times 151 \Rightarrow S_{22} = 1661$$

Hence, $S_{22} = 1661$.

Q.8. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Sol. We have, $n = 51$, $a_2 = 14$, $a_3 = 18$

$$\therefore d = a_3 - a_2$$

$$\Rightarrow d = 18 - 14 \Rightarrow d = 4$$

$$\text{Therefore, } a = a_2 - d = 14 - 4 = 10$$

$$\text{Now, } S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$S_{51} = \frac{51}{2}[2 \times 10 + (51 - 1) \times 4]$$

$$\Rightarrow S_{51} = \frac{51}{2} [20 + 50 \times 4]$$

$$\Rightarrow S_{51} = \frac{51}{2} [20 + 200]$$

$$\Rightarrow S_{51} = \frac{51}{2} [220]$$

$$\Rightarrow S_{51} = 51 \times 110 = 5610$$

Hence, $S_{51} = 5610$.

Q.9. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289. Find the sum of first n terms. **[CBSE 2008C]**

Sol. We have, $S_7 = 49$

$$\Rightarrow \frac{7}{2} [2a + (7 - 1)d] = 49$$

$$\left[\because S_n = \frac{n}{2} \{2a + (n - 1)d\} \right]$$

$$\Rightarrow 2a + 6d = \frac{49 \times 2}{7} \Rightarrow 2a + 6d = 14$$

$$\Rightarrow a + 3d = 7 \quad \dots(1)$$

and $S_{17} = 289$

$$\Rightarrow \frac{17}{2} [2a + (17 - 1)d] = 289$$

$$\Rightarrow 2a + 16d = \frac{289 \times 2}{17} \Rightarrow 2a + 16d = 34$$

$$\Rightarrow a + 8d = 17 \quad \dots(2)$$

Subtracting equation (2) from equation (1), we get

$$\begin{array}{r} a + 3d = 7 \\ a + 8d = 17 \\ \hline -5d = -10 \end{array}$$

$$\Rightarrow d = \frac{-10}{-5} = 2$$

Putting the value of d in equation (1), we get

$$a + 3 \times 2 = 7 \Rightarrow a + 6 = 7$$

$$\Rightarrow a = 7 - 6 = 1$$

Now, $S_n = \frac{n}{2} [2a + (n - 1)d]$

$$\Rightarrow S_n = \frac{n}{2} [2 \times 1 + (n - 1) \times 2]$$

$$\Rightarrow S_n = \frac{n}{2} [2 + 2n - 2] \Rightarrow S_n = \frac{n}{2} [2n]$$

$$\Rightarrow S_n = n^2$$

Hence $S_n = n^2$.

Q.10. Show that $a_1, a_2, \dots, a_n$ form an AP, where a_n is defined as below :

(i) $a_n = 3 + 4n$ (ii) $a_n = 9 - 5n$.

Also find the sum of the first 15 terms in each case.

Sol. (i) We have, $a_n = 3 + 4n$

Putting $n = 1, 2, 3, 4, \dots$ in this equation, we get $a_1 = 3 + 4 \times 1$

$$\Rightarrow a_1 = 7 \quad a_2 = 3 + 4 \times 2$$

$$\Rightarrow a_2 = 3 + 8 \Rightarrow a_2 = 11 \quad a_3 = 3 + 4 \times 3$$

$$\Rightarrow a_3 = 3 + 12 \Rightarrow a_3 = 15$$

$$a_4 = 3 + 4 \times 4$$

$$\Rightarrow a_4 = 3 + 16 \Rightarrow a_4 = 19 \quad \dots \text{and so on.}$$

Therefore, the sequence is : 7, 11, 15, 19, ...

$$a_2 - a_1 = 11 - 7 = 4$$

$$a_3 - a_2 = 15 - 11 = 4$$

$$a_4 - a_3 = 19 - 15 = 4$$

$$\therefore a_2 - a_1 = a_3 - a_2 = a_4 - a_3$$

$\therefore$ The given sequence forms an AP

Proved

Here $a = 7$, $d = a_2 - a_1 = 11 - 7 = 4$,
 $n = 15$

Now, $S_n = \frac{n}{2} [2a + (n - 1)d]$

$$\Rightarrow S_{15} = \frac{15}{2} [2 \times 7 + (15 - 1) \times 4]$$

$$\Rightarrow S_{15} = \frac{15}{2} [14 + 14 \times 4]$$

$$\Rightarrow S_{15} = \frac{15}{2} [14 + 56]$$

$$\Rightarrow S_{15} = \frac{15}{2} \times 70$$

$$\Rightarrow S_{15} = 15 \times 35$$

$$\Rightarrow S_{15} = 525$$

Hence, $S_{15} = 525$.

(ii) We have, $a_n = 9 - 5n$

Putting $n = 1, 2, 3, 4, \dots$ in this equation, we get $a_1 = 9 - 5 \times 1$

$$\Rightarrow a_1 = 9 - 5 \Rightarrow a_1 = 4 \quad a_2 = 9 - 5 \times 2$$

$$\Rightarrow a_2 = 9 - 10 \Rightarrow a_2 = -1 \quad a_3 = 9 - 5 \times 3$$

$$\Rightarrow a_3 = 9 - 15 \Rightarrow a_3 = -6$$

$$a_4 = 9 - 5 \times 4$$

$$\Rightarrow a_4 = 9 - 20 \Rightarrow a_4 = -11$$

Therefore, the sequence is : 4, -1, -6, -11,

...

$$a_2 - a_1 = -1 - 4 = -5$$

$$a_3 - a_2 = -6 - (-1)$$

$$= -6 + 1 = -5$$

$$a_4 - a_3 = -11 - (-6)$$

$$= -11 + 6 = -5$$

$$\therefore a_2 - a_1 = a_3 - a_2 = a_4 - a_3$$

$\therefore$ The given sequence forms an AP.

Proved

Here, $a = 4$, $d = a_2 - a_1 = -1 - 4 = -5$, $n = 15$

$$\begin{aligned}\text{Now, } S_n &= \frac{n}{2} [2a + (n-1)d] \\ \Rightarrow S_{15} &= \frac{15}{2} [2 \times 4 + (15-1) \times (-5)] \\ \Rightarrow S_{15} &= \frac{15}{2} [8 + 14 \times (-5)] \\ \Rightarrow S_{15} &= \frac{15}{2} [8 - 70] \\ \Rightarrow S_{15} &= \frac{15}{2} \times (-62) \\ \Rightarrow S_{15} &= 15 \times (-31) \\ \Rightarrow S_{15} &= -465 \\ \text{Hence, } S_{15} &= -465.\end{aligned}$$

Q.11. If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (i.e., S_1)? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and n^{th} terms.

Sol. We have, $S_n = 4n - n^2$

Putting $n = 1$, We get

$$\begin{aligned}S_1 &= 4 \times 1 - (1)^2 \\ \Rightarrow S_1 &= 4 - 1 \Rightarrow S_1 = 3 \Rightarrow a_1 = 3 \\ \text{Putting } n &= 2, \text{ we get } S_2 = 4 \times 2 - (2)^2 \\ \Rightarrow S_2 &= 8 - 4 = 4 \Rightarrow a_1 + a_2 = 4 \\ \Rightarrow 3 + a_2 &= 4 \Rightarrow a_2 = 4 - 3 = 1 \\ \text{Now, } d &= a_2 - a_1 = 1 - 3 = -2 \\ a_3 &= 3 + (3-1) \times (-2) \quad [\because a_n = a + (n-1)d] \\ \Rightarrow a_3 &= 3 + 2 \times (-2) \Rightarrow a_3 = 3 - 4 \\ \Rightarrow a_3 &= -1 \quad a_{10} = 3 + (10-1) \times (-2) \\ \Rightarrow a_{10} &= 3 + 9 \times (-2) \Rightarrow a_{10} = 3 - 18 = -15 \\ \text{And } a_n &= 3 + (n-1) \times (-2) \\ \Rightarrow a_n &= 3 - 2n + 2 \Rightarrow a_n = 5 - 2n \\ \text{Hence, } a_1 &= 3, S_2 = 4, a_2 = 1, a_3 = -1, a_{10} = -15 \text{ and } a_n = 5 - 2n.\end{aligned}$$

Q.12. Find the sum of the first 40 positive integers divisible by 6.

Sol. The first 40 positive integers divisible by 6 are : 6, 12, 18, 24, ..., 240.

Since, each number is divisible by 6. So it is an AP in which

$a = 6$, $d = a_2 - a_1 = 12 - 6 = 6$, $n = 40$ and $l = 240$

$$\begin{aligned}\text{Now, } S_{40} &= \frac{40}{2} [6 + 240] \quad \left[\because S_n = \frac{n}{2} (a + l) \right] \\ \Rightarrow S_{40} &= 20 \times 246 = 4920\end{aligned}$$

Hence, the sum of the first 40 positive integers divisible by 6 = 4920.

Q.13. Find the sum of the first 15 multiples of 8.

Sol. The first 15 multiples of 8 are 8, 16, 24, 32, ..., 120.

Since each number is multiple of 8, So, it is an AP in which

$a = 8$, $d = a_2 - a_1 = 16 - 8 = 8$, $n = 15$ and $l = 120$

$$\text{Now, } S_{15} = \frac{15}{2} [8 + 120] \quad \left[\because S_n = \frac{n}{2} (a + l) \right]$$

$$\Rightarrow S_{15} = \frac{15}{2} \times 128$$

$$\Rightarrow S_{15} = 15 \times 64 = 960$$

Hence, the sum of the first 15 multiples of 8 = 960.

Q.14. Find the sum of the odd numbers between 0 and 50.

Sol. The odd numbers between 0 and 50 are : 1, 3, 5, 7, ..., 49.

Since, difference between each two consecutive odd numbers is 2. So it is an AP.

In which $a = 1$, $d = a_2 - a_1 = 3 - 1 = 2$,

$l = a_n = 49$

$$\Rightarrow a + (n-1)d = 49 \quad [\because a + (n-1)d = a_n]$$

$$\Rightarrow 1 + (n-1) \times 2 = 49 \Rightarrow 1 + 2n - 2 = 49$$

$$\Rightarrow 2n - 1 = 49 \Rightarrow 2n = 49 + 1 = 50$$

$$\Rightarrow n = \frac{50}{2} = 25$$

$$\text{Now, } S_n = \frac{n}{2} [a + l]$$

$$\Rightarrow S_{25} = \frac{25}{2} [1 + 49] \Rightarrow S_{25} = \frac{25}{2} \times 50$$

$$\Rightarrow S_{25} = 25 \times 25 = 625$$

Hence, the sum of odd numbers between 0 and 50 = 625.

Q.15. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows : ₹ 200 for the first day, ₹ 250 for the second day, ₹ 300 for the third day, etc ..., the penalty for each succeeding day being ₹ 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?

Sol. We have,

Penalty for the first day = ₹ 200

Penalty for the second day = ₹ 250

Penalty for the third day = ₹ 300

So, the sequence of penalty is : 200, 250, 300, ...

Here, $a = 200$

The penalty for each succeeding day is ₹ 50 more than for each the preceding day.

So, $d = 250 - 200 = 50$

and $n = 30$

Now, $S_{30} = \frac{30}{2} [2 \times 200 + (30 - 1) \times 50]$

$$\left[\because S_n = \frac{n}{2} \{2a + (n - 1)d\} \right]$$

$$\Rightarrow S_{30} = 15[400 + 29 \times 50]$$

$$\Rightarrow S_{30} = 15 [400 + 1450]$$

$$\Rightarrow S_{30} = 15 \times 1850 = 27750$$

Hence, the penalty paid for 30 days = ₹ 27750.

Q.16. A sum of ₹ 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is ₹ 20 less than its preceding prize, find the value of each of the prizes.

Sol. Let 1st Prize be ₹ a

Then 2nd prize = ₹ $(a - 20)$

3rd prize = ₹ $(a - 20 - 20)$
 $= (a - 40)$

Thus, the seven prize are a ,

$(a - 20)$, $(a - 40)$, ..., $(a - 120)$

Since, each prize is Rs. 20 less than its preceding prize. So it is an AP.

Here first term $(a) = a$

Common difference $(d) = a_2 - a_1$
 $= a - 20 - a = -20$

Number of prizes (n)

$= 7$ and $a_n = l = a - 120$

Sum of prizes $(S_n) = 700$

Now, $S_7 = 700$

$$\Rightarrow \frac{7}{2} [a + a - 120] = 700$$

$$\left[\because S_n = \frac{n}{2} (a + l) \right]$$

$$\Rightarrow [2a - 120] = \frac{700 \times 2}{7}$$

$$\Rightarrow 2a - 120 = 200$$

$$\Rightarrow 2a = 200 + 120$$

$$\Rightarrow a = \frac{320}{2} = 160$$

So, given prizes are 160, $160 - 20$, $160 - 40$, $160 - 60$, $160 - 80$, $160 - 100$, $160 - 120$, i.e. 160, 140, 120, 100, 80, 60, 40.

Hence, seven prizes are ₹ 160, ₹ 140, ₹ 120, ₹ 100, ₹ 80, ₹ 60, ₹ 40.

Q.17. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g. a section of class I will plant 1 tree, a section of class II will plant 2 trees and so on till class XII. There are three sections of each class. How many trees will be planted by the students?

Sol. Trees planted by three sections of class I = $3 \times 1 = 3$

Trees planted by three sections of Class

$$\text{II} = 3 \times 2 = 6$$

Trees planted by three sections of Class

$$\text{III} = 3 \times 3 = 9$$

Trees planted by three sections of class

$$\text{IV} = 3 \times 4 = 12$$

Trees planted by three section of class

$$\text{XII} = 3 \times 12 = 36$$

$$\begin{array}{ccc} : & & : \\ : & & : \\ : & & : \end{array}$$

$\therefore$ The sequence of trees is : 3, 6, 9, 12, ..., 36

$$a_2 - a_1 = 6 - 3 = 3$$

$$a_3 - a_2 = 9 - 6 = 3$$

$$a_4 - a_3 = 12 - 9 = 3$$

$$\therefore a_2 - a_1 = a_3 - a_2 = a_4 - a_3$$

$\therefore$ The given sequence of trees forms an AP.

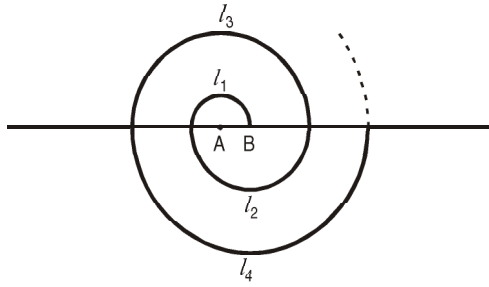
Here $a = 3$, $d = a_2 - a_1 = 6 - 3 = 3$, $a_n = l = 36$, $n = 12$

Total number of trees will be planted
 $= S_n$

$$\begin{aligned} &= \frac{n}{2} [a + l] = \frac{12}{2} [3 + 36] \\ &= 6 \times 39 = 234 \end{aligned}$$

Hence, the total number of trees will be planted = 234.

Q.18. A spiral is made up of successive semi-circles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm ... as shown in figure. What is the total length of such a spiral made up of thirteen consecutive semicircles? (Take $\pi = \frac{22}{7}$)



Sol. Let $l_1, l_2, l_3, l_4, \dots, l_{13}$ be the lengths (Circumferences) of semicircles of radii $r_1 = 0.5$ cm, $r_2 = 1.0$ cm, $r_3 = 1.5$ cm, $r_4 = 2.0$ cm, ... respectively. Then

$$l_1 = \pi r_1 = \pi \times 0.5 = \frac{\pi}{2} \text{ cm} = \frac{22}{14} \text{ cm} = \frac{11}{7} \text{ cm}$$

$$l_2 = \pi r_2 = \pi \times 1.0 = \pi \text{ cm} = \frac{22}{7} \text{ cm}$$

$$l_3 = \pi r_3 = \pi \times 1.5 = 1.5\pi \text{ cm}$$

$$= 1.5 \times \frac{22}{7} = \frac{33}{7} \text{ cm}$$

$$l_4 = \pi r_4 = \pi \times 2.0 = 2\pi \text{ cm} = \frac{44}{7} \text{ cm}$$

$$\vdots$$

$$l_{13} = \pi r_{13} = \pi \times 6.5 = 6.5\pi \text{ cm}$$

$$= 6.5 \times \frac{22}{7} \text{ cm}$$

$$= \frac{143}{7} \text{ cm.}$$

The sequence of length of spiral is :

$$\frac{11}{7}, \frac{22}{7}, \frac{33}{7}, \frac{44}{7}, \dots, \frac{143}{7}$$

$$a_2 - a_1 = \frac{22}{7} - \frac{11}{7} = \frac{11}{7}$$

$$a_3 - a_2 = \frac{33}{7} - \frac{22}{7} = \frac{11}{7}$$

$$a_4 - a_3 = \frac{44}{7} - \frac{33}{7} = \frac{11}{7}$$

$$\therefore a_2 - a_1 = a_3 - a_2 = a_4 - a_3$$

$\therefore$ The sequence of length of spiral forms an AP.

Here $a = \frac{11}{7}$,

$$d = a_2 - a_1 = \frac{22}{7} - \frac{11}{7} = \frac{11}{7},$$

$$l = \frac{143}{7}, \text{ and } n = 13$$

Now, the length of such a spiral made up of thirteen consecutive semicircles = S_{13}

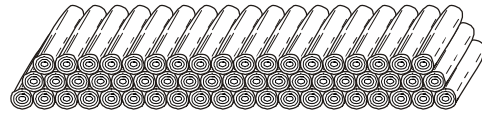
$$\Rightarrow S_{13} = \frac{13}{2} \left[\frac{11}{7} + \frac{143}{7} \right] \left[\because S_n = \frac{n}{2}(a+l) \right]$$

$$\Rightarrow S_{13} = \frac{13}{2} \left[\frac{154}{7} \right] \Rightarrow S_{13} = \frac{13}{2} \times 22$$

$$\Rightarrow S_{13} = 13 \times 11 = 143$$

Hence, the length of a spiral made up of thirteen consecutive semicircles = 143 cm.

Q.19. 200 logs are stacked in the following manner : 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on (see in figure). In how many rows are the 200 logs placed and how many logs are in the top row?



Sol. Sequence of logs row-wise is as follows:

$$20, 19, 18, 17, \dots$$

$$a_2 - a_1 = 19 - 20 = -1$$

$$a_3 - a_2 = 18 - 19 = -1$$

$$a_4 - a_3 = 17 - 18 = -1$$

$$\therefore a_2 - a_1 = a_3 - a_2 = a_4 - a_3$$

$\therefore$ Sequence of logs row-wise forms an AP.

Here, $a = 20$, $d = a_2 - a_1 = 19 - 20 = -1$, $S_n = 200$

$$\Rightarrow \frac{n}{2} [2a + (n-1)d] = 200$$

$$\Rightarrow n[2 \times 20 + (n-1) \times (-1)] = 200 \times 2$$

$$\Rightarrow n[40 - n + 1] = 400$$

$$\Rightarrow n[41 - n] = 400$$

$$\Rightarrow 41n - n^2 = 400$$

$$\Rightarrow n^2 - 41n + 400 = 0$$

$$\Rightarrow n^2 - 25n - 16n + 400 = 0$$

$$\Rightarrow n(n-25) - 16(n-25) = 0$$

$$\Rightarrow (n-25)(n-16) = 0$$

$$\Rightarrow n - 25 = 0 \text{ or } n - 16 = 0$$

$$\Rightarrow n = 25 \text{ or } n = 16$$

If $n = 25$

Number of logs in top row

$$= a_{25} = 20 + (25-1) \times (-1)$$

$$[\because a_n = a + (n-1)d]$$

$$= 20 - 24 = -4$$

-4 is not meaningful

So, we reject $n = 25$.

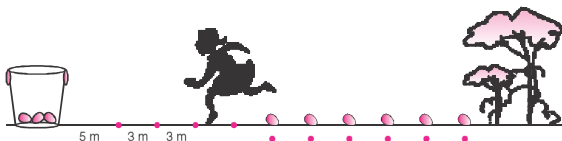
$$\therefore n = 16$$

∴ Number of logs in top row

$$= a_{16} = 20 + (16 - 1) \times (-1) \\ = 20 - 15 = 5$$

Hence, the number of rows = 16 and the number of logs in top row = 5.

Q.20. In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line (see figure).



A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are dropped in the bucket. What is the total distance the competitor has to run?

Sol. Distance run to pick up the 1st potato
 $= 2 \times 5 = 10$ m

Distance run to pick up the 2nd potato
 $= 2 \times (5 + 3) = 16$ m

Distance run to pick up the 3rd potato
 $= 2 \times (5 + 3 + 3) = 22$ m

Distance run to pick up the 4th potato
 $= 2 \times (5 + 3 + 3 + 3) = 28$ m

The sequence of distances is 10, 16, 22, 28.

$$a_2 - a_1 = 16 - 10 = 6$$

$$a_3 - a_2 = 22 - 16 = 6$$

$$a_4 - a_3 = 28 - 22 = 6$$

$$\therefore a_2 - a_1 = a_3 - a_2 = a_4 - a_3$$

∴ The sequence forms an AP.

Here, $a = 10$, $d = a_2 - a_1 = 16 - 10 = 6$,

$n = 10$

Now, total distance run by a competitor = S_{10}

$$= \frac{10}{2} [2 \times 10 + (10 - 1) \times 6]$$

$$\left[\because S_n = \frac{n}{2} \{2a + (n - 1)d\} \right]$$

$$= 5[20 + 9 \times 6]$$

$$= 5[20 + 54] = 5[74] = 370 \text{ m}$$

Hence, the total distance run by competitor = 370 m.

Textbook Exercise 5.4

(Optional)*

* These exercises are not from the examination point of view.

Q.1. Which term of the AP : 121, 117, 113, ... is its first negative term?

Sol. The given sequence of AP is 121, 117, 113, ...

Here

$$a = 121$$

$$d = a_2 - a_1$$

$$= 117 - 121 = -4$$

Let n^{th} term of AP be the first negative term, then

$$a_n = a + (n - 1)d < 0$$

$$\Rightarrow a + (n - 1)d < 0$$

$$\Rightarrow 121 + (n - 1) \times (-4) < 0$$

$$\Rightarrow 121 - 4n + 4 < 0$$

$$\Rightarrow 125 - 4n < 0 \Rightarrow 125 < 4n$$

$$\Rightarrow 4n > 125 \Rightarrow n > \frac{125}{4}$$

$$\Rightarrow n > 31.25$$

Hence, 32th term of the AP is the first negative term.

Q.2. The sum of the third and the seventh terms of an AP is 6 and their product is 8. Find the sum of first sixteen terms of the AP.

Sol. Let a be the first term and d be the common difference of AP. Then, According to question :

$$a_3 + a_7 = 6$$

$$\Rightarrow a + (3 - 1)d + a + (7 - 1)d = 6$$

$$\Rightarrow a + 2d + a + 6d = 6$$

$$\Rightarrow 2a + 8d = 6$$

$$\Rightarrow a + 4d = 3$$

$$\Rightarrow a = 3 - 4d \quad \dots(1)$$

And

$$a_3 \times a_7 = 8$$

$$\Rightarrow (a + 2d)(a + 6d) = 8$$

$$\Rightarrow (3 - 4d + 2d)(3 - 4d + 6d) = 8$$

[From equation (1) put $a = 3 - 4d$]

$$\Rightarrow (3 - 2d)(3 + 2d) = 8$$

$$\Rightarrow 9 - 4d^2 = 8$$

$$\Rightarrow 4d^2 = 9 - 8$$

$$\Rightarrow 4d^2 = 1$$

$$\Rightarrow d^2 = \frac{1}{4} = \pm \frac{1}{2}$$

Case I : If

$$d = \frac{1}{2}$$

Putting the value of d in equation (1), we get

$$a = 3 - 4 \times \frac{1}{2} \Rightarrow a = 3 - 2 \Rightarrow a = 1$$

$$\text{Now } S_{16} = \frac{16}{2} \left[2 \times 1 + (16 - 1) \times \frac{1}{2} \right]$$

$$\Rightarrow S_{16} = 8 \left[2 + \frac{15}{2} \right] \Rightarrow S_{16} = 8 \times \frac{19}{2} = 76$$

Case II : If $d = -\frac{1}{2}$

Putting the value of d in equation (1), we get

$$\begin{aligned} a &= 3 - 4 \times \left(-\frac{1}{2}\right) \\ \Rightarrow a &= 3 + 2 = 5 \end{aligned}$$

$$\text{Now, } S_{16} = \frac{16}{2} \left[2 \times 5 + (16 - 1) \times \left(-\frac{1}{2}\right) \right]$$

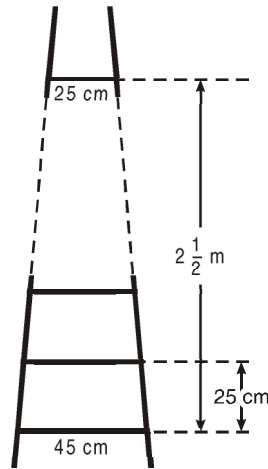
$$\Rightarrow S_{16} = 8 \left[10 - \frac{15}{2} \right]$$

$$\Rightarrow S_{16} = 8 \times \frac{5}{2} = 20$$

Hence, $S_{16} = 76$ or $S_{16} = 20$.

Q.3. A ladder has rungs 25 cm apart (See figure). The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top.

If the top and the bottom rungs are $2\frac{1}{2}$ m apart, what is the length of the wood required for the rungs?



Sol. We have,

Gap between two consecutive rungs = 25 cm
Distance between bottom and top rungs

$$= 2\frac{1}{2} \text{ m} = 250 \text{ cm}$$

$$\text{Number of rungs} = \frac{250}{25} + 1 = 11$$

Since, rungs are decreasing uniformly in length from 45 cm at the bottom to 25 cm at

the top.

Therefore, the lengths of rungs form an AP in which $a = 45$, $a_{11} = 25$

$\therefore$ Required length of wood for rungs

$$= S_{11}$$

$$= \frac{11}{2} (45 + 25) \left[\because S_n = \frac{n}{2} (a + l) \right]$$

$$= \frac{11}{2} \times 70$$

$$= 385 \text{ cm or } 3.85 \text{ m}$$

Hence, required length of wood for rungs = 3.85 m.

Q.4. The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to sum of numbers of the houses following it. Find this value of x .

Sol. Let there be a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it *i.e.*:

House No. 1, 2, 3, ..., $(x - 1)$, x ,

$(x + 1)$, ..., 49.

$$\text{Now, } S_{x-1} = S_{49} - S_x$$

$$\Rightarrow \frac{x-1}{2} [1 + (x-1)] = \frac{49}{2} [1 + 49] - \frac{x}{2} [1 + x]$$

$$\left[S_n = \frac{n}{2} (a + l) \right]$$

$$\Rightarrow \frac{x-1}{2} (x) = \frac{49}{2} \times 50 - \frac{x}{2} [1 + x]$$

$$\Rightarrow \frac{x^2 - x}{2} + \frac{x^2 + x}{2} = 49 \times 25$$

$$\Rightarrow \frac{x^2 - x + x^2 + x}{2} = 1225$$

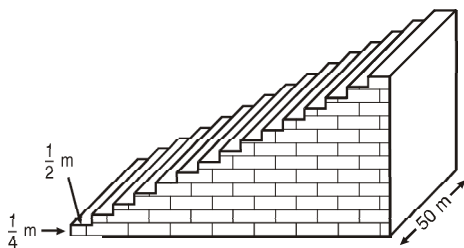
$$\Rightarrow \frac{2x^2}{2} = 1225 \Rightarrow x^2 = 1225$$

$$\Rightarrow x = \sqrt{1225} \Rightarrow x = \pm 35.$$

Since, x can't be negative. Therefore, $x = 35$.

Q.5. A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete.

Each step has a rise of $\frac{1}{4}$ m and a tread of $\frac{1}{2}$ m (see figure). Calculate the total volume of concrete required to build the terrace.



Sol. We have, Length and breadth of each step are 50 m and $\frac{1}{2}$ m respectively.

Height of first step = $\frac{1}{4}$ m

Height of second step = $\frac{1}{4} + \frac{1}{4} = \frac{2}{4}$ m

Height of third step = $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$ m and so on.

Volume of concrete to build the first step

$$= 50 \times \frac{1}{2} \times \frac{1}{4} = \frac{25}{4} \text{ m}^3$$

[$\because$ Volume of cuboidal step = $L \times B \times H$]

Volume of concrete to build the second step

$$= 50 \times \frac{1}{2} \times \frac{2}{4} \\ = \frac{50}{4} \text{ m}^3$$

Volume of concrete to build the third step

$$= 50 \times \frac{1}{2} \times \frac{3}{4} \\ = \frac{75}{4} \text{ m}^3 \text{ and so on.}$$

The sequence of volume of concrete of steps are : $\frac{25}{4}, \frac{50}{4}, \frac{75}{4}, \dots$

$$a_2 - a_1 = \frac{50}{4} - \frac{25}{4} = \frac{25}{4}$$

$$a_3 - a_2 = \frac{75}{4} - \frac{50}{4} = \frac{25}{4}$$

$$\therefore a_3 - a_2 = a_2 - a_1$$

$\therefore$ The sequence forms an AP in which $a = \frac{25}{4}$

and $d = \frac{25}{4}$.

Now, total volume of concrete

= Sum of volumes of concrete to build 15 steps
= S_{15}

$$= \frac{15}{2} \left[2 \times \frac{25}{4} + (15 - 1) \times \frac{25}{4} \right]$$

$$= \frac{15}{2} \left[\frac{25}{2} + 14 \times \frac{25}{4} \right] = \frac{15}{2} \left[\frac{25}{2} + \frac{175}{2} \right]$$

$$= \frac{15}{2} \times \frac{200}{2} = 15 \times 50 = 750 \text{ m}^3$$

Hence, the total volume of concrete
= 750 m^3 .

6

Triangles

Textbook Exercise 6.1

Q.1. Fill in the blanks using the correct word given in the brackets :

- (i) All circles are (congruent, similar)
- (ii) All squares are (similar, congruent)
- (iii) All triangles are similar (isosceles, equilateral)
- (iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are and (b) their corresponding sides are (equal, proportional)

Sol. (i) **similar.** (ii) **similar.**

(iii) **equilateral**

(iv) (a) **equal** and (b) **proportional.**

Q.2. Give two different examples of pair of :

- (i) Similar figures (ii) non-similar figures

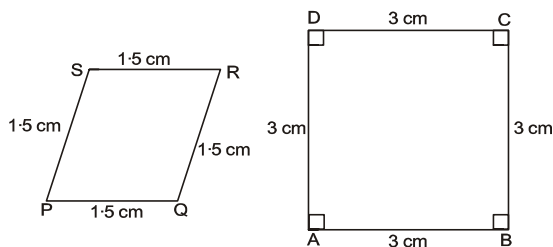
Sol. (i) (a) Pair of equilateral triangles are similar figures.

(b) Pair of circles are similar figures.

(ii) (a) A pair of a triangle and a square are non-similar figures.

(b) A pair of a square and a rectangle are non-similar figures.

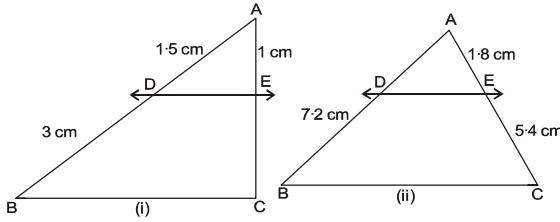
3. State whether the following quadrilaterals are similar or not.



Sol. The two quadrilaterals PQRS and ABCD are not similar because their corresponding angles are not equal.

Textbook Exercise 6.2

Q.1. In given figure (i) and (ii), $DE \parallel BC$. Find EC in (i) and AD in (ii)



Sol. (i) we have, $AD = 1.5$ cm, $AE = 1$ cm, $DB = 3$ cm

and $DE \parallel BC$

$$\therefore \frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{1.5}{3.0} = \frac{1}{EC}$$

$$\Rightarrow EC = \frac{3.0}{1.5} = 2$$

Hence, $EC = 2$ cm.

(ii) We have, $DB = 7.2$ cm, $AE = 1.8$ cm, $EC = 5.4$ cm and $DE \parallel BC$

$$\therefore \frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{AD}{7.2} = \frac{1.8}{5.4}$$

$$\Rightarrow AD = \frac{7.2 \times 1.8}{5.4} = 2.4$$

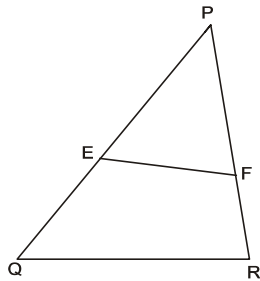
Hence, $AD = 2.4$ cm.

Q.2. E and F are points on the sides PQ and PR respectively of a $\triangle PQR$. For each of the following cases, state whether $EF \parallel QR$:

(i) $PE = 3.9$ cm, $EQ = 3$ cm, $PF = 3.6$ cm and $FR = 2.4$ cm.

(ii) $PE = 4$ cm, $QE = 4.5$ cm, $PF = 8$ cm and $RF = 9$ cm.

(iii) $PQ = 1.28$ cm, $PR = 2.56$ cm, $PE = 0.18$ cm and $PF = 0.36$ cm.



Sol. (i) we have $PE = 3.9$ cm, $EQ = 3$ cm, $PF = 3.6$ cm and $FR = 2.4$ cm

$$\text{Now, } \frac{PE}{EQ} = \frac{3.9}{3.0} = 1.3 \text{ and } \frac{PF}{FR} = \frac{3.6}{2.4} = 1.5$$

$$\therefore \frac{PE}{EQ} \neq \frac{PF}{FR}$$

$\therefore EF$ is not parallel to QR .

(ii) We have,

$PE = 4$ cm, $QE = 4.5$ cm, $PF = 8$ cm and $RF = 9$ cm

$$\text{Now, } \frac{PE}{EQ} = \frac{4}{4.5} = \frac{8}{9} \text{ and } \frac{PF}{FR} = \frac{8}{9}$$

$$\therefore \frac{PE}{EQ} = \frac{PF}{FR}$$

$\therefore EF \parallel QR$

(iii) We have,

$PQ = 1.28$ cm, $PR = 2.56$ cm, $PE = 0.18$ cm and $PF = 0.36$ cm,

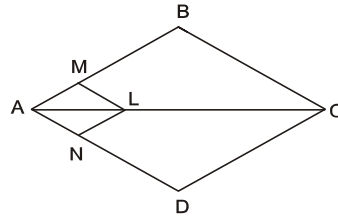
$EQ = PQ - PE = 1.28 - 0.18 = 1.1$ cm, $R = PR - PF = 2.56 - 0.36 = 2.2$ cm.

$$\text{Now, } \frac{PE}{EQ} = \frac{0.18}{1.1} = \frac{9}{55} \text{ and } \frac{PF}{FR} = \frac{0.36}{2.2} = \frac{9}{55}$$

$$\therefore \frac{PE}{EQ} = \frac{PF}{FR} \therefore EF \parallel QR$$

Q.3. In given figure, if $LM \parallel CB$ and $LN \parallel CD$,

prove that $\frac{AM}{AB} = \frac{AN}{AD}$.



Sol. Given : In $\square ABCD$,

$LM \parallel CB$ and $LN \parallel CD$

To prove : $\frac{AM}{AB} = \frac{AN}{AD}$

Solution : In $\triangle ACB$

$LM \parallel CB$ (given)

$$\therefore \frac{AM}{MB} = \frac{AL}{LC} \quad \dots(1)$$

In $\triangle ACD$ $LN \parallel CD$ (given)

$$\therefore \frac{AN}{ND} = \frac{AL}{LC} \quad \dots(2)$$

From (1) and (2), we get

$$\frac{AM}{MB} = \frac{AN}{ND} \text{ or, } \frac{MB}{AM} = \frac{ND}{AN}$$

$$\Rightarrow \frac{MB}{AM} + 1 = \frac{ND}{AN} + 1 \quad [\text{Adding 1 both sides}]$$

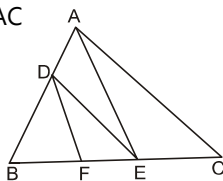
$$\Rightarrow \frac{MB + AM}{AM} = \frac{ND + AN}{AN}$$

$$\Rightarrow \frac{AB}{AM} = \frac{AD}{AN} \text{ or, } \frac{AM}{AB} = \frac{AN}{AD} \text{ Hence Proved}$$

Q.4. In given figure $DE \parallel AC$

and $DF \parallel AE$. Prove

$$\text{that } \frac{BF}{FE} = \frac{BE}{EC}.$$



Sol. Given : In $\triangle ABC$ $DE \parallel AC$ and $DF \parallel AE$

To prove : $\frac{BF}{FE} = \frac{BE}{EC}$

Proof : In $\triangle BAE$,

$$DF \parallel AE \text{ (given)}$$

$$\therefore \frac{BD}{AD} = \frac{BF}{FE} \quad \dots(1)$$

In $\triangle ABC$, $DE \parallel AC$ (given)

$$\therefore \frac{BD}{AD} = \frac{BE}{EC} \quad \dots(2)$$

From (1) and (2), we get

$$\frac{BF}{FE} = \frac{BE}{EC} \quad \text{Hence Proved}$$

Q.5. In given figure $DE \parallel OQ$ and $DF \parallel OR$. Show that $EF \parallel QR$.

Sol. Given : In $\triangle POQ$, $DE \parallel OQ$ and in $\triangle POR$, $DF \parallel OR$

To prove : $EF \parallel QR$

Proof : In $\triangle POQ$ $DE \parallel OQ$ (given)

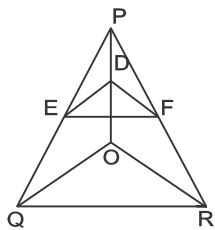
$$\therefore \frac{PD}{DO} = \frac{PE}{EQ} \quad \dots(1)$$

In $\triangle POR$ $DF \parallel OR$ (given)

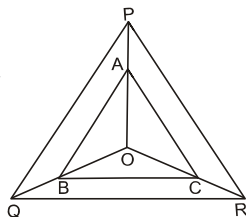
$$\therefore \frac{PD}{DO} = \frac{PF}{FR} \quad \dots(2)$$

From (1) and (2), we get $\frac{PE}{EQ} = \frac{PF}{FR}$

$\therefore EF \parallel QR$ [By converse of BPT]
Hence proved



Q.6. In given figure, A, B, and C are points on OP, OQ and OR respectively such that $AB \parallel PQ$ and $AC \parallel PR$. Show that $BC \parallel QR$.



Sol. Given : In figure A, B and C are points on OP, OQ and OR respectively such that $AB \parallel PQ$ and $AC \parallel PR$.

To prove : $BC \parallel QR$

Proof : In $\triangle OPQ$

$$AB \parallel PQ \quad \text{(given)}$$

$$\frac{OA}{AP} = \frac{OB}{BQ} \quad \dots(1)$$

In $\triangle OPR$,

$$AC \parallel PR \text{ (given)}$$

$$\frac{OA}{AP} = \frac{OR}{CR} \quad \dots(2)$$

From (1) and (2) we get $\frac{OB}{BQ} = \frac{OR}{CR}$

Thus B and C are respectively points on sides OQ and OR of $\triangle OQR$, such that $\frac{OB}{BQ} = \frac{OR}{CR}$

$\Rightarrow BC \parallel QR$ Hence Proved

Q.7. Prove that a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in class IX).

Sol. Given : A $\triangle ABC$ in which D is the mid-point of AB and $DE \parallel BC$ and meeting AC at E.

To Prove : $AE = EC$

Proof : Since, $DE \parallel BC$

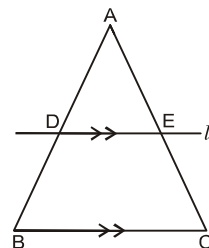
$$\therefore \frac{AD}{DB} = \frac{AE}{EC} \quad \dots(1)$$

But $AD = DB$

[$\because$ D is the mid point of AB]

$$\therefore \frac{AD}{AD} = \frac{AE}{EC} \Rightarrow \frac{AE}{EC} = 1$$

$\Rightarrow AE = EC.$ Hence Proved



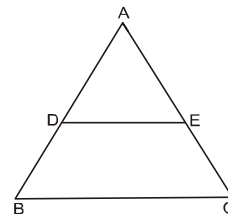
Q.8. Prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. (Recall that you have done it in class IX).

Sol. Given : A $\triangle ABC$ in which D is the mid-point of AB and E is the mid-point of AC

To prove : $DE \parallel BC$

Proof : Since D and E are the mid-points of AB and AC respectively
Therefore,

$$AD = DB \Rightarrow \frac{AD}{DB} = 1$$



and $AE = EC \Rightarrow \frac{AE}{EC} = 1$

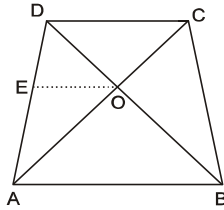
$$\therefore \frac{AD}{DB} = \frac{AE}{EC}$$

$\Rightarrow DE \parallel BC$ **Hence Proved**

Q.9. ABCD is a trapezium in which $AB \parallel DC$ and its diagonals intersect each other at the point O.

Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

Sol. Given : A trapezium ABCD in which $AB \parallel DC$ and its diagonals AC and BD intersect at O.



To Prove : $\frac{AO}{BO} = \frac{CO}{DO}$

Construction : Draw $OE \parallel AB$ through O, which meets AD at E.

Proof : We have,

$$EO \parallel AB \quad (\text{By construction}) \dots (1)$$

$$DC \parallel AB \quad (\text{given}) \dots (2)$$

From (1) and (2), we get $EO \parallel DC$

$$\therefore \frac{AE}{ED} = \frac{AO}{OC} \dots (3)$$

In $\triangle DAB$ $EO \parallel AB$

$$\frac{DE}{EA} = \frac{DO}{OB}$$

$$\therefore \frac{AE}{ED} = \frac{BO}{OD} \dots (4)$$

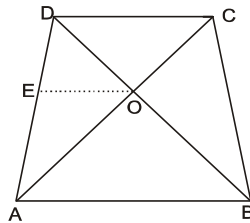
From (3), (4), we get $\frac{AO}{OC} = \frac{BO}{OD}$

$$\Rightarrow \frac{AO}{BO} = \frac{CO}{DO} \quad \text{Hence Proved}$$

Q.10. The diagonals of a quadrilateral ABCD intersect each other at the point O such that

$\frac{AO}{BO} = \frac{CO}{DO}$. Show that ABCD is a trapezium.

Sol. Given : A quadrilateral ABCD whose diagonals AC and BD intersect at a point O such that



$$\frac{AO}{BO} = \frac{CO}{DO}$$

To Prove : ABCD is a trapezium i.e. $AB \parallel DC$.

Construction : Draw $OE \parallel DC$ which meets AD at E

Proof : In $\triangle ACD$, $OE \parallel DC$ (By construction)

$$\therefore \frac{AO}{OC} = \frac{AE}{ED} \dots (1)$$

$$\text{But, } \frac{AO}{BO} = \frac{CO}{DO} \quad (\text{given})$$

$$\Rightarrow \frac{AO}{OC} = \frac{BO}{DO} \dots (2)$$

From (1) and (2), we get

$$\frac{AE}{ED} = \frac{BO}{DO} \Rightarrow EO \parallel AB \dots (3)$$

But, $OE \parallel DC$ (By construction) $\dots (4)$

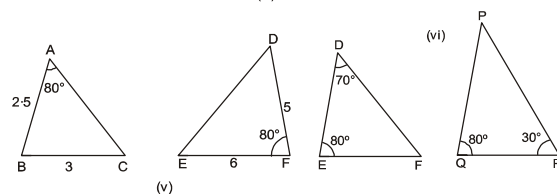
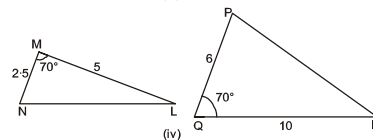
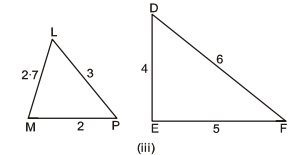
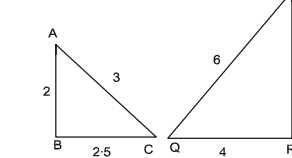
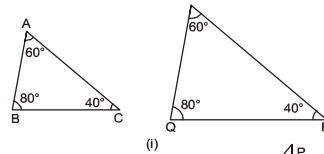
From (3) and (4), we get

$$AB \parallel DC$$

Hence, ABCD is a trapezium. *Hence Proved*

Textbook Exercise 6.3

- State which pairs of triangles in below figures are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form :



Sol. (i) In $\triangle ABC$ and $\triangle PQR$

$$\angle A = \angle P = 60^\circ \quad \angle B = \angle Q = 80^\circ$$

$$\angle C = \angle R = 40^\circ$$

By AAA similarity criterion. $\triangle ABC \sim \triangle PQR$.

(ii) In $\triangle ABC$ and $\triangle QRP$

$$\frac{AB}{QR} = \frac{2}{4} = \frac{1}{2} \Rightarrow \frac{BC}{RP} = \frac{2.5}{5} = \frac{1}{2}$$

$$\frac{AC}{PQ} = \frac{3}{6} = \frac{1}{2}$$

$$\therefore \frac{AB}{QR} = \frac{BC}{RP} = \frac{AC}{PQ}$$

$\therefore$ By SSS similarity criterion

$$\triangle ABC \sim \triangle QRP$$

(iii) In $\triangle LMP$ and $\triangle DEF$

$$\frac{LM}{DE} = \frac{2.7}{4} = 0.675 \quad \frac{MP}{EF} = \frac{2}{5} = 0.4$$

$$\frac{LP}{DF} = \frac{3}{6} = 0.5$$

$$\therefore \frac{LM}{DF} \neq \frac{MP}{EF} \neq \frac{LP}{DF}$$

$\therefore$ These two triangles are not similar because they do not satisfy the SSS similarity criterion.

(iv) In $\triangle MNL$ and $\triangle QPR$ $\angle M = \angle Q = 70^\circ$

$$\frac{MN}{PQ} = \frac{2.5}{6} = 0.42 \text{ (approx)}$$

$$\frac{ML}{QR} = \frac{5}{10} = 0.5$$

$$\therefore \frac{MN}{PQ} \neq \frac{ML}{QR}$$

$\therefore$ These two triangles are not similar because they do not satisfy the SAS similarity criterion.

(v) In $\triangle ABC$ and $\triangle FDE$ $\angle A = \angle F = 80^\circ$

$$\frac{AB}{DF} \neq \frac{AC}{EF} \quad [\because AC \text{ is not given}]$$

$\therefore$ These two triangles are not similar because they do not satisfy the SAS similarity criterion.

(vi) In $\triangle DEF$ and $\triangle PQR$

$$\angle P = 180^\circ - (80^\circ + 30^\circ)$$

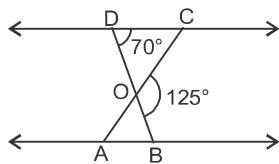
$$\Rightarrow \angle P = 180^\circ - 110^\circ \Rightarrow \angle P = 70^\circ$$

$$\text{Now, } \angle D = \angle P = 70^\circ \quad \angle E = \angle Q = 80^\circ$$

By AA similarity criterion

$$\triangle DEF \sim \triangle PQR.$$

Q.2. In given figure $\triangle ODC \sim \triangle OBA$, $\angle BOC = 125^\circ$ and $\angle CDO = 70^\circ$. Find $\angle DOC$, $\angle DCO$ and $\angle OAB$.



Sol. We have, $\angle CDO = 70^\circ$ and $\angle COB = 125^\circ$

$\therefore$ BD is a line and CO ray on it

$$\angle DOC + \angle COB = 180^\circ$$

$$\Rightarrow \angle DOC + 125^\circ = 180^\circ \Rightarrow \angle DOC = 180^\circ - 125^\circ$$

$$\Rightarrow \angle DOC = 55^\circ$$

In $\triangle COD$, we have

$$\angle DOC + \angle DCO + \angle CDO = 180^\circ$$

[Sum of $\angle$ s of a triangle is 180°]

$$\Rightarrow 55^\circ + \angle DCO + 70^\circ = 180^\circ$$

$$\Rightarrow 125^\circ + \angle DCO = 180^\circ$$

$$\Rightarrow \angle DCO = 180^\circ - 125^\circ$$

$$\Rightarrow \angle DCO = 55^\circ$$

Now $\triangle ODC \sim \triangle OBA$ (given)

$$\therefore \angle OAB = \angle DCO$$

(Corresponding $\angle$ s of similar $\triangle$ s)

$$\Rightarrow \angle OAB = 55^\circ$$

$$\text{Hence, } \angle DOC = 55^\circ, \angle DCO = 55^\circ, \angle OAB = 55^\circ.$$

Q.3. Diagonals AC and BD of a trapezium ABCD with $AB \parallel DC$ intersect each other at the point O. Using a similarity criterion for two triangles, show that $\frac{OA}{OC} = \frac{OB}{OD}$.

Sol. Given : ABCD is a trapezium in which $AB \parallel DC$. Its diagonals AC and BD meet at O.

To Prove : $\frac{OA}{OC} = \frac{OB}{OD}$

Proof : In $\triangle OAB$ and $\triangle OCD$

$$\angle OAB = \angle OCD$$

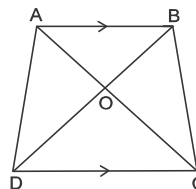
$[\because AB \parallel DC]$

$\therefore$ Alternate $\angle$ s are equal]

$$\angle AOB = \angle COD \quad (\text{Vertically opposite } \angle\text{s})$$

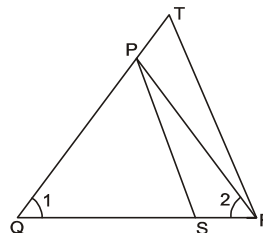
$\therefore \triangle OAB \sim \triangle OCD$ (By AA similarity criterion)

$$\therefore \frac{OA}{OC} = \frac{OB}{OD} \quad \text{Hence Proved}$$



4. In given figure $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$.

Show that $\triangle PQS \sim \triangle TQR$.



Sol. Given : $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$

To Prove : $\triangle PQS \sim \triangle TQR$

Proof : We have $\frac{QR}{QS} = \frac{QT}{PR}$... (i)

and $\angle 1 = \angle 2$
 $\Rightarrow PR = PQ$, ... (ii)

From (i), (ii), we get

$$\frac{QR}{QS} = \frac{QT}{PQ} \Rightarrow \frac{QS}{QR} = \frac{PQ}{QT}$$

[Taking reciprocals of both sides]

Thus in $\triangle PQS$ and $\triangle TQR$, we have

$$\angle Q = \angle Q \quad (\text{Common})$$

$$\text{and } \frac{QS}{QR} = \frac{PQ}{QT}$$

$$\therefore \triangle PQS \sim \triangle TQR$$

[By SAS similarity criterion]

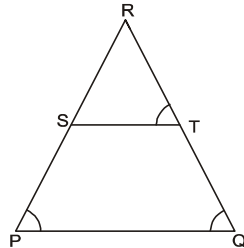
Hence Proved

5. S and T are points on sides PR and QR of $\triangle PQR$ such that $\angle P = \angle RTS$. Show that $\triangle RPQ \sim \triangle RTS$.

Sol. Given : S and T are points on sides PR and QR of $\triangle PQR$ such that $\angle P = \angle RTS$

To Prove :

$\triangle RPQ \sim \triangle RTS$.



Proof : In $\triangle RPQ$ and $\triangle RTS$.

$$\angle P = \angle RTS \quad (\text{given})$$

$$\angle R = \angle R \quad (\text{common})$$

$$\therefore \triangle RPQ \sim \triangle RTS$$

(By AA similarity criterion)

Hence Proved

Q.6. In given figure, If $\triangle ABE \cong \triangle ACD$, show that $\triangle ADE \sim \triangle ABC$.

Sol. Given :

$$\triangle ABE \cong \triangle ACD$$

To Prove :

$$\triangle ADE \sim \triangle ABC$$

Proof :

$$\therefore \triangle ABE \cong \triangle ACD$$

$$\therefore AB = AC \text{ and }$$

$$AD = AE \quad [\text{CPCT}]$$

$$\Rightarrow \frac{AB}{AC} = 1 \text{ and } \frac{AD}{AE} = 1 \Rightarrow \frac{AB}{AC} = \frac{AD}{AE}$$

Now, In $\triangle ADE$ and $\triangle ABC$, we have

$$\frac{AB}{AC} = \frac{AD}{AE} \Rightarrow \frac{AB}{AD} = \frac{AC}{AE}$$

and

$$\angle DAE = \angle BAC \quad (\text{each} = \angle A)$$

$$\therefore \triangle ADE \sim \triangle ABC.$$

[By AA similarity criterion]

Hence Proved

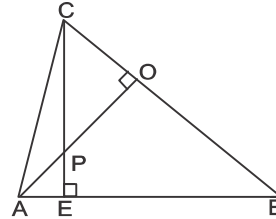
Q.7. In given figure altitudes AD and CE of $\triangle ABC$ intersect each other at the point P. Show that :

$$(i) \triangle AEP \sim \triangle CDP$$

$$(ii) \triangle ABD \sim \triangle CBE$$

$$(iii) \triangle AEP \sim \triangle ADB$$

$$(iv) \triangle PDC \sim \triangle BEC.$$



Sol. Given : $CE \perp AB$ and $AD \perp BC$.

To Prove : (i) $\triangle AEP \sim \triangle CDP$

$$(ii) \triangle ABD \sim \triangle CBE$$

$$(iii) \triangle AEP \sim \triangle ADB$$

$$(iv) \triangle PDC \sim \triangle BEC.$$

Proof : (i) In $\triangle AEP$ and $\triangle CDP$

$$\angle AEP = \angle CDP \quad (\text{each} = 90^\circ)$$

$$\angle APE = \angle CPD$$

(vertically opposite $\angle$ s)

$$\therefore \triangle AEP \sim \triangle CDP$$

(By AA similarity criterion)

Hence Proved

(ii) In $\triangle ABD$ and $\triangle CBE$

$$\angle ADB = \angle CEB \quad (\text{each} = 90^\circ)$$

$$\angle B = \angle B \quad (\text{common})$$

$$\therefore \triangle ABD \sim \triangle CBE$$

(By AA similarity criterion)

Hence Proved

(iii) In $\triangle AEP$ and $\triangle ADB$

$$\angle AEP = \angle ADB \quad (\text{each} = 90^\circ)$$

$$\angle PAE = \angle DAB \quad (\text{same angle})$$

$$\therefore \triangle AEP \sim \triangle ADB$$

(By AA similarity criterion)

Hence Proved

(iv) In $\triangle PDC$ and $\triangle BEC$

$$\angle PDC = \angle BEC \quad (\text{each} = 90^\circ)$$

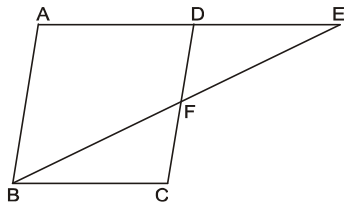
$$\angle PCD = \angle BCE \quad (\text{same angle})$$

$$\therefore \triangle PDC \sim \triangle BEC$$

(By AA similarity criterion)

Hence Proved

Q.8. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that $\triangle ABE \sim \triangle CFB$.



Sol. Given : E is a point on side AD produced of a || gm ABCD and BE intersects CD at F.

To Prove : $\triangle ABE \sim \triangle CFB$.

Proof : In $\triangle ABE$ and $\triangle CFB$.

$$\angle AEB = \angle CBF$$

(alternate interior $\angle$ s)

$$\angle A = \angle C$$

(opposite $\angle$ s of a ||gm)

$$\therefore \triangle ABE \sim \triangle CFB$$

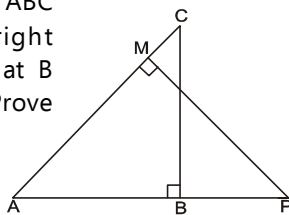
(By AA similarity criterion)

Hence Proved

Q.9. In given figure, ABC and AMP are two right triangles, right angled at B and M respectively. Prove that :

(i) $\triangle ABC \sim \triangle AMP$.

(ii) $\frac{CA}{PA} = \frac{BC}{MP}$.



Sol.

Given : $\triangle ABC$ and $\triangle AMP$ are two right angled triangles right angles at B and M respectively.

To Prove : (i) $\triangle ABC \sim \triangle AMP$

$$(ii) \frac{CA}{PA} = \frac{BC}{MP}$$

Proof : (i) In $\triangle ABC$ and $\triangle AMP$.

$$\angle ABC = \angle AMP \text{ (each} = 90^\circ \text{)}$$

$$\angle A = \angle A \text{ (common)}$$

$$\therefore \triangle ABC \sim \triangle AMP$$

(By AA similarity criterion)

Hence Proved

(ii) $\therefore \triangle ABC \sim \triangle AMP$

$$\therefore \frac{CA}{PA} = \frac{BC}{MP}$$

(corresponding sides of similar triangles)

Hence Proved

Q.10. CD and GH are respectively the bisectors of $\angle ACB$ and $\angle EGF$ such that D and H lie on sides AB and FE of $\triangle ABC$ and $\triangle EFG$ respectively. If $\triangle ABC \sim \triangle FEG$, show that:

$$(i) \frac{CD}{GH} = \frac{AC}{FG} \quad (ii) \triangle DCB \sim \triangle HGE$$

$$(iii) \triangle DCA \sim \triangle HGF$$

Sol.

Given : CD and GH are respectively the bisectors of $\angle ACB$ and $\angle EGF$ such that D and H lie on the sides AB and EF of $\triangle ABC$ and $\triangle EFG$ and $\triangle ABC \sim \triangle FEG$.

To Prove : (i) $\triangle DCB \sim \triangle HGE$

$$(ii) \triangle DCA \sim \triangle HGF \quad (iii) \frac{CD}{GH} = \frac{AC}{FG}$$

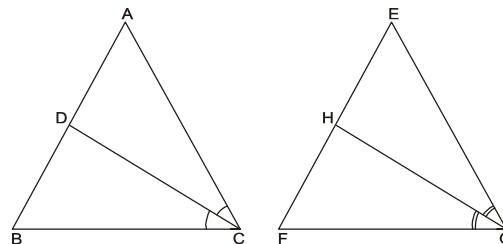
Proof : (i) $\triangle ABC \sim \triangle FEG$ (given)

$$\angle ACB = \angle EGF$$

(corresponding $\angle$ s of similar Δ s)

$$\Rightarrow \frac{1}{2} \angle ACB = \frac{1}{2} \angle EGF \text{ [CD and GH}$$

are bisectors of $\angle C$ and $\angle G$.]



$$\Rightarrow \angle DCB = \angle HGE \quad \dots(1)$$

$$\text{and } \angle B = \angle E$$

(corresponding $\angle$ s of similar Δ s)

$$\Rightarrow \angle DBC = \angle HEG \quad \dots(2)$$

From (1) and (2), we have

$$\triangle DCB \sim \triangle HGE \text{ Hence Proved}$$

(ii) In $\triangle DCA$ and $\triangle HGF$ $\angle C = \angle G$

(corresponding $\angle$ s of similar Δ s)

$$\angle ACD = \angle HGF$$

[CD and GH are bisectors of $\angle C$ and $\angle G$]

$$\dots(1)$$

$$\angle A = \angle F$$

(corresponding $\angle$ s of similar Δ s) $\dots(2)$

$$\angle ADC = \angle FHG$$

(third $\angle$ s of two triangles) $\dots(3)$

$$\therefore \triangle DCA \sim \triangle HGF.$$

(By AAA similarity criterion)

Hence Proved

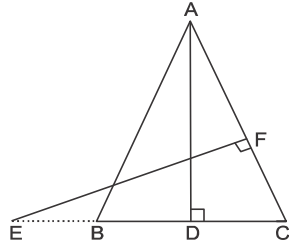
(iii) $\therefore \triangle DCA \sim \triangle HGF.$

$$\therefore \frac{CD}{GH} = \frac{AC}{FG}$$

(corresponding sides of similar Δ s)

Hence Proved

- 11.** In given figure, E is a point on side CB produced of an isosceles triangle ABC with $AB = AC$. If $AD \perp BC$ and $EF \perp AC$, prove that $\triangle ABD \sim \triangle ECF$.



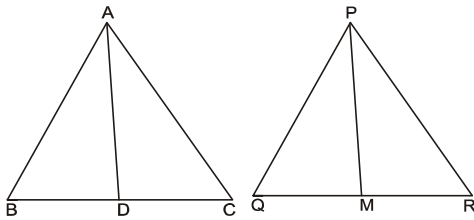
Sol. Given : $AB = AC$, $AD \perp BC$ and $EF \perp AC$.

To Prove : $\triangle ABD \sim \triangle ECF$

Proof : $AB = AC$ (given)
 $\angle B = \angle C$ [$\because$ $\angle$ s opposite to equal side are equal]

Now, In $\triangle ABD$ and $\triangle ECF$,
 $\angle ABD = \angle ECF$ [$\because \angle B = \angle C$]
 $\angle ADB = \angle EFC = 90^\circ$
 $[\because AD \perp BC \text{ and } EF \perp AC]$
 $\therefore \triangle ABD \sim \triangle ECF$
 (By AA similarity criterion)
Hence Proved

- Q.12.** Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of $\triangle PQR$ (see figure below). Show that $\triangle ABC \sim \triangle PQR$.



Sol. Given : $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$

To Prove : $\triangle ABC \sim \triangle PQR$.

Proof : $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$ (given)

$\Rightarrow \frac{AB}{PQ} = \frac{2BD}{2QM} = \frac{AD}{PM}$
 $[\because BD = CD \text{ and } QM = MR]$
 $\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$
 $\therefore \triangle ABD \sim \triangle PQM$

(By SSS similarity criterion)

$$\angle B = \angle Q$$

(Corresponding $\angle$ s of Similar Δ s)

Now, in $\triangle ABC$ and $\triangle PQR$.

$$\frac{AB}{PQ} = \frac{BC}{QR} \quad (\text{given})$$

$$\angle B = \angle Q \quad (\text{Proved above})$$

$$\therefore \triangle ABC \sim \triangle PQR$$

[By SAS similarity criterion]

Hence Proved

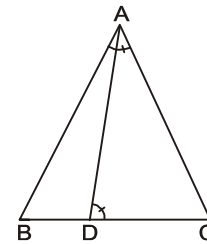
- Q.13.** D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$. Show that $CA^2 = CB \cdot CD$

Sol.

Given : $\angle ADC = \angle BAC$

To prove : $CA^2 = CB \cdot CD$

Proof : In $\triangle ABC$ and $\triangle DAC$
 $\angle ADC = \angle BAC$ (given)
 $\angle C = \angle C$ (common)
 $\therefore \triangle ABC \sim \triangle DAC$
 (By AA similarity criterion)



$$\Rightarrow \frac{AB}{DA} = \frac{BC}{AC} = \frac{AC}{DC}$$

$$\Rightarrow \frac{CB}{CA} = \frac{CA}{CD}$$

$$\Rightarrow CA^2 = CB \times CD. \quad \text{Hence Proved}$$

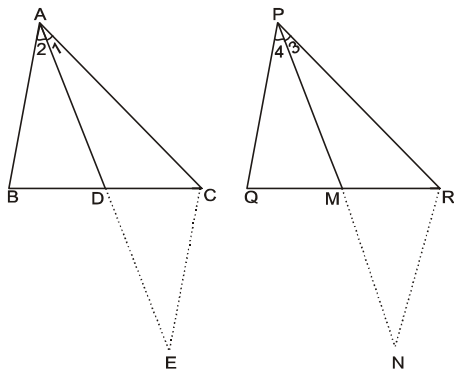
- Q.14..** Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\triangle ABC \sim \triangle PQR$.

Sol. Given : In $\triangle ABC$ and $\triangle PQR$

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$

To Prove : $\triangle ABC \sim \triangle PQR$

Construction : Produce AD to E such that $AD = DE$ and PM to N such that $PM = MN$ Join CE and NR.



Proof : In $\triangle ADB$ and $\triangle EDC$

$$AD = ED \quad (\text{By construction})$$

$$BD = CD \quad (\text{AD is median})$$

$$\angle ADB = \angle CDE$$

(vertically opposite $\angle$ s)

$$\therefore \triangle ADB \cong \triangle EDC$$

(By SAS Congruence criterion)

$$\Rightarrow AB = EC \quad (\text{By CPCT}) \quad \dots(1)$$

Similarly,

$$\triangle PMQ \cong \triangle NMR$$

(By SAS congruence criterion)

$$\Rightarrow PQ = NR \quad (\text{By CPCT}) \quad \dots(2)$$

Now,

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM} \quad (\text{given})$$

$$\Rightarrow \frac{EC}{NR} = \frac{AC}{PR} = \frac{AD}{PM}$$

[From (1) and (2), $AB = EC$
and $PQ = NR$]

$$\Rightarrow \frac{EC}{NR} = \frac{AC}{PR} = \frac{2AD}{2PM} \Rightarrow \frac{EC}{NR} = \frac{AC}{PR} = \frac{AE}{PN}$$

[By construction $AD = DE$ and $PM = MN$]

$$\therefore \triangle ACE \sim \triangle PRN$$

(By SSS similarity criterion)

$$\angle 1 = \angle 3 \quad \dots(3)$$

(Corresponding $\angle$ s of similar triangles)

Similarly $\angle 2 = \angle 4 \quad \dots(4)$

Adding (3) and (4) we get

$$\angle 1 + \angle 2 = \angle 3 + \angle 4.$$

$$\Rightarrow \angle A = \angle P.$$

Now $\triangle ABC$ and $\triangle PQR$.

$$\angle A = \angle P \quad (\text{Proved above})$$

$$\frac{AB}{PQ} = \frac{AC}{PR} \quad (\text{given})$$

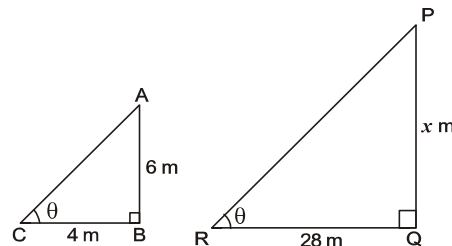
$$\therefore \triangle ABC \sim \triangle PQR$$

(By SAS Similarity criterion)

Hence Proved

Q.15. A vertical pole of length 6m casts a shadow 4m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower. **(CBSE 2023)**

Sol.



Let AB be the vertical pole of length 6m casts a shadow BC and PQ be the tower casts a shadow QR. Let height of tower be x m. Join AC and PR.

In $\triangle ABC$ and $\triangle PQR$ $\angle B = \angle Q$ (each = 90°)

$$\angle C = \angle R \quad (\text{Angle of elevation of the Sun})$$

$$\therefore \triangle ABC \sim \triangle PQR \quad (\text{By AA similarity criterion})$$

$$\therefore \frac{AB}{PQ} = \frac{BC}{QR}$$

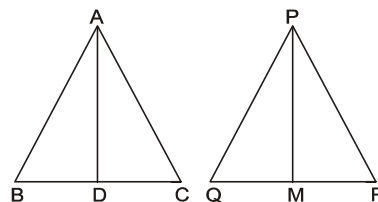
[Corresponding sides of similar triangles]

$$\Rightarrow \frac{6}{x} = \frac{4}{28} \Rightarrow x = \frac{6 \times 28}{4} \Rightarrow x = 42.$$

Hence, height of tower = 42 m.

Q.16. If AD and PM are medians of triangles ABC and PQR, respectively where $\triangle ABC \sim \triangle PQR$,

prove that $\frac{AB}{PQ} = \frac{AD}{PM}$



Sol. Given : AD and PM are medians of $\triangle ABC$ and $\triangle PQR$ respectively and $\triangle ABC \sim \triangle PQR$.

To Prove : $\frac{AB}{PQ} = \frac{AD}{PM}$

Proof : $\triangle ABC \sim \triangle PQR$

$$\therefore \frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR} \quad \dots(1)$$

(corresponding sides of similar triangles)

$$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} \Rightarrow \frac{AB}{PQ} = \frac{2BD}{2QM}$$

$\therefore AD$ and PM are medians $\therefore BD = CD$ and $QM = MR$

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} \text{ and } \angle B = \angle Q$$

[Corresponding $\angle$ s of similar Δ s ABC and PQR]

$$\Delta ADB \sim \Delta PMQ$$

[By SAS similarity criterion]

$$\frac{AB}{PQ} = \frac{AD}{PM}$$

(Corresponding sides of similar Δ s.)
Hence Proved

7

Coordinate Geometry

Textbook Exercise 7.1

Q.1. Find the distance between the following pairs of points :

(i) (2, 3), (4, 1) (ii) (-5, 7), (-1, 3)

(iii) (a, b), (-a, -b)

Sol. (i) Let the given points be $A(2, 3)$ and $B(4, 1)$, then $AB = \sqrt{(4-2)^2 + (1-3)^2}$

$$\Rightarrow AB = \sqrt{(2)^2 + (-2)^2} \Rightarrow AB = \sqrt{4+4} = \sqrt{8}$$

$$\Rightarrow AB = \sqrt{2 \times 2 \times 2} = 2\sqrt{2}$$

Hence, distance (AB) = $2\sqrt{2}$

(ii) Let the given points be $P(-5, 7)$ and

$Q(-1, 3)$, then $PQ = \sqrt{(-1+5)^2 + (3-7)^2}$

$$\Rightarrow PQ = \sqrt{(4)^2 + (-4)^2} \Rightarrow PQ = \sqrt{16+16}$$

$$\Rightarrow PQ = \sqrt{32} \Rightarrow PQ = \sqrt{4 \times 4 \times 2}$$

$$\Rightarrow PQ = 4\sqrt{2}$$

Hence, distance (PQ) = $4\sqrt{2}$

(iii) Let the given points be $P(a, b)$ and $Q(-$

$a, -b)$, then $PQ = \sqrt{(-a-a)^2 + (-b-b)^2}$

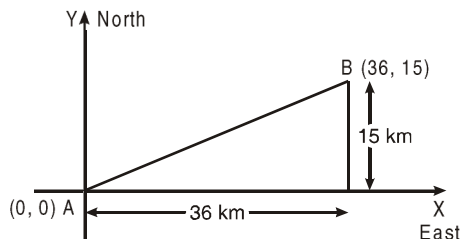
$$\Rightarrow PQ = \sqrt{(-2a)^2 + (-2b)^2}$$

$$\Rightarrow PQ = \sqrt{4a^2 + 4b^2} \Rightarrow PQ = \sqrt{4(a^2 + b^2)}$$

$$\Rightarrow PQ = 2\sqrt{a^2 + b^2}$$

Hence, the distance (PQ) = $2\sqrt{a^2 + b^2}$.

Q.2. Find the distance between the points (0, 0) and (36, 15). Can you now find the distance between the two towns A and B.



Sol. Let the given points be $A(0, 0)$ and $B(36, 15)$ then $AB = \sqrt{(36-0)^2 + (15-0)^2}$

$$\Rightarrow AB = \sqrt{(36)^2 + (15)^2} \Rightarrow AB$$

$$= \sqrt{1296 + 225} \Rightarrow AB = \sqrt{1521}$$

$$\Rightarrow AB = 39$$

Hence, distance between two towns = 39 km.

Q.3. Determine if the points (1, 5), (2, 3) and (-2, -11) are collinear.

Sol. Let the given points be $A(1, 5)$, $B(2, 3)$ and

$C(-2, -11)$. Then, $AB = \sqrt{(2-1)^2 + (3-5)^2}$

$$\Rightarrow AB = \sqrt{(1)^2 + (-2)^2} \Rightarrow AB = \sqrt{1+4} = \sqrt{5} = 2.24 \text{ (approx)}$$

$$BC = \sqrt{(-2-2)^2 + (-11-3)^2}$$

$$\Rightarrow BC = \sqrt{(-4)^2 + (-14)^2} \Rightarrow BC = \sqrt{16+196}$$

$$\Rightarrow BC = \sqrt{212} = 2\sqrt{53} = 14.56 \text{ (approx)}$$

$$\text{and, } AC = \sqrt{(-2-1)^2 + (-11-5)^2}$$

$$= \sqrt{(-3)^2 + (-16)^2} = \sqrt{9+256}$$

$$\Rightarrow AC = \sqrt{265} = 16.28 \text{ (approx)}$$

$$\text{Now, } AB + BC = 2.24 + 14.56 = 16.8$$

$$AC = 16.28$$

$$\therefore AB + BC \neq AC$$

Hence, the given points (1, 5), (2, 3) and (-2, -11) are not collinear.

Q.4. Check whether (5, -2), (6, 4) and (7, -2) are the vertices of an isosceles triangle.

Sol. Let the given points be $A(5, -2)$, $B(6, 4)$ and $C(7, -2)$, then

$$AB = \sqrt{(6-5)^2 + (4+2)^2}$$

$$= \sqrt{(1)^2 + (6)^2} = \sqrt{37}$$

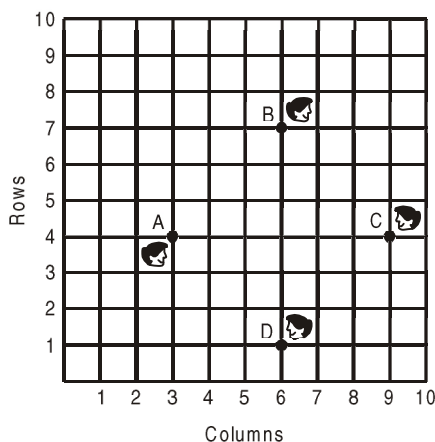
$$BC = \sqrt{(7-6)^2 + (-2-4)^2}$$

$$= \sqrt{(1)^2 + (-6)^2} = \sqrt{37}$$

Since, $AB = BC$

Therefore, the given points are the vertices of an isosceles triangle.

Q.5. In a classroom, 4 friends are seated at the points A, B, C and D as shown in figure. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, "Don't you think ABCD is a square?" Chameli disagrees. Using distance formula, find which of them is correct?



Sol. From the figure, coordinates of A, B, C, D, are respectively (3, 4), (6, 7), (9, 4), (6, 1)

$$AB = \sqrt{(6-3)^2 + (7-4)^2}$$

$$\Rightarrow AB = \sqrt{(3)^2 + (3)^2}$$

$$= \sqrt{9+9}$$

$$\Rightarrow AB = \sqrt{18} = \sqrt{3 \times 3 \times 2} = 3\sqrt{2}$$

$$BC = \sqrt{(9-6)^2 + (4-7)^2}$$

$$\Rightarrow BC = \sqrt{(3)^2 + (-3)^2} = \sqrt{9+9}$$

$$\Rightarrow BC = \sqrt{18} = \sqrt{3 \times 3 \times 2} = 3\sqrt{2}$$

$$CD = \sqrt{(6-9)^2 + (1-4)^2}$$

$$\Rightarrow CD = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9}$$

$$\Rightarrow CD = \sqrt{18} = \sqrt{3 \times 3 \times 2} = 3\sqrt{2}$$

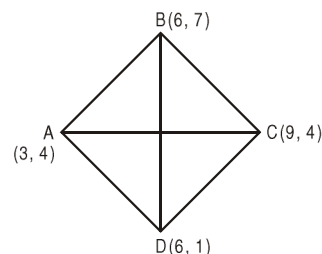
$$DA = \sqrt{(3-6)^2 + (4-1)^2}$$

$$\Rightarrow DA = \sqrt{(-3)^2 + (3)^2}$$

$$\Rightarrow DA = \sqrt{9+9}$$

$$\Rightarrow DA = \sqrt{18} = \sqrt{3 \times 3 \times 2} = 3\sqrt{2}$$

Now, $AC = \sqrt{(9-3)^2 + (4-4)^2}$



$$\Rightarrow AC = \sqrt{(6)^2 + 0}$$

$$\Rightarrow AC = \sqrt{36} = 6$$

and, $BD = \sqrt{(6-6)^2 + (1-7)^2}$

$$\Rightarrow BD = \sqrt{36}$$

$$\Rightarrow BD = 6.$$

Since, $AB = BC = CD = DA$ and diagonal $AC =$ diagonal BD .

Therefore, ABCD is a square.

Hence, Champa is correct.

Q.6. Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer:

(i) $(-1, -2)$, $(1, 0)$, $(-1, 2)$, $(-3, 0)$

(ii) $(-3, 5)$, $(3, 1)$, $(0, 3)$, $(-1, -4)$

(iii) $(4, 5)$, $(7, 6)$, $(4, 3)$, $(1, 2)$

Sol. (i) Let the given points of the quadrilateral be $A(-1, -2)$, $B(1, 0)$, $C(-1, 2)$ and $D(-3, 0)$. Join AC and BD

$$AB = \sqrt{(1+1)^2 + (0+2)^2}$$

$$\Rightarrow AB = \sqrt{(2)^2 + (2)^2}$$

$$= \sqrt{4+4} = \sqrt{8}$$

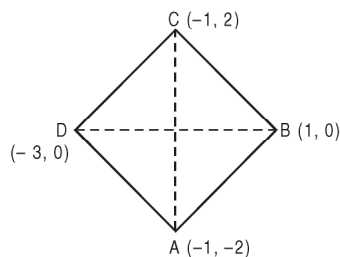
$$= \sqrt{2 \times 2 \times 2} = 2\sqrt{2}$$

$$BC = \sqrt{(-1-1)^2 + (2-0)^2}$$

$$\Rightarrow BC = \sqrt{(-2)^2 + (2)^2}$$

$$\Rightarrow BC = \sqrt{4+4} = \sqrt{8}$$

$$= \sqrt{2 \times 2 \times 2} = 2\sqrt{2}$$



$$\begin{aligned}
 CD &= \sqrt{(-3+1)^2 + (0-2)^2} \\
 \Rightarrow CD &= \sqrt{(-2)^2 + (-2)^2} \\
 \Rightarrow CD &= \sqrt{4+4} = \sqrt{8} = \sqrt{2 \times 2 \times 2} = 2\sqrt{2} \\
 DA &= \sqrt{(-1+3)^2 + (-2-0)^2} \\
 \Rightarrow DA &= \sqrt{(2)^2 + (-2)^2} \\
 \Rightarrow DA &= \sqrt{4+4} = \sqrt{8} = \sqrt{2 \times 2 \times 2} = 2\sqrt{2} \\
 \text{Thus, } AB &= BC = CD = DA.
 \end{aligned}$$

$$\begin{aligned}
 \text{Now, } AC &= \sqrt{(-1+1)^2 + (2+2)^2} \\
 &= \sqrt{0+(4)^2} = \sqrt{16} = 4
 \end{aligned}$$

$$\begin{aligned}
 \text{and } BD &= \sqrt{(-3-1)^2 + (0-0)^2} \\
 &= \sqrt{(-4)^2 + 0}
 \end{aligned}$$

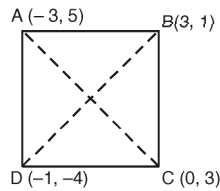
$$\Rightarrow BD = \sqrt{16} = 4$$

Since, $AB = BC = CA = DA$ and diagonal $AC =$ diagonal BD

Therefore, given points form a square.

(ii) Let the given points of the quadrilateral be $A(-3, 5)$, $B(3, 1)$, $C(0, 3)$ and $D(-1, -4)$.

Join AC and BD



$$\begin{aligned}
 \therefore AB &= \sqrt{(3+3)^2 + (1-5)^2} \\
 &= \sqrt{(6)^2 + (-4)^2} = \sqrt{36+16} \\
 \Rightarrow AB &= \sqrt{52} = \sqrt{2 \times 2 \times 13} = 2\sqrt{13} \\
 BC &= \sqrt{(0-3)^2 + (3-1)^2} \\
 &= \sqrt{(-3)^2 + (2)^2} \\
 \Rightarrow BC &= \sqrt{9+4} = \sqrt{13} \\
 CD &= \sqrt{(-1-0)^2 + (-4-3)^2} \\
 &= \sqrt{(-1)^2 + (-7)^2} \\
 CD &= \sqrt{1+49} = \sqrt{50} \\
 \Rightarrow CD &= \sqrt{5 \times 5 \times 2} \Rightarrow CD = 5\sqrt{2} \\
 AD &= \sqrt{(-1+3)^2 + (-4-5)^2} \\
 &= \sqrt{(2)^2 + (-9)^2} = \sqrt{4+81} \\
 \Rightarrow AD &= \sqrt{85} \\
 AC &= \sqrt{(0+3)^2 + (3-5)^2}
 \end{aligned}$$

$$\Rightarrow AC = \sqrt{(3)^2 + (-2)^2} = \sqrt{9+4}$$

$$\Rightarrow AC = \sqrt{13}$$

$$\begin{aligned}
 BD &= \sqrt{(-1-3)^2 + (-4-1)^2} \\
 &= \sqrt{(-4)^2 + (-5)^2} \\
 &= \sqrt{16+25}
 \end{aligned}$$

$$\Rightarrow BD = \sqrt{41}$$

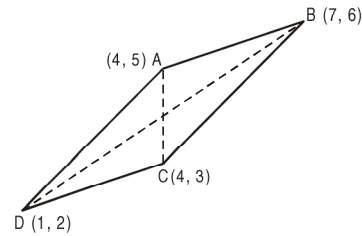
$$\text{Now, } AC + BC = \sqrt{13} + \sqrt{13} = 2\sqrt{13}$$

$$AB = 2\sqrt{13}$$

Since, $AC + BC = AB$.

Therefore, A, B, C are collinear. So, the given points do not form any quadrilateral.

(iii) The given points of a quadrilateral be $A(4, 5)$, $B(7, 6)$, $C(4, 3)$ and $D(1, 2)$. Join AC and BD



$$\therefore AB = \sqrt{(7-4)^2 + (6-5)^2}$$

$$\Rightarrow AB = \sqrt{(3)^2 + (1)^2}$$

$$\Rightarrow AB = \sqrt{9+1} = \sqrt{10}$$

$$BC = \sqrt{(4-7)^2 + (3-6)^2}$$

$$\Rightarrow BC = \sqrt{(-3)^2 + (-3)^2}$$

$$\Rightarrow BC = \sqrt{9+9} = \sqrt{18}$$

$$= \sqrt{3 \times 3 \times 2} = 3\sqrt{2}$$

$$CD = \sqrt{(1-4)^2 + (2-3)^2}$$

$$\Rightarrow CD = \sqrt{(-3)^2 + (-1)^2}$$

$$\Rightarrow CD = \sqrt{9+1} = \sqrt{10}$$

$$DA = \sqrt{(4-1)^2 + (5-2)^2}$$

$$\Rightarrow DA = \sqrt{(3)^2 + (3)^2}$$

$$\Rightarrow DA = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

Thus, $AB = CD$ and $BC = DA$

$$\text{Now, } AC = \sqrt{(4-4)^2 + (3-5)^2}$$

$$\Rightarrow AC = \sqrt{0+(-2)^2}$$

$$\Rightarrow AC = \sqrt{4} = 2$$

$$BD = \sqrt{(1-7)^2 + (2-6)^2}$$

$$\begin{aligned}\Rightarrow BD &= \sqrt{(-6)^2 + (-4)^2} \\ \Rightarrow BD &= \sqrt{36 + 16} \\ \Rightarrow BD &= \sqrt{52} = \sqrt{2 \times 2 \times 13} \\ &= 2\sqrt{13}\end{aligned}$$

$\therefore$ Diagonal AC $\neq$ diagonal BD.

Since, opposite sides are equal but diagonals are not equal.

Therefore, the given points form a parallelogram.

Q.7. Find the point on the x -axis which is equidistant from $(2, -5)$ and $(-2, 9)$. (CBSE 2022)

Sol. Let the given points be $A(2, -5)$ and $B(-2, 9)$ and let the required point be $P(x, 0)$, then

$$\begin{aligned}PA &= PB \\ PA^2 &= PB^2 \\ \Rightarrow (x-2)^2 + (0+5)^2 &= (x+2)^2 + (0-9)^2 \\ \Rightarrow x^2 + 4 - 4x + 25 &= x^2 + 4 + 4x + 81 \\ \Rightarrow x^2 - 4x + 29 &= x^2 + 4x + 85 \\ \Rightarrow x^2 - 4x - x^2 - 4x &= 85 - 29 \\ \Rightarrow -8x &= 56. \Rightarrow x = \frac{56}{-8} \Rightarrow x = -7\end{aligned}$$

Hence, the required point = $(-7, 0)$

Q.8. Find the values of y for which the distance between the points $P(2, -3)$ and $Q(10, y)$ is 10 units.

Sol. The given points are $P(2, -3)$ and $Q(10, y)$

$$\begin{aligned}\text{and } PQ &= 10 \text{ units} \\ \therefore PQ &= \sqrt{(10-2)^2 + (y+3)^2} \\ \Rightarrow 10 &= \sqrt{(8)^2 + y^2 + 9 + 6y} \\ \Rightarrow 10 &= \sqrt{64 + y^2 + 9 + 6y} \\ \Rightarrow 10 &= \sqrt{73 + y^2 + 6y} \\ \Rightarrow 100 &= 73 + y^2 + 6y \\ \Rightarrow y^2 + 6y + 73 - 100 &= 0 \\ \Rightarrow y^2 + 6y - 27 &= 0 \\ \Rightarrow y^2 + (9-3)y - 27 &= 0 \\ \Rightarrow y^2 + 9y - 3y - 27 &= 0 \\ \Rightarrow y(y+9) - 3(y+9) &= 0 \\ \Rightarrow (y+9)(y-3) &= 0 \\ \Rightarrow y+9 = 0 \text{ or } y-3 &= 0 \\ \Rightarrow y = -9 \text{ or } y = 3.\end{aligned}$$

Hence, $y = -9$ or 3 .

Q.9. If $Q(0, 1)$ is equidistant from $P(5, -3)$ and $R(x, 6)$, find the values of x . Also find the distances QR and PR.

Sol. The given points are $P(5, -3)$, $Q(0, 1)$ and $R(x, 6)$

$\therefore Q$ is the equidistant from P and R .

$$\begin{aligned}\therefore PQ &= RQ \\ \Rightarrow PQ^2 &= RQ^2 \\ \Rightarrow (0-5)^2 + (1+3)^2 &= (0-x)^2 + (1-6)^2 \\ \Rightarrow (-5)^2 + (4)^2 &= (-x)^2 + (-5)^2 \\ \Rightarrow 25 + 16 &= x^2 + 25 \\ \Rightarrow 25 - 25 + 16 &= x^2 \\ \Rightarrow x^2 &= 16 \\ \Rightarrow x &= \pm 4\end{aligned}$$

Case I If $x = 4$

$$\begin{aligned}\text{Now, } QR &= \sqrt{(x-0)^2 + (6-1)^2} \\ \Rightarrow QR &= \sqrt{x^2 + (5)^2} \\ \Rightarrow QR &= \sqrt{(4)^2 + 25} \\ &= \sqrt{16 + 25} \\ &= \sqrt{41} \quad [\because x = 4]\end{aligned}$$

$$\begin{aligned}\text{and } PR &= \sqrt{(x-5)^2 + (6+3)^2} \\ &= \sqrt{(4-5)^2 + (9)^2} \quad [\because x = 4] \\ \Rightarrow PR &= \sqrt{(-1)^2 + 81} \\ \Rightarrow PR &= \sqrt{1 + 81} = \sqrt{82}\end{aligned}$$

Case II If $x = -4$

$$\begin{aligned}QR &= \sqrt{(x-0)^2 + (6-1)^2} \\ \Rightarrow QR &= \sqrt{x^2 + (5)^2} \\ &= \sqrt{(-4)^2 + 25} \\ &= \sqrt{16 + 25} = \sqrt{41} \quad [\because x = -4] \\ \Rightarrow QR &= \sqrt{16 + 25} = \sqrt{41} \\ \text{and } PR &= \sqrt{(x-5)^2 + (6+3)^2} \\ \Rightarrow PR &= \sqrt{(-4-5)^2 + (9)^2} \\ \Rightarrow PR &= \sqrt{(-9)^2 + 81} \\ \Rightarrow PR &= \sqrt{81 + 81} \\ \Rightarrow PR &= \sqrt{162} = \sqrt{9 \times 9 \times 2} \\ &= 9\sqrt{2}\end{aligned}$$

Hence, $x = \pm 4$, $QR = \sqrt{41}$ and $PR = \sqrt{82}$ or $9\sqrt{2}$.

Q.10. Find a relation between x and y such that the point (x, y) is equidistant from the points $(3, 6)$ and $(-3, 4)$.

Sol. Let $P(x, y)$ be the equidistant from the points $A(3, 6)$ and $B(-3, 4)$, then

$$\begin{aligned} PA &= PB \\ \Rightarrow PA^2 &= PB^2 \\ \Rightarrow (3-x)^2 + (6-y)^2 &= (-3-x)^2 + (4-y)^2 \\ \Rightarrow 9 + x^2 - 6x + 36 + y^2 - 12y &= 9 + x^2 + 6x + 16 + y^2 - 8y \\ \Rightarrow x^2 + y^2 + 6x - 8y + 6x + 12y - x^2 - y^2 &= 45 - 25 \\ \Rightarrow 12x + 4y &= 20 \\ \Rightarrow 3x + y &= 5 \\ \Rightarrow 3x + y - 5 &= 0 \end{aligned}$$

Hence, the required relation is $3x + y - 5 = 0$

Textbook Exercise 7.2

Q.1. Find the co-ordinates of the point which divides the join of $(-1, 7)$ and $(4, -3)$ in the ratio $2 : 3$.

Sol. Let $P(x, y)$ be the required point

By section formula co-ordinates of P are

$$\begin{aligned} A(-1, 7) & \quad P(x, y) \quad B(4, -3) \\ \text{Lengths: } 2 & \quad 3 \end{aligned}$$

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$$

$$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

Here, $x_1 = -1$, $y_1 = 7$, $x_2 = 4$, $y_2 = -3$ and $m_1 : m_2 = 2 : 3$

$$\Rightarrow x = \frac{2 \times 4 + 3 \times (-1)}{2 + 3}$$

$$y = \frac{2 \times (-3) + 3 \times 7}{2 + 3}$$

$$\Rightarrow x = \frac{8-3}{5}; y = \frac{-6+21}{5}$$

$$\Rightarrow x = \frac{5}{5}, y = \frac{15}{5} \Rightarrow x = 1, y = 3$$

Hence, the co-ordinates of required point are $(1, 3)$.

Q.2. Find the co-ordinates of the points of trisection of the line segment joining $(4, -1)$ and $(-2, -3)$.

Sol. Let P and Q be the points of trisection of AB .

$$\text{i.e.} \quad AP = PQ = QB$$

$$\text{Let} \quad AP = PQ = QB = K$$

$$A(4, -1) \quad P \quad Q \quad B(-2, -3)$$

$$PB = PQ + QB = 2K$$

$$\text{and,} \quad AQ = AP + PQ = 2K$$

$$\therefore AP : PB = K : 2K = 1 : 2$$

$$\text{and,} \quad AQ : QB = 2K : K = 2 : 1.$$

Therefore, P divides AB in the ratio $1 : 2$.

By section formula the co-ordinates of P are

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$$

$$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

$$\Rightarrow x = \frac{1 \times (-2) + 2 \times (4)}{1 + 2}$$

$$y = \frac{1 \times (-3) + 2 \times (-1)}{1 + 2}$$

$$\Rightarrow x = \frac{-2+8}{3}, y = \frac{-3-2}{3}$$

$$\Rightarrow x = \frac{6}{3}, y = -\frac{5}{3}$$

$$\Rightarrow x = 2, y = -\frac{5}{3}$$

So, the co-ordinates of P are $(2, -\frac{5}{3})$

Now, Q divides AB in the ratio $2 : 1$.

By section formula, co-ordinates of Q are

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$$

$$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

$$\Rightarrow x = \frac{2 \times (-2) + 1 \times (4)}{2 + 1}$$

$$y = \frac{2 \times (-3) + 1 \times (-1)}{2 + 1}$$

$$\Rightarrow x = \frac{-4+4}{3}; y = \frac{-6-1}{3}$$

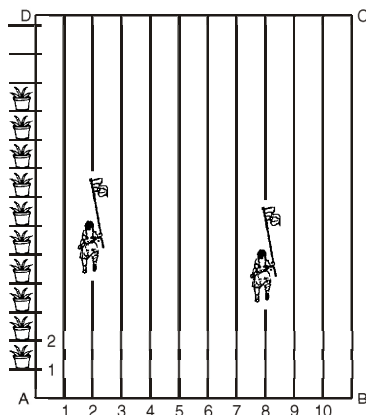
$$\Rightarrow x = 0, y = -\frac{7}{3}$$

So, the co-ordinates of Q are $(0, -\frac{7}{3})$

Hence, co-ordinates of trisection are $(2, -\frac{5}{3})$

and $(0, -\frac{7}{3})$

Q.3. To conduct sports day activities in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1m each. 100 flower pots have been placed at a distance of 1m from each other along AD, as shown in figure. Niharika runs $\frac{1}{4}$ th the distance of AD on the 2nd line and posts a green flag. Preet runs $\frac{1}{5}$ th the distance AD on the eighth line and posts a red flag. What is the distance between



both the flags? If Rashmi has to post a blue flag exactly half way between the line segment joining the two flags, where should she post her flag?

Sol. From the figure taking A as origin (0, 0), x -axis along AB and y -axis along AD, we will determine the co-ordinates of the green flag and red flag.

Now, the green flag in 2nd line and its distance parallel to AD.

$$AD = \frac{1}{4} \times 100 = 25 \text{ m.}$$

So, the co-ordinates of green flag are (2, 25) we mark this position as G.

Similarly, co-ordinates of red flag = (8, 20), we mark this position as R.

Now, the distance between G and R

$$= \sqrt{(8-2)^2 + (20-25)^2} = \sqrt{(6)^2 + (-5)^2}$$

$$= \sqrt{36 + 25} = \sqrt{61}$$

Position of the blue flag is the mid point of GR.

Co-ordinates of blue flag

$$= \left(\frac{2+8}{2}, \frac{25+20}{2} \right) = \left(\frac{10}{2}, \frac{45}{2} \right) = (5, 22.5)$$

It means that blue flag is in the 5th line and at a distance of 22.5 m along the direction parallel to AD.

Hence, distance between G and R = $\sqrt{61}$ m, and blue flag in 5th line at a distance of 22.5 m.

Q.4. Find the ratio in which the line segment joining the points $(-3, 10)$ and $(6, -8)$ is divided by $(-1, 6)$.

Sol. Let the required ratio be $m_1 : m_2$

By section formula, we have

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$$

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

$$\Rightarrow -1 = \frac{m_1 \times 6 + m_2 \times (-3)}{m_1 + m_2}$$

$$6 = \frac{m_1 \times (-8) + m_2 \times 10}{m_1 + m_2}$$

$$\Rightarrow -1 = \frac{6m_1 - 3m_2}{m_1 + m_2}$$

$$6 = \frac{-8m_1 + 10m_2}{m_1 + m_2}$$

$$\text{Now, } \frac{6m_1 - 3m_2}{m_1 + m_2} = -1$$

$$\Rightarrow 6m_1 - 3m_2 = -m_1 - m_2$$

$$\Rightarrow 6m_1 + m_1 = 3m_2 - m_2$$

$$\Rightarrow 7m_1 = 2m_2 \Rightarrow \frac{m_1}{m_2} = \frac{2}{7}$$

$$\Rightarrow m_1 : m_2 = 2 : 7$$

Hence, the required ratio is 2 : 7

Q.5. Find the ratio in which the line segment joining A(1, -5) and B(-4, 5) is divided by the x -axis. Also find the coordinates of the point of division.

Sol. Let the required ratio be $k : 1$

By section formula, we have

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$$

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

$$\Rightarrow x = \frac{k \times (-4) + 1 \times 1}{k + 1}$$

$$y = \frac{k \times 5 + 1 \times (-5)}{k + 1}$$

$$\Rightarrow x = \frac{-4k + 1}{k + 1}; \quad y = \frac{5k - 5}{k + 1}$$

Since, point lies on x-axis, so its ordinate is zero.

$$\Rightarrow 0 = \frac{5k - 5}{k + 1}$$

$$\Rightarrow 5k - 5 = 0$$

$$\Rightarrow k = \frac{5}{5} = 1$$

So, the required ratio is 1 : 1

Putting the value of k we get co-ordinates of the point of division.

$$x = \frac{-4 \times 1 + 1}{1 + 1}; \quad y = \frac{5 \times 1 - 5}{1 + 1}$$

$$\Rightarrow x = \frac{-4 + 1}{2}; \quad y = \frac{5 - 5}{2}$$

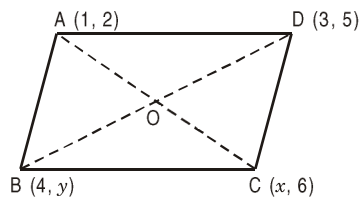
$$\Rightarrow x = \frac{-3}{2}; \quad y = 0$$

Hence, required ratio = 1 : 1 and co-ordinates of the point of division = $\left(\frac{-3}{2}, 0\right)$

Q.6. If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y .

Sol. Let given vertices of a parallelogram be A(1, 2), B(4, y), C(x, 6) and D(3, 5). Join AC and BD.

We know that diagonals of a parallelogram bisect each other



So, O is the mid point of AC as well as that of BD. Co-ordinates of mid point of AC = Co-ordinates of mid point of BD.

$$\Rightarrow \left(\frac{1+x}{2}, \frac{2+6}{2}\right) = \left(\frac{4+3}{2}, \frac{y+5}{2}\right)$$

$$\Rightarrow \left(\frac{1+x}{2}, \frac{8}{2}\right) = \left(\frac{7}{2}, \frac{y+5}{2}\right)$$

$$\Rightarrow \left(\frac{1+x}{2}, 4\right) = \left(\frac{7}{2}, \frac{y+5}{2}\right)$$

$$\Rightarrow \frac{1+x}{2} = \frac{7}{2} \text{ and } \frac{y+5}{2} = 4$$

$$\Rightarrow 1+x = \frac{7 \times 2}{2} \text{ and } y+5 = 8$$

$$\Rightarrow x = 7-1 \text{ and } y = 8-5$$

$$\Rightarrow x = 6 \text{ and } y = 3$$

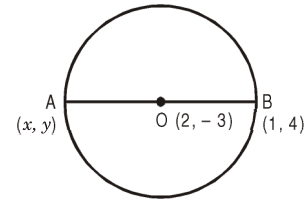
Hence, $x = 6$ and $y = 3$

Q.7. Find the co-ordinates of a point A, where AB is the diameter of a circle whose centre is (2, -3) and B is (1, 4).

Sol. Let the co-ordinates of point A be (x, y).

$\therefore$ O is the mid point of AB

$\therefore$ Co-ordinates of O are



$$2 = \frac{x+1}{2} \text{ and } -3 = \frac{y+4}{2}$$

$$\Rightarrow 4 = x + 1$$

$$\text{and } -6 = y + 4$$

$$\Rightarrow 3 = x \text{ and } -10 = y$$

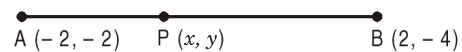
$$\Rightarrow x = 3, \text{ and } y = -10$$

Hence, co-ordinates of point A are (3, -10).

Q.8. If A and B are (-2, -2) and (2, -4), respectively, find the co-ordinates of P such that

$AP = \frac{3}{7} AB$ and P lies on the line segment AB.

Sol. Let the co-ordinates of P be (x, y), we have



$$AP = \frac{3}{7} AB$$

$$AP + PB = AB$$

$$\Rightarrow \frac{3}{7} AB + PB = AB \Rightarrow PB = \frac{AB}{1} - \frac{3}{7} AB.$$

$$\Rightarrow PB = \frac{4}{7} AB$$

$$\therefore \frac{AP}{PB} = \frac{\frac{3}{7} AB}{\frac{4}{7} AB} \Rightarrow \frac{AP}{PB} = \frac{3}{7} \times \frac{7}{4}$$

$$\Rightarrow \frac{AP}{PB} = \frac{3}{4} \Rightarrow AP : PB = 3 : 4$$

By section formula, the co-ordinates of point P are

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}; y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

$$\Rightarrow x = \frac{3 \times 2 + 4 \times (-2)}{3 + 4}$$

$$y = \frac{3 \times (-4) + 4 \times (-2)}{3 + 4}$$

$$\Rightarrow x = \frac{6 - 8}{7}; y = \frac{-12 - 8}{7}$$

$$\Rightarrow x = \frac{-2}{7}; y = -\frac{20}{7}$$

Hence, co-ordinates of P are $\left(-\frac{2}{7}, -\frac{20}{7}\right)$.

Q.9. Find the co-ordinates of the points which divide the line segment joining A(-2, 2) and B(2, 8) into four equal parts.

Sol. Let P, Q, R be three points that divides the line segment joining A(-2, 2) and B(2, 8) in four equal parts.



Since Q is the mid point of AB.

Therefore, co-ordinates of point Q are

$$\left(-\frac{2+2}{2}, \frac{2+8}{2}\right) \text{ i.e. } (0, 5)$$

Similarly, P is the mid point of AQ.

Therefore, co-ordinates of P are

$$\left(\frac{-2+0}{2}, \frac{2+5}{2}\right) \text{ i.e., } \left(-1, \frac{7}{2}\right)$$

and R is the mid point of QB.

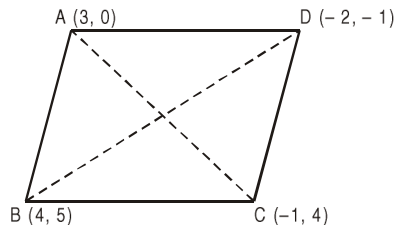
Therefore, co-ordinates of R are

$$\left(\frac{0+2}{2}, \frac{5+8}{2}\right) \text{ i.e., } \left(1, \frac{13}{2}\right)$$

Hence, the required co-ordinates are

$$\left(-1, \frac{7}{2}\right), (0, 5) \text{ and } \left(1, \frac{13}{2}\right).$$

Q.10. Find the area of a rhombus if its vertices are (3, 0), (4, 5), (-1, 4) and (-2, -1) taken in order.



Sol. Let co-ordinates of vertices of a rhombus be A(3, 0), B(4, 5), C(-1, 4) and D(-2, -1), then

$$AC = \sqrt{(-1-3)^2 + (4-0)^2}$$

$$\Rightarrow AC = \sqrt{(-4)^2 + (4)^2}$$

$$\Rightarrow AC = \sqrt{16 + 16}$$

$$\Rightarrow AC = \sqrt{32} = \sqrt{2 \times 2 \times 2 \times 2 \times 2}$$

$$= 4\sqrt{2}$$

and,

$$BD = \sqrt{(-2-4)^2 + (-1-5)^2}$$

$$\Rightarrow BD = \sqrt{(-6)^2 + (-6)^2}$$

$$\Rightarrow BD = \sqrt{36 + 36}$$

$$\Rightarrow BD = \sqrt{72}$$

$$\Rightarrow BD = \sqrt{6 \times 6 \times 2}$$

$$\Rightarrow BD = 6\sqrt{2}$$

Now, area of rhombus

$$= \frac{1}{2} (\text{Product of diagonals})$$

$$= \frac{1}{2} \times AC \times BD$$

$$= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$$

$$= 24 \text{ square units}$$

Hence, area of rhombus = 24 square units.

Textbook Exercises 8.1

Q.1. In $\triangle ABC$, right-angled at B, $AB = 24$ cm, $BC = 7$ cm. Determine :

- (i) $\sin A$, $\cos A$ (ii) $\sin C$, $\cos C$

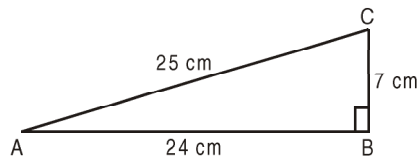
Sol. (i) In a right triangle ABC we have $\angle B = 90^\circ$, $AB = 24$ cm and $BC = 7$ cm.

By Pythagoras theorem, we have

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow AC^2 = 24^2 + 7^2 = 576 + 49$$

$$\Rightarrow AC^2 = 625 \Rightarrow AC = \sqrt{625} = 25$$



$$\therefore \sin A = \frac{BC}{AC} = \frac{7}{25} \text{ and } \cos A = \frac{AB}{AC} = \frac{24}{25}$$

$$\text{Hence, } \sin A = \frac{7}{25}, \cos A = \frac{24}{25}$$

- (ii) For trigonometric ratio of $\angle C$,

We have base = $BC = 7$ cm

Perpendicular = $AB = 24$ cm

Hypotenuse = $AC = 25$ cm

$$\therefore \sin C = \frac{AB}{AC} = \frac{24}{25} \text{ and } \cos C = \frac{BC}{AC} = \frac{7}{25}$$

$$\text{Hence, } \sin C = \frac{24}{25}, \cos C = \frac{7}{25}$$

Q.2. In figure, find $\tan P - \cot R$.

Sol. We have, $PQ = 12$ cm, $PR = 13$ cm.

In right $\triangle PQR$, $\angle Q = 90^\circ$

By Pythagoras theorem, we have

$$PR^2 = QR^2 + PQ^2$$

$$\Rightarrow (13)^2 = QR^2 + (12)^2$$

$$\Rightarrow QR^2 = (13)^2 - (12)^2$$

$$\Rightarrow QR^2 = (13 + 12)(13 - 12)$$

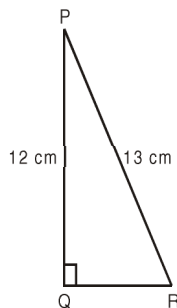
$$\Rightarrow QR^2 = 25$$

$$\Rightarrow QR = \sqrt{25} = 5$$

$$\cot R = \frac{QR}{PQ} = \frac{5}{12}$$

For trigonometric ratio of $\angle P$, we have

$$\text{Base} = PQ = 12 \text{ cm}$$



Perpendicular = $QR = 5$ cm

Hypotenuse = $PR = 13$ cm

$$\therefore \tan P = \frac{QR}{PQ} = \frac{5}{12}$$

$$\text{Now, } \tan P - \cot R = \frac{5}{12} - \frac{5}{12} = 0$$

Hence, $\tan P - \cot R = 0$.

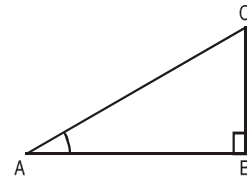
Q.3. If $\sin A = \frac{3}{4}$. Calculate $\cos A$ and $\tan A$.

Sol. Consider a right triangle ABC in which $\angle B = 90^\circ$

$$\text{Then } \sin A = \frac{BC}{AC} = \frac{3}{4}$$

Let $BC = 3k$ and $AC = 4k$, where k is a positive integer.

By Pythagoras theorem, we have



$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow (4k)^2 = AB^2 + (3k)^2 \Rightarrow AB^2 = (4k)^2 - (3k)^2$$

$$\Rightarrow AB^2 = 16k^2 - 9k^2 \Rightarrow AB^2 = 7k^2$$

$$\Rightarrow AB = \sqrt{7}k$$

$$\therefore \cos A = \frac{AB}{AC} = \frac{\sqrt{7}k}{4k} = \frac{\sqrt{7}}{4}$$

$$\text{and } \tan A = \frac{BC}{AB} = \frac{3k}{\sqrt{7}k} = \frac{3}{\sqrt{7}}$$

$$\text{Hence, } \cos A = \frac{\sqrt{7}}{4}, \tan A = \frac{3}{\sqrt{7}}$$

Q.4. Given $15 \cot A = 8$, find $\sin A$ and $\sec A$.

Sol. we have,

$$15 \cot A = 8;$$

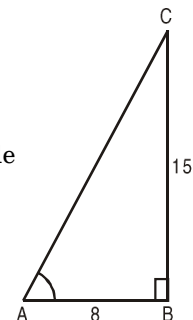
$$\Rightarrow \cot A = \frac{8}{15}$$

Now consider a right triangle

ABC, in which

$$\angle B = 90^\circ$$

$$\cot A = \frac{8}{15}$$



$$\Rightarrow \frac{AB}{BC} = \frac{8}{15}$$

Let $AB = 8k$ and $BC = 15k$, where k is a positive integer.

By Pythagoras theorem, we have

$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ \Rightarrow AC^2 &= (8k)^2 + (15k)^2 \Rightarrow AC^2 = 64k^2 + 225k^2 \\ \Rightarrow AC^2 &= 289k^2 \Rightarrow AC = \sqrt{289k^2} = 17k. \end{aligned}$$

$$\therefore \sin A = \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17}$$

$$\text{and } \sec A = \frac{AC}{AB} = \frac{17k}{8k} = \frac{17}{8}$$

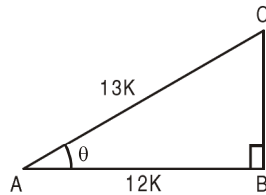
$$\text{Hence, } \sin A = \frac{15}{17} \text{ and } \sec A = \frac{17}{8}.$$

Q.5. Given $\sec \theta = \frac{13}{12}$, calculate all other trigonometric ratios.

Sol. Consider a right triangle ABC in which $\angle B = 90^\circ$,

$$\angle CAB = \theta, \sec \theta = \frac{AC}{AB} = \frac{13}{12}$$

Let $AC = 13k$ and $AB = 12k$, where k is a positive integer.



By Pythagoras theorem, we have

$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ \Rightarrow (13k)^2 &= (12k)^2 + BC^2 \\ \Rightarrow BC^2 &= (13k)^2 - (12k)^2 \\ \Rightarrow BC^2 &= (13k + 12k)(13k - 12k) \\ \Rightarrow BC^2 &= 25k \times k = 25k^2 \\ \Rightarrow BC &= \sqrt{25k^2} = 5k \end{aligned}$$

$$\therefore \sin \theta = \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}$$

$$\cos \theta = \frac{AB}{AC} = \frac{12k}{13k} = \frac{12}{13}$$

$$\tan \theta = \frac{BC}{AB} = \frac{5k}{12k} = \frac{5}{12}$$

$$\cot \theta = \frac{AB}{BC} = \frac{12k}{5k} = \frac{12}{5}$$

$$\operatorname{cosec} \theta = \frac{AC}{BC} = \frac{13k}{5k} = \frac{13}{5}$$

$$\text{Hence, } \sin \theta = \frac{5}{13}, \cos \theta = \frac{12}{13},$$

$$\tan \theta = \frac{5}{12}, \cot \theta = \frac{12}{5}, \operatorname{cosec} \theta = \frac{13}{5}.$$

Q.6. If $\angle A$ and $\angle B$ are acute angles such that $\cos A = \cos B$, then show that $\angle A = \angle B$.

Sol. Let us consider right triangle ABC in which $\angle C = 90^\circ$ and $\cos A = \cos B$.

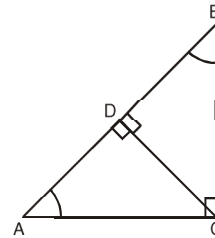
Draw $CD \perp AB$.

In right triangle ADC, $\angle ADC = 90^\circ$.

$$\cos A = \frac{AD}{AC} \quad \dots(1)$$

In right $\triangle BDC$, $\angle BDC = 90^\circ$

$$\cos B = \frac{BD}{BC} \quad \dots(2)$$



According to question, $\cos A = \cos B$

$$\Rightarrow \frac{AD}{AC} = \frac{BD}{BC} \Rightarrow \frac{AD}{BD} = \frac{AC}{BC} \quad [\text{each} = \frac{CD}{CD}]$$

$\Rightarrow \triangle ADC \sim \triangle BDC$ [By SSS similarity criterion]

$\Rightarrow \angle A = \angle B$ [corresponding $\angle$ s of two similar Δ s are equal]

Hence Proved

Q.7. If $\cot \theta = \frac{7}{8}$, evaluate :

$$(i) \frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)}$$

$$(ii) \cot^2 \theta$$

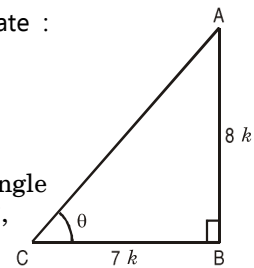
Sol. (i) Consider a triangle ABC in which $\angle B = 90^\circ$,

$\angle ACB = \theta$

$$\text{and } \cot \theta = \frac{7}{8}.$$

$$\Rightarrow \frac{BC}{AB} = \frac{7}{8}$$

Let $BC = 7k$ and $AB = 8k$, where k is a positive integer,



By Pythagoras theorem, we have

$$\begin{aligned} AC^2 &= BC^2 + AB^2 \\ \Rightarrow AC^2 &= (7k)^2 + (8k)^2 \Rightarrow AC^2 = 49k^2 + 64k^2 \\ \Rightarrow AC^2 &= 113k^2 \Rightarrow AC = \sqrt{113} k \end{aligned}$$

$$\therefore \sin \theta = \frac{AB}{AC} = \frac{8k}{\sqrt{113} k} = \frac{8}{\sqrt{113}}$$

$$\text{and } \cos \theta = \frac{BC}{AC} = \frac{7k}{\sqrt{113} k} = \frac{7}{\sqrt{113}}$$

$$\text{Now, } \frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)} = \frac{1 - \sin^2 \theta}{1 - \cos^2 \theta}$$

$$\begin{aligned} &= \frac{(1)^2 - \left(\frac{8}{\sqrt{113}}\right)^2}{(1)^2 - \left(\frac{7}{\sqrt{113}}\right)^2} \\ &[\because (a + b)(a - b) = a^2 - b^2] \\ &= \frac{1 - \frac{64}{113}}{1 - \frac{49}{113}} = \frac{\frac{113 - 64}{113}}{\frac{113 - 49}{113}} = \frac{49}{64} \end{aligned}$$

$$\begin{aligned} \text{(ii) } \cot^2 \theta &= (\cot \theta)^2 = \left(\frac{BC}{AB}\right)^2 = \left(\frac{7k}{8k}\right)^2 \\ &= \left(\frac{7}{8}\right)^2 = \frac{49}{64} \end{aligned}$$

$$\text{Hence, (i) } \frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)} = \frac{49}{64}$$

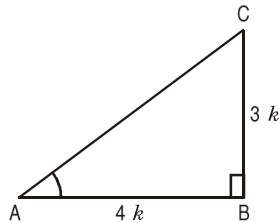
$$\text{(ii) } \cot^2 \theta = \frac{49}{64}$$

Q.8. If $3 \cot A = 4$, check whether $\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$ or not.

Sol. Consider a triangle ABC in which $\angle B = 90^\circ$ and $3 \cot A = 4$

$$\Rightarrow \cot A = \frac{4}{3} \Rightarrow \frac{AB}{BC} = \frac{4}{3}$$

Let $AB = 4k$ and $BC = 3k$, where k is a positive integer.



By Pythagoras theorem, we have

$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ \Rightarrow AC^2 &= (4k)^2 + (3k)^2 \Rightarrow AC^2 = 16k^2 + 9k^2 \\ \Rightarrow AC^2 &= 25k^2 \Rightarrow AC = \sqrt{25k^2} \Rightarrow AC = 5k \end{aligned}$$

$$\therefore \sin A = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5}$$

$$\cos A = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$$

$$\tan A = \frac{BC}{AB} = \frac{3k}{4k} = \frac{3}{4}$$

$$\begin{aligned} \text{L. H. S.} &= \frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - \left(\frac{3}{4}\right)^2}{1 + \left(\frac{3}{4}\right)^2} \\ &= \frac{1 - \frac{9}{16}}{1 + \frac{9}{16}} = \frac{\frac{16 - 9}{16}}{\frac{16 + 9}{16}} \\ &= \frac{7}{25} = \frac{7}{16} \times \frac{16}{25} = \frac{7}{25} \end{aligned}$$

and, R.H.S. = $\cos^2 A - \sin^2 A$

$$\begin{aligned} &= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} \\ &= \frac{16 - 9}{25} = \frac{7}{25} \therefore \text{L.H.S.} = \text{R.H.S.} \end{aligned}$$

$$\text{Hence, } \frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$$

Q.9. In triangle ABC, right angled at B, if

$\tan A = \frac{1}{\sqrt{3}}$, find the value of :

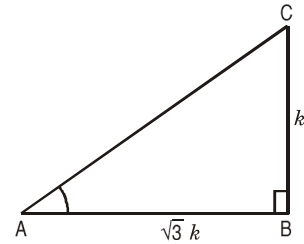
(i) $\sin A \cos C + \cos A \sin C$

(ii) $\cos A \cos C - \sin A \sin C$

Sol. In right triangle ABC, $\angle B = 90^\circ$

$$\text{and } \tan A = \frac{1}{\sqrt{3}} \Rightarrow \frac{BC}{AB} = \frac{1}{\sqrt{3}}$$

Let $BC = k$ and $AB = \sqrt{3}k$, where k is positive integer.



By Pythagoras theorem, we have

$$AC^2 = AB^2 + BC^2 \Rightarrow AC^2 = (\sqrt{3}k)^2 + k^2$$

$$\Rightarrow AC^2 = 3k^2 + k^2 = 4k^2$$

$$\Rightarrow AC = \sqrt{4k^2} = 2k$$

$$\therefore \sin A = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

$$\text{and } \cos A = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

Now, for trigonometric ratio of $\angle C$, we have

$$\text{Base} = BC = k$$

$$\text{Perpendicular} = AB = \sqrt{3}k$$

$$\text{Hypotenuse} = AC = 2k$$

$$\sin C = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\cos C = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

$$(i) \sin A \cos C + \cos A \sin C$$

$$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$$

$$(ii) \cos A \cos C - \sin A \sin C$$

$$= \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0.$$

Hence,

$$(i) \sin A \cos C + \cos A \sin C = 1$$

$$(ii) \cos A \cos C - \sin A \sin C = 0.$$

Q.10. In $\triangle PQR$, right angled at Q, $PR + QR = 25$ cm and $PQ = 5$ cm. Determine the values of $\sin P$, $\cos P$ and $\tan P$.

Sol. In right $\triangle PQR$, $\angle Q = 90^\circ$, $PQ = 5$ cm and $PR + QR = 25$ cm $\Rightarrow PR = 25 - QR$

By Pythagoras theorem, we have

$$PR^2 = PQ^2 + QR^2$$

$$(25 - QR)^2 = (5)^2 + QR^2$$

$$\Rightarrow 625 + QR^2 - 50QR = 25 + QR^2$$

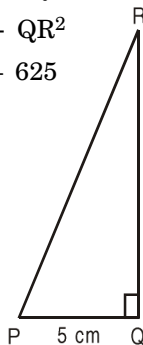
$$\Rightarrow QR^2 - 50QR - QR^2 = 25 - 625$$

$$\Rightarrow -50QR = -600 \Rightarrow QR$$

$$= \frac{-600}{-50} = 12 \text{ cm}$$

$$\text{and } PR = 25 - 12 = 13 \text{ cm}$$

$$\therefore \sin P = \frac{QR}{PR} = \frac{12}{13}$$



$$\cos P = \frac{PQ}{PR} = \frac{5}{13}$$

$$\tan P = \frac{QR}{PQ} = \frac{12}{5}$$

$$\text{Hence, } \sin P = \frac{12}{13}, \cos P = \frac{5}{13} \text{ and}$$

$$\tan P = \frac{12}{5}$$

Q.11. State whether the following are true or false. Justify your Answer.

(i) The value of $\tan A$ is always less than 1.

(ii) $\sec A = \frac{12}{5}$ for some value of $\angle A$.

(iii) $\cos A$ is the abbreviation used, for the cosecant of $\angle A$.

(iv) $\cot A$ is the product of $\cot$ and A .

(v) $\sin \theta = \frac{4}{3}$ for some angle θ .

Sol. (i) False. Because $\tan 60^\circ = \sqrt{3} > 1$

(ii) True. Because value of $\sec A$ is always ≥ 1 .

(iii) False. Because $\cos A$ is abbreviation used for cosine A .

(iv) False. Because $\cot$ is meaningless without an angle.

(v) False. Because $\sin \theta > 1$.

Textbook Exercises 8.2

Q.1. Evaluate the following :

$$(i) \sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$$

$$(ii) 2 \tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$$

$$(iii) \frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$$

$$(iv) \frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$$

$$(v) \frac{5 \cos^2 60^\circ + 4 \sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$$

Sol. (i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{3}{4} + \frac{1}{4} = 1$$

$$\text{Hence, } \sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$$

$$= \cos 60^\circ = 1.$$

(ii) $2 \tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

$$= 2 \times (1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= 2 \times 1 + \frac{3}{4} - \frac{3}{4} = 2$$

Hence, $2 \tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ = 2$.

$$(iii) \frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ} = \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + 2}$$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{2 + 2\sqrt{3}}{\sqrt{3}}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2 + 2\sqrt{3}}$$

$$= \frac{\sqrt{3}}{2\sqrt{2} + 2\sqrt{6}} = \frac{\sqrt{3}}{2(\sqrt{2} + \sqrt{6})}$$

$$= \frac{\sqrt{3} \times (\sqrt{2} - \sqrt{6})}{2(\sqrt{2} + \sqrt{6})(\sqrt{2} - \sqrt{6})}$$

[Multiplying numerator and denominator by $(\sqrt{2} - \sqrt{6})$]

$$= \frac{\sqrt{6} - \sqrt{18}}{2[(\sqrt{2})^2 - (\sqrt{6})^2]} = \frac{\sqrt{6} - 3\sqrt{2}}{2(2 - 6)}$$

$$= \frac{\sqrt{6} - 3\sqrt{2}}{2 \times (-4)} = \frac{(3\sqrt{2} - \sqrt{6})}{8}$$

Hence, $\frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

$$= \frac{1}{8}(3\sqrt{2} - \sqrt{6})$$

$$(iv) \frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$$

$$= \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1} = \frac{\frac{3}{2} - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{3}{2}} = \frac{\frac{3\sqrt{3} - 4}{2}}{\frac{4 + 3\sqrt{3}}{2\sqrt{3}}}$$

$$= \frac{3\sqrt{3} - 4}{2\sqrt{3}} \times \frac{2\sqrt{3}}{4 + 3\sqrt{3}}$$

$$= \frac{(3\sqrt{3} - 4)(4 - 3\sqrt{3})}{(4 + 3\sqrt{3})(4 - 3\sqrt{3})}$$

[Multiplying numerator and denominator by $(4 - 3\sqrt{3})$]

$$= \frac{-(4 - 3\sqrt{3})(4 - 3\sqrt{3})}{(4)^2 - (3\sqrt{3})^2} = \frac{-(4 - 3\sqrt{3})^2}{16 - 27}$$

$$= \frac{-(16 + 27 - 24\sqrt{3})}{-11} = \frac{43 - 24\sqrt{3}}{11}$$

Hence, $\frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

$$= \frac{43 - 24\sqrt{3}}{11}$$

$$(v) \frac{5 \cos^2 60^\circ + 4 \sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$$

$$= \frac{5 \times \left(\frac{1}{2}\right)^2 + 4 \times \left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{5 \times \frac{1}{4} + 4 \times \frac{4}{3} - 1}{\frac{1}{4} + \frac{3}{4}}$$

$$= \frac{\frac{5}{4} + \frac{16}{3} - 1}{\frac{1 + 3}{4}} = \frac{\frac{15 + 64 - 12}{12}}{\frac{4}{4}} = \frac{67}{12} = \frac{67}{12}$$

Hence,

$$\frac{5 \cos^2 60^\circ + 4 \sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ} = \frac{67}{12}$$

Q.2. Choose the correct option and justify your choice:

$$(i) \frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ} =$$

- (a) $\sin 60^\circ$ (b) $\cos 60^\circ$
(c) $\tan 60^\circ$ (d) $\sin 30^\circ$

$$(ii) \frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} =$$

- (a) $\tan 90^\circ$ (b) 1
(c) $\sin 45^\circ$ (d) 0

$$(iii) \sin 2A = 2 \sin A \text{ is true when } A =$$

- (a) 0° (b) 30°
(c) 45° (d) 60°

$$(iv) \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} =$$

- (a) $\cos 60^\circ$ (b) $\sin 60^\circ$
(c) $\tan 60^\circ$ (d) $\sin 30^\circ$

Sol. (i) (a)

$$\therefore \frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ} = \frac{2 \times \frac{1}{\sqrt{3}}}{1 + \left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{\frac{2}{\sqrt{3}}}{1 + \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}} = \frac{2}{\sqrt{3}} \times \frac{3}{4} = \frac{\sqrt{3}}{2} = \sin 60^\circ$$

(ii) (d)

$$\begin{aligned}\therefore \frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} &= \frac{1 - (1)^2}{1 + (1)^2} \\ &= \frac{1 - 1}{1 + 1} = \frac{0}{2} = 0\end{aligned}$$

(iii) (a) $\therefore$ When $A = 0^\circ$

$$\begin{aligned}\text{L.H.S.} &= \sin 2A = \sin 2 \times 0^\circ \\ &= \sin 0^\circ = 0\end{aligned}$$

$$\begin{aligned}\text{R.H.S.} &= 2 \sin A = 2 \times \sin 0^\circ \\ &= 2 \times 0 = 0\end{aligned}$$

in other option we shall find that

$$\text{L.H.S.} \neq \text{R.H.S.}$$

(iv) (c)

$$\begin{aligned}\therefore \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} &= \frac{2 \times \frac{1}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} \\ &= \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \frac{2}{\sqrt{3}} \times \frac{3}{2} = \sqrt{3} = \tan 60^\circ\end{aligned}$$

Q.3. If $\tan(A + B) = \sqrt{3}$ and $\tan(A - B) = \frac{1}{\sqrt{3}}$; $0^\circ < (A + B) \leq 90^\circ$, $A > B$, find A and B .

Sol. We have

$$\begin{aligned}\tan(A + B) &= \sqrt{3} \\ \Rightarrow \tan(A + B) &= \tan 60^\circ \\ \Rightarrow A + B &= 60^\circ \quad \dots(1)\end{aligned}$$

$$\begin{aligned}\text{and } \tan(A - B) &= \frac{1}{\sqrt{3}} \\ \Rightarrow \tan(A - B) &= \tan 30^\circ \\ \Rightarrow A - B &= 30^\circ \quad \dots(2)\end{aligned}$$

Adding equation (1) and (2), we get

$$\begin{aligned}A + B &= 60^\circ \\ A - B &= 30^\circ \\ 2A &= 90^\circ\end{aligned}$$

$$\Rightarrow A = \frac{90^\circ}{2} \Rightarrow A = 45^\circ$$

Substituting the value of A in equation (1), we get $45^\circ + B = 60^\circ$

$$\Rightarrow B = 60^\circ - 45^\circ = 15^\circ$$

Hence, $A = 45^\circ$ and $B = 15^\circ$.

Q.4. State whether the following are true or false. Justify your answer.

(i) $\sin(A + B) = \sin A + \sin B$.(ii) The value of $\sin \theta$ increases as θ increases.(iii) The value of $\cos \theta$ increases as θ increases.(iv) $\sin \theta = \cos \theta$ for all values of θ .(v) $\cot A$ is not defined for $A = 0^\circ$.**Sol.** (i) False. Taking $A = 30^\circ$, $B = 60^\circ$

$$\text{LHS} = \sin(A + B) = \sin(30^\circ + 60^\circ)$$

$$= \sin 90^\circ = 1$$

$$\text{RHS} = \sin A + \sin B = \sin 30^\circ + \sin 60^\circ$$

$$= \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1 + \sqrt{3}}{2}$$

$$\therefore \text{LHS} \neq \text{RHS}$$

(ii) True.

(iii) False. Because value of $\cos \theta$ decreases as θ increases.(iv) False. Because $\sin 30^\circ$

$$= \frac{1}{2} \text{ and } \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\text{i.e., } \sin 30^\circ \neq \cos 30^\circ$$

(v) True. Because $\cot 0^\circ = \infty$ (not defined)

Textbook Exercises 8.3

Q.1. Express the trigonometric ratios $\sin A$, $\sec A$ and $\tan A$ in terms of $\cot A$.

Write the trigonometric ratio of $\sin A$ in terms of $\cot A$.**Sol.** We have, $\text{cosec}^2 A = 1 + \cot^2 A$

$$\Rightarrow \text{cosec } A = \sqrt{1 + \cot^2 A}$$

$$\Rightarrow \frac{1}{\sin A} = \sqrt{1 + \cot^2 A}$$

$$\Rightarrow \sin A = \frac{1}{\sqrt{1 + \cot^2 A}}$$

[Taking reciprocal of both sides]

$$\text{We have, } \sec^2 A = 1 + \tan^2 A$$

$$\Rightarrow \sec^2 A = 1 + \frac{1}{\cot^2 A} \Rightarrow \sec^2 A = \frac{\cot^2 A + 1}{\cot^2 A}$$

$$\Rightarrow \sec A = \sqrt{\frac{\cot^2 A + 1}{\cot^2 A}} = \frac{\sqrt{\cot^2 A + 1}}{\cot A}$$

$$\text{and } \tan A = \frac{1}{\cot A}$$

$$\text{Hence, } \sin A = \frac{1}{\sqrt{1 + \cot^2 A}},$$

$$\sec A = \frac{\sqrt{\cot^2 A + 1}}{\cot A} \text{ and } \tan A = \frac{1}{\cot A}.$$

Q.2. Write all the other trigonometric ratios of A in terms of sec A.

Express the trigonometric ratio of tan A in terms of sec A.

Sol. We have, $\sin^2 A + \cos^2 A = 1$

$$\sin^2 A = 1 - \cos^2 A$$

$$\Rightarrow \sin^2 A = 1 - \frac{1}{\sec^2 A}$$

$$\Rightarrow \sin^2 A = \frac{\sec^2 A - 1}{\sec^2 A}$$

$$\Rightarrow \sin A = \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}}$$

$$\Rightarrow \sin A = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

$$\therefore \cos A = \frac{1}{\sec A}$$

$$\text{and } \tan A = \frac{\sin A}{\cos A} = \frac{\frac{\sqrt{\sec^2 A - 1}}{\sec A}}{\frac{1}{\sec A}}$$

$$\Rightarrow \tan A = \frac{\sqrt{\sec^2 A - 1}}{\sec A} \times \frac{\sec A}{1} = \sqrt{\sec^2 A - 1}$$

$$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{1}{\frac{\sqrt{\sec^2 A - 1}}{\sec A}}$$

$$\Rightarrow \operatorname{cosec} A = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

$$\cot A = \frac{1}{\tan A} = \frac{1}{\sqrt{\sec^2 A - 1}}$$

$$\text{Hence, } \sin A = \frac{\sqrt{\sec^2 A - 1}}{\sec A},$$

$$\cos A = \frac{1}{\sec A}, \tan A = \sqrt{\sec^2 A - 1},$$

$$\cot A = \frac{1}{\sqrt{\sec^2 A - 1}}, \text{ and}$$

$$\operatorname{cosec} A = \frac{\sec A}{\sqrt{\sec^2 A - 1}}.$$

Q.3. Choose the correct option. Justify your choice :

(i) $9 \sec^2 A - 9 \tan^2 A =$

- (A) 1 (B) 9 (C) 8 (D) 0

(ii) $(1 + \tan \theta + \sec \theta)(1 + \cot \theta - \operatorname{cosec} \theta) =$

- (A) 0 (B) 1 (C) 2 (D) -1

(iii) $(\sec A + \tan A)(1 - \sin A) =$

- (A) sec A (B) sin A
(C) cosec A (D) cos A

(iv) $\frac{1 + \tan^2 A}{1 + \cot^2 A} =$

- (A) $\sec^2 A$ (B) -1
(C) $\cot^2 A$ (D) $\tan^2 A$

Sol. (i) (B)

$$\text{Because; } 9 \sec^2 A - 9 \tan^2 A$$

$$= 9 (\sec^2 A - \tan^2 A)$$

$$= 9 \times 1$$

$$[\because \sec^2 A - \tan^2 A = 1]$$

$$= 9$$

(ii) (C) Because ;

$$(1 + \tan \theta + \sec \theta)(1 + \cot \theta - \operatorname{cosec} \theta)$$

$$= \left(1 + \frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta}\right) \left(1 + \frac{\cos \theta}{\sin \theta} - \frac{1}{\sin \theta}\right)$$

$$= \left(\frac{\cos \theta + \sin \theta + 1}{\cos \theta}\right) \left(\frac{\sin \theta + \cos \theta - 1}{\sin \theta}\right)$$

$$= \frac{(\cos \theta + \sin \theta)^2 - (1)^2}{\sin \theta \cos \theta}$$

$$= \frac{\cos^2 \theta + \sin^2 \theta + 2 \sin \theta \cos \theta - 1}{\sin \theta \cos \theta}$$

$$= \frac{1 + 2 \sin \theta \cos \theta - 1}{\sin \theta \cos \theta}$$

$$= \frac{2 \sin \theta \cos \theta - 1}{\sin \theta \cos \theta} = 2$$

(iii) (D) Because; $(\sec A + \tan A)$

$$(1 - \sin A)$$

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$$

$$= \frac{(1 + \sin A)(1 - \sin A)}{\cos A}$$

$$= \frac{1 - \sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A} = \cos A$$

$$[\because 1 - \sin^2 A = \cos^2 A]$$

(iv) (D) Because;

$$\frac{1 + \tan^2 A}{1 + \cot^2 A} = \frac{\sec^2 A}{\operatorname{cosec}^2 A}$$

$$\left[\because \begin{aligned} 1 + \tan^2 A &= \sec^2 A \\ 1 + \cot^2 A &= \operatorname{cosec}^2 A \end{aligned} \right]$$

$$\begin{aligned}
 &= \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}} = \frac{1}{\cos^2 A} \times \frac{\sin^2 A}{1} \\
 &= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A
 \end{aligned}$$

$$\left[\because \frac{\sin A}{\cos A} = \tan A \right]$$

Q.4. Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

(i) $(\operatorname{cosec} \theta - \cot \theta)^2 = \frac{1 - \cos \theta}{1 + \cos \theta}$

(ii) $\frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2 \sec A$

(iii) $\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \operatorname{cosec} \theta$

(iv) $\frac{1 + \sec A}{\sec A} = \frac{\sin^2 A}{1 - \cos A}$

(v) $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$,

using the identity

$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

(vi) $\sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$

(vii) $\frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta} = \tan \theta$

(viii) $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$. **[CBSE 2019]**

(ix) $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$

(x) $\left(\frac{1 + \tan^2 A}{1 + \cot^2 A} \right) = \left(\frac{1 - \tan A}{1 - \cot A} \right)^2 = \tan^2 A$

Sol. (i) We have,

$$\begin{aligned}
 \text{LHS} &= (\operatorname{cosec} \theta - \cot \theta)^2 \\
 &= \left(\frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} \right)^2 \\
 &= \left(\frac{1 - \cos \theta}{\sin \theta} \right)^2 = \frac{(1 - \cos \theta)^2}{\sin^2 \theta} \\
 &= \frac{(1 - \cos \theta)^2}{1 - \cos^2 \theta} \\
 &\quad [\because \sin^2 \theta = 1 - \cos^2 \theta]
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{(1 - \cos \theta)^2}{(1 + \cos \theta)(1 - \cos \theta)} \\
 &= \frac{1 - \cos \theta}{1 + \cos \theta} = \text{R.H.S.}
 \end{aligned}$$

$\therefore$ L.H.S. = R.H.S. *Hence proved*

(ii) We have,

$$\begin{aligned}
 \text{L.H.S.} &= \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} \\
 &= \frac{\cos^2 A + (1 + \sin A)^2}{(1 + \sin A) \cos A} \\
 &= \frac{\cos^2 A + 1 + \sin^2 A + 2 \sin A}{(1 + \sin A) \cos A} \\
 &= \frac{\sin^2 A + \cos^2 A + 1 + 2 \sin A}{(1 + \sin A) \cos A} \\
 &= \frac{1 + 1 + 2 \sin A}{(1 + \sin A) \cos A} \\
 &\quad [\because \sin^2 \theta + \cos^2 \theta = 1] \\
 &= \frac{2 + 2 \sin A}{(1 + \sin A) \cos A} \\
 &= \frac{2(1 + \sin A)}{(1 + \sin A) \cos A} \\
 &= \frac{2}{\cos A} = 2 \sec A \\
 &= \text{R.H.S.} \quad \left[\because \frac{1}{\cos A} = \sec A \right]
 \end{aligned}$$

$\therefore$ L.H.S. = R.H.S. *Hence proved*

(iii) We have,

$$\begin{aligned}
 \text{L.H.S.} &= \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} \\
 &= \frac{\frac{\sin \theta}{\cos \theta}}{1 - \frac{\cos \theta}{\sin \theta}} + \frac{\frac{\cos \theta}{\sin \theta}}{1 - \frac{\sin \theta}{\cos \theta}} \\
 &= \frac{\frac{\sin \theta}{\cos \theta}}{\frac{\sin \theta - \cos \theta}{\sin \theta}} + \frac{\frac{\cos \theta}{\sin \theta}}{\frac{\cos \theta - \sin \theta}{\cos \theta}} \\
 &= \frac{\sin \theta}{\cos \theta} \times \frac{\sin \theta}{\sin \theta - \cos \theta} + \frac{\cos \theta}{\sin \theta} \times \frac{\cos \theta}{\cos \theta - \sin \theta} \\
 &= \frac{\sin^2 \theta}{\cos \theta(\sin \theta - \cos \theta)} - \frac{\cos^2 \theta}{\sin \theta(\sin \theta - \cos \theta)} \\
 &= \frac{\sin^3 \theta - \cos^3 \theta}{\sin \theta \cos \theta(\sin \theta - \cos \theta)} \\
 &= \frac{(\sin \theta - \cos \theta)(\sin^2 \theta + \cos^2 \theta \sin \theta \cos \theta)}{\sin \theta \cos \theta(\sin \theta - \cos \theta)} \\
 &\quad [\because a^3 - b^3 = (a - b)(a^2 + b^2 + ab)]
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{(1 + \sin \theta \cdot \cos \theta)}{\sin \theta \cos \theta} \\
 &\quad [\because \sin^2 \theta + \cos^2 \theta = 1] \\
 &= \frac{1}{\sin \theta \cos \theta} + \frac{\sin \theta \cdot \cos \theta}{\sin \theta \cos \theta} \\
 &= \operatorname{cosec} \theta \sec \theta + 1 \\
 &= 1 + \sec \theta \operatorname{cosec} \theta = \text{R.H.S.} \\
 \therefore \text{L.H.S.} &= \text{R.H.S.} \quad \text{Hence proved}
 \end{aligned}$$

(iv) We have, $\text{L.H.S.} = \frac{1 + \sec A}{\sec A}$

$$\begin{aligned}
 &= \frac{1 + \frac{1}{\cos A}}{\frac{1}{\cos A}} = \frac{\frac{\cos A + 1}{\cos A}}{\frac{1}{\cos A}} \\
 &= \frac{1 + \cos A}{\cos A} \times \frac{\cos A}{1} = 1 + \cos A \\
 &= \frac{(1 + \cos A)(1 - \cos A)}{(1 - \cos A)} \\
 &\quad [\text{Multiplying numerator and denominator by } (1 - \cos A)] \\
 &= \frac{1 - \cos^2 A}{(1 - \cos A)} = \frac{\sin^2 A}{1 - \cos A} = \text{R.H.S.} \\
 &\quad [\because 1 - \cos^2 A = \sin^2 A] \\
 \therefore \text{L.H.S.} &= \text{R.H.S.} \quad \text{Hence proved}
 \end{aligned}$$

(v) We have, $\text{L.H.S.} = \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$

$$\begin{aligned}
 &= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} - \frac{1}{\sin A}} \\
 &\quad [\text{Dividing numerator and denominator by } \sin A] \\
 &= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A} \\
 &= \frac{\operatorname{cosec} A + \cot A - 1}{(1 + \cot A - \operatorname{cosec} A)} \\
 &= \frac{(\operatorname{cosec} A + \cot A) - (\operatorname{cosec}^2 A - \cot^2 A)}{(1 + \cot A - \operatorname{cosec} A)} \\
 &\quad [\because 1 = \operatorname{cosec}^2 A - \cot^2 A] \\
 &= \frac{(\operatorname{cosec} A + \cot A) - (\operatorname{cosec} A + \cot A)}{(1 + \cot A - \operatorname{cosec} A)} \\
 &= \frac{(\operatorname{cosec} A + \cot A)[1 - (\operatorname{cosec} A - \cot A)]}{(1 - \operatorname{cosec} A + \cot A)}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{(\operatorname{cosec} A + \cot A)(1 - \operatorname{cosec} A + \cot A)}{(1 - \operatorname{cosec} A + \cot A)} \\
 &= \operatorname{cosec} A + \cot A = \text{R.H.S.} \\
 \therefore \text{L.H.S.} &= \text{R.H.S.} \quad \text{Hence proved}
 \end{aligned}$$

(vi) We have,

$$\begin{aligned}
 \text{L.H.S.} &= \sqrt{\frac{1 + \sin A}{1 - \sin A}} \\
 &= \sqrt{\frac{(1 + \sin A)(1 + \sin A)}{(1 - \sin A)(1 + \sin A)}} \\
 &\quad [\text{Multiplying numerator and denominator by } \sqrt{(1 + \sin A)}] \\
 &= \sqrt{\frac{(1 + \sin A)^2}{1 - \sin^2 A}} = \sqrt{\frac{(1 + \sin A)^2}{\cos^2 A}} \\
 &= \frac{1 + \sin A}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A} \\
 &= \sec A + \tan A = \text{R.H.S.} \\
 \therefore \text{L.H.S.} &= \text{R.H.S.} \quad \text{Hence proved}
 \end{aligned}$$

(vii) We have, $\text{L.H.S.} = \frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta}$

$$\begin{aligned}
 &= \frac{\sin \theta(1 - 2 \sin^2 \theta)}{\cos \theta(2 \cos^2 \theta - 1)} \\
 &= \frac{\sin \theta(1 - \sin^2 \theta - \sin^2 \theta)}{\cos \theta(\cos^2 \theta + \cos^2 \theta - 1)} \\
 &= \frac{\sin \theta(\cos^2 \theta - \sin^2 \theta)}{\cos \theta(\cos^2 \theta + 1 - \sin^2 \theta - 1)} \\
 &\quad [\because 1 - \sin^2 \theta = \cos^2 \theta] \\
 &= \frac{\sin \theta(\cos^2 \theta - \sin^2 \theta)}{\cos \theta(\cos^2 \theta - \sin^2 \theta)} \\
 &= \tan \theta = \text{R.H.S.} \\
 \therefore \text{L.H.S.} &= \text{R.H.S.} \quad \text{Hence proved}
 \end{aligned}$$

(viii) We have,

$$\begin{aligned}
 \text{L.H.S.} &= (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 \\
 &= \sin^2 A + \operatorname{cosec}^2 A + 2 \sin A \operatorname{cosec} A \\
 &\quad + \cos^2 A + \sec^2 A + 2 \cos A \sec A \\
 &= (\sin^2 A + \cos^2 A) + (\operatorname{cosec}^2 A) \\
 &\quad + (\sec^2 A) + 2 \sin A \times \frac{1}{\sin A} \\
 &\quad + 2 \cos A \times \frac{1}{\cos A} \\
 &= 1 + 1 + \cot^2 A + 1 + \tan^2 A + 2 + 2 \\
 &= 7 + \tan^2 A + \cot^2 A = \text{R.H.S.} \\
 \therefore \text{L.H.S.} &= \text{R.H.S.} \quad \text{Hence proved}
 \end{aligned}$$

(ix) We have,

$$\begin{aligned}
 \text{L.H.S.} &= (\operatorname{cosec} A - \sin A)(\sec A - \cos A) \\
 &= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right) \\
 &= \left(\frac{1 - \sin^2 A}{\sin A}\right)\left(\frac{1 - \cos^2 A}{\cos A}\right) \\
 &= \frac{\cos^2 A}{\sin A} \cdot \frac{\sin^2 A}{\cos A} \\
 &= \sin A \cos A
 \end{aligned}$$

$$\begin{aligned}
 \text{R.H.S.} &= \frac{1}{\tan A + \cot A} \\
 &= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}} \\
 &= \frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}} \\
 &= \frac{\sin A \cos A}{\sin^2 A + \cos^2 A} \\
 &= \frac{\sin A \cos A}{1} = \sin A \cos A
 \end{aligned}$$

$\therefore$ L.H.S. = R.H.S. *Hence proved*

(x) We have, L.H.S. = $\left(\frac{1 + \tan^2 A}{1 + \cot^2 A}\right)$

$$\begin{aligned}
 &= \frac{\sec^2 A}{\operatorname{cosec}^2 A} \\
 &\left[\begin{array}{l} \because 1 + \tan^2 A = \sec^2 A \\ 1 + \cot^2 A = \operatorname{cosec}^2 A \end{array} \right] \\
 &= \frac{1}{\frac{\cos^2 A}{\sin^2 A}} = \frac{\sin^2 A}{\cos^2 A} = \tan^2 A
 \end{aligned}$$

$\therefore$ L.H.S. = R.H.S. *Hence proved*

$$\begin{aligned}
 \text{Now, } \left(\frac{1 - \tan A}{1 - \cot A}\right)^2 &= \left(\frac{1 - \frac{\sin A}{\cos A}}{1 - \frac{\cos A}{\sin A}}\right)^2 \\
 &= \left(\frac{\frac{\cos A - \sin A}{\cos A}}{\frac{\sin A - \cos A}{\sin A}}\right)^2 \\
 &= \left[-\frac{(\sin A - \cos A)}{\cos A} \times \frac{\sin A}{(\sin A - \cos A)}\right]^2 \\
 &= \left(-\frac{\sin A}{\cos A}\right)^2 \\
 &= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A
 \end{aligned}$$

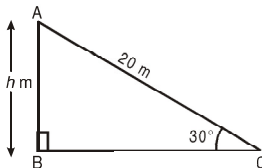
$\therefore$ LHS = RHS *Hence proved*

Textbook Exercises 9.1

Q.1. A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30° (See figure).

Sol. Let height of the pole be h m.

In right $\triangle ABC$, we have

$$\begin{aligned}\sin 30^\circ &= \frac{AB}{AC} \\ \Rightarrow \frac{1}{2} &= \frac{h}{20} \\ \Rightarrow h &= \frac{20}{2} = 10 \text{ m.}\end{aligned}$$


Hence, height of the pole is 10 m.

Q.2. A tree breaks due to storm and the broken part bends so that top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

Sol. Let AB be a tree of height h metres which is broken due to storm at point C and part AC assumes the position CD meets the grounds at D.

$\angle BDC = 30^\circ$ (given)

In right $\triangle CBD$, we have $\cos 30^\circ = \frac{BD}{CD}$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{8}{CD}$$

$$\Rightarrow \sqrt{3} CD = 8 \times 2$$

$$\Rightarrow CD = \frac{8 \times 2}{\sqrt{3}}$$

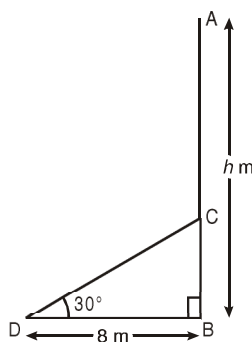
$$\Rightarrow CD = \frac{16\sqrt{3}}{3}$$

[Multiply Nr. and Dr. by $\sqrt{3}$]

$$\Rightarrow CD = \frac{16 \times 1.732}{3}$$

$$\Rightarrow CD = 9.24 \text{ m.}$$

$$\text{and } \tan 30^\circ = \frac{BC}{BD}$$



$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{BC}{8} \Rightarrow BC = \frac{8}{\sqrt{3}}$$

$$\Rightarrow BC = \frac{8\sqrt{3}}{3}$$

[Multiply Nr. and Dr. by $\sqrt{3}$]

$$\Rightarrow BC = \frac{8 \times 1.732}{3} \Rightarrow BC = 4.62$$

Height of the tree (h) = $BC + AC$

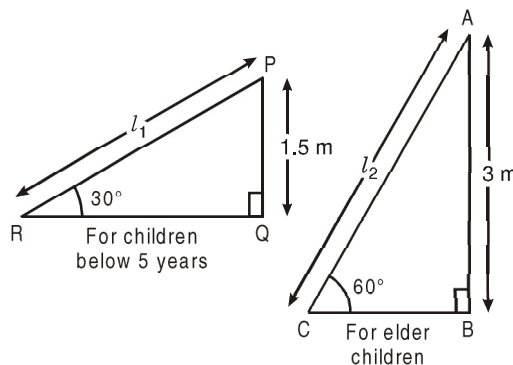
$$= BC + CD \quad [\because AC = CD]$$

$$= 4.62 + 9.24 = 13.86 \text{ m.}$$

Hence, height of the tree is 13.86 m.

Q.3. A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m, and is inclined at an angle of 30° to the ground, whereas for elder children, she wants to have a steep slide at a height of 3 m, and inclined at an angle of 60° to the ground. What should be length of the slide in each case?

Sol. Let l_1 is the length of slide for children below the age of 5 years and l_2 is the length of the slide for elder children.



Point P represents the top of slide l_1 and inclined at an angle of 30° to the ground.

i.e. $PQ = 1.5$ m and $\angle PRQ = 30^\circ$

and point A represents the top of slide l_2 and inclined at an angle of 60° to the ground.

i.e. $AB = 3$ m and $\angle ACB = 60^\circ$

In right $\triangle PQR$, we have

$$\sin 30^\circ = \frac{PQ}{PR}$$

$$\Rightarrow \frac{1}{2} = \frac{1.5}{l_1} \Rightarrow l_1 = 2 \times 1.5 = 3.0 \text{ m}$$

and in right $\triangle ABC$, we have

$$\sin 60^\circ = \frac{AB}{AC} \Rightarrow \frac{\sqrt{3}}{2} = \frac{3}{l_2}$$

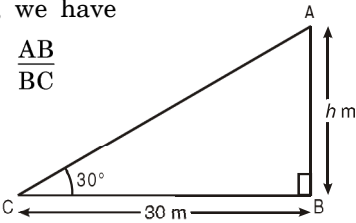
$$\Rightarrow l_2 = \frac{3 \times 2}{\sqrt{3}} \Rightarrow l_2 = 2\sqrt{3}$$

Hence, lengths of the slides are 3 m and $2\sqrt{3}$ m.

Q.4. The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of tower, is 30° . Find the height of tower.

Sol. Let AB be the tower of height h m and C be a point on the ground such that angle of elevation of the top A of tower is 30° i.e. $\angle ACB = 30^\circ$ and $BC = 30$ m.

In right $\triangle ABC$, we have

$$\begin{aligned} \tan 30^\circ &= \frac{AB}{BC} \\ \Rightarrow \frac{1}{\sqrt{3}} &= \frac{h}{30} \\ \Rightarrow h &= \frac{30}{\sqrt{3}} \\ \Rightarrow h &= \frac{30\sqrt{3}}{3} \Rightarrow h = 10\sqrt{3} \end{aligned}$$


Hence, height of the tower is $10\sqrt{3}$ metres.

Q.5. A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60° . Find the length of the string, assuming that there is no slack in the string.

Sol. Let A be the position of kite. Let C be the position of observer and AC be length of string. Then $\angle ACB = 60^\circ$, height of kite = 60 m.

In right $\triangle ABC$, we have

$$\begin{aligned} \sin 60^\circ &= \frac{AB}{AC} \Rightarrow \frac{\sqrt{3}}{2} = \frac{60}{AC} \\ \Rightarrow AC &= \frac{60 \times 2}{\sqrt{3}} \Rightarrow AC = \frac{120\sqrt{3}}{3} \\ \Rightarrow AC &= 40\sqrt{3}. \end{aligned}$$

Hence, length of string = $40\sqrt{3}$ metres.

Q.6. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building

increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

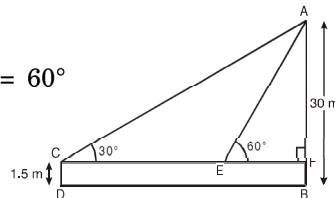
Sol. Let AB be the building of height 30 m and CD be the boy of height 1.5 m

i.e. $AB = 30$ m and $CD = 1.5$ m.

The angle of elevation from the eyes of the boy to the top of building is 30° and as the boy walks towards the building the angle of elevation becomes 60° .

Then $\angle ACF$

= 30° and $\angle AEF = 60^\circ$



In right $\triangle AFC$, we have

$$\begin{aligned} \tan 30^\circ &= \frac{AF}{FC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{AB - BF}{FC} \\ \Rightarrow \frac{1}{\sqrt{3}} &= \frac{30 - 1.5}{FC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{28.5}{FC} \\ \Rightarrow FC &= 28.5\sqrt{3} \end{aligned}$$

and in right $\triangle AFE$, we have

$$\begin{aligned} \therefore AF &= AB - BF = 30 - 1.5 \\ &= AF = 28.5 \text{ m} \end{aligned}$$

$$\begin{aligned} \tan 60^\circ &= \frac{AF}{FE} \\ \Rightarrow \sqrt{3} &= \frac{28.5}{FE} \Rightarrow FE = \frac{28.5}{\sqrt{3}} \\ \Rightarrow FE &= \frac{28.5\sqrt{3}}{3} \Rightarrow FE = 9.5\sqrt{3} \end{aligned}$$

The distance walked by the boy towards the building = $CE = FC - FE$

$$= 28.5\sqrt{3} - 9.5\sqrt{3} = 19\sqrt{3}$$

Hence, distance walked by the boy towards the building = $19\sqrt{3}$ m.

Q.7. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower. **[CBSE 2020]**

Sol. Let PQ be the transmission tower of height h m fixed at the top of 20 m height building (QR), angles of elevation of the bottom and top of a transmission tower are 45° and 60° respectively i.e. $\angle QSR = 45^\circ$ and $\angle PSR = 60^\circ$.

In right $\triangle QSR$, we have

$$\tan 45^\circ = \frac{QR}{SR}$$

$$\Rightarrow 1 = \frac{20}{SR} \Rightarrow SR = 20 \text{ m}$$

In right $\triangle PSR$,

$$\text{we have } \tan 60^\circ = \frac{PR}{SR}$$

$$\Rightarrow \sqrt{3} = \frac{PQ + QR}{20}$$

$$\Rightarrow \sqrt{3} = \frac{h + 20}{20}$$

$$\Rightarrow 20\sqrt{3} = h + 20 \Rightarrow h = 20\sqrt{3} - 20$$

$$\Rightarrow h = 20(\sqrt{3} - 1)$$

Hence, height of the tower is $20(\sqrt{3} - 1)$ m.

Q.8. A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45° . Find the height of the pedestal.

Sol. Let BC be the pedestal of height h m and AB be the statue.

Let D be the point of observation.

Then $\angle BDC = 45^\circ$, $\angle ADC = 60^\circ$,

$$AB = 1.6 \text{ m}$$

In right $\triangle BCD$, we have

$$\tan 45^\circ = \frac{BC}{CD}$$

$$\Rightarrow 1 = \frac{h}{CD}$$

$$\Rightarrow CD = h \quad \dots(1)$$

and right $\triangle ACD$, we have

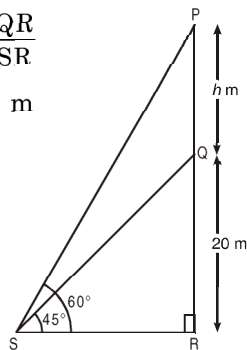
$$\tan 60^\circ = \frac{AC}{CD}$$

$$\Rightarrow \sqrt{3} = \frac{AB + BC}{CD}$$

$$\Rightarrow \sqrt{3} = \frac{1.6 + h}{h} \quad [\text{Using (1)}]$$

$$\Rightarrow \sqrt{3}h = 1.6 + h$$

$$\Rightarrow \sqrt{3}h - h = 1.6$$



$$\Rightarrow h(\sqrt{3} - 1) = 1.6$$

$$\Rightarrow h = \frac{1.6}{\sqrt{3} - 1} \Rightarrow h = \frac{1.6(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)}$$

[Multiplying numerator and denominator by $(\sqrt{3} + 1)$]

$$\Rightarrow h = \frac{1.6(\sqrt{3} + 1)}{(\sqrt{3})^2 - (1)^2} \Rightarrow h = \frac{1.6(\sqrt{3} + 1)}{2}$$

$$\Rightarrow h = 0.8(\sqrt{3} + 1)$$

Hence, height of pedestal is $0.8(\sqrt{3} + 1)$ m.

Q.9. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60° . If the tower is 50m high, find the height of the building.

[CBSE 2020]

Sol. Let AB be tower of height 50m and PQ be the building of height h m.

The angle of elevation of the top of the tower from the foot of the building is 60° i.e. $\angle AQB = 60^\circ$ and angle of elevation of the top of building from the foot of the tower is 30° , i.e., $\angle PBQ = 30^\circ$

In right triangle ABQ, we have

$$\tan 60^\circ = \frac{AB}{BQ}$$

$$\Rightarrow \sqrt{3} = \frac{50}{BQ}$$

$$\Rightarrow BQ = \frac{50}{\sqrt{3}}$$

$$\Rightarrow BQ = \frac{50\sqrt{3}}{3} \quad \dots(1)$$

and in right $\triangle PQB$,

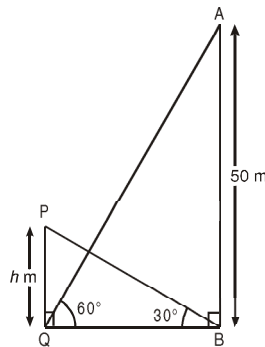
$$\text{we have } \tan 30^\circ = \frac{PQ}{BQ}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{\frac{50\sqrt{3}}{3}} \quad [\text{using (1)}]$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{3h}{50\sqrt{3}} \Rightarrow 3\sqrt{3}h = 50\sqrt{3}$$

$$\Rightarrow h = \frac{50\sqrt{3}}{3\sqrt{3}} \Rightarrow h = \frac{50}{3} = 16\frac{2}{3} \text{ m.}$$

Hence, height of the building is $16\frac{2}{3}$ m.



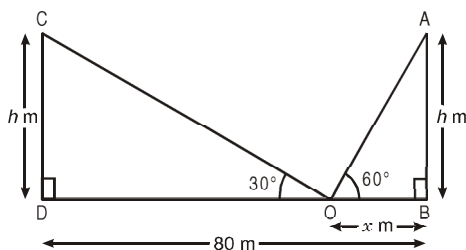
Q.10. Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30° respectively. Find the height of the poles and the distances of the point from the poles. **[CBSE 2019]**

Sol. Let AB and CD be two poles of equal height h m, angles of elevation of the top of the poles are 60° and 30° respectively i.e. $\angle AOB = 60^\circ$, $\angle COD = 30^\circ$ and $BD = 80$ m.

Let $OB = x$ m, then $DO = (80 - x)$ m

In right $\triangle ABO$, we have

$$\tan 60^\circ = \frac{AB}{BO}$$



$$\Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow h = \sqrt{3}x \Rightarrow x = \frac{h}{\sqrt{3}} \quad \dots(1)$$

and right $\triangle CDO$, we have $\tan 30^\circ = \frac{CD}{DO}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{80 - x} \Rightarrow \sqrt{3}h = 80 - x$$

$$\Rightarrow \sqrt{3}h = 80 - \frac{h}{\sqrt{3}} \quad [\text{Using (1)}]$$

$$\Rightarrow \sqrt{3}h + \frac{h}{\sqrt{3}} = 80 \Rightarrow \frac{3h + h}{\sqrt{3}} = 80$$

$$\Rightarrow 4h = 80\sqrt{3} \Rightarrow h = \frac{80\sqrt{3}}{4}$$

$$\Rightarrow h = 20\sqrt{3} \text{ m}$$

Putting the value of h in (1), we get $x = \frac{20\sqrt{3}}{\sqrt{3}}$

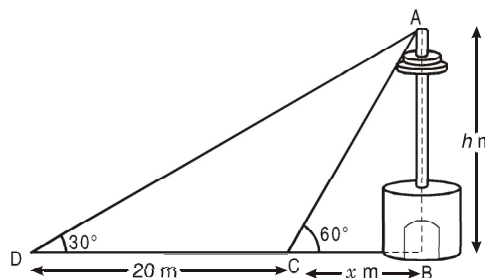
$$\Rightarrow x = 20 \text{ m}$$

$$DO = 80 - x = 80 - 20 = 60 \text{ m}$$

Hence height of each pole is $20\sqrt{3}$ m and distances of point from pole (AB) is 20 m and from pole (CD) is 60 m.

Q.11. A TV Tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60° . From another point 20

m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30° . (See in figure). Find the height of tower and width of the canal.



Sol. Let height of tower be h m. and width of canal be x m. Then $\angle ACB = 60^\circ$, $\angle ADB = 30^\circ$ and $CD = 20$ m.

In right $\triangle ABC$, we have

$$\tan 60^\circ = \frac{AB}{BC} \Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow h = \sqrt{3}x \quad \dots(1)$$

In right $\triangle ABD$, we have $\tan 30^\circ = \frac{AB}{BD}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{BC + CD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x + 20}$$

$$\Rightarrow \sqrt{3}h = x + 20$$

$$\Rightarrow \sqrt{3} \times \sqrt{3}x = x + 20 \quad [\text{Using (1)}]$$

$$\Rightarrow 3x = x + 20 \Rightarrow 2x = 20$$

$$\Rightarrow x = \frac{20}{2} = 10$$

Putting the value of x in (1), we get

$$h = \sqrt{3} \times 10$$

$$\Rightarrow h = 10\sqrt{3}$$

Hence, height of tower is $10\sqrt{3}$ m and width of canal is 10 m.

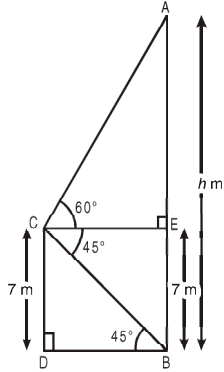
Q.12. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45° . Determine the height of the tower.

[CBSE 2020]

Sol. Let AB be cable tower of height h m. and CD be building of height 7 m the angle of elevation of the top of the cable tower from the top of a building is 60° i.e. $\angle ACE = 60^\circ$ and

angle of depression of its foot from the top of a building is 45° i.e. $\angle ECB = 45^\circ$

$$\begin{aligned}\angle CBD &= \angle ECB \text{ [Alternate interior } \angle\text{s]} \\ \angle CBD &= 45^\circ\end{aligned}$$



In right $\triangle AEC$, we have

$$\begin{aligned}\tan 60^\circ &= \frac{AE}{CE} \Rightarrow \sqrt{3} = \frac{AB - EB}{CE} \\ \Rightarrow \sqrt{3} &= \frac{h - 7}{CE} \Rightarrow CE = \frac{h - 7}{\sqrt{3}} \quad \dots(1)\end{aligned}$$

and in right $\triangle CDB$, we have

$$\begin{aligned}\tan 45^\circ &= \frac{CD}{DB} \\ \Rightarrow 1 &= \frac{7}{CE} \quad [\because DB = CE] \\ \Rightarrow CE &= 7.\end{aligned}$$

Putting the value of CE in (1), we get

$$\begin{aligned}7 &= \frac{h - 7}{\sqrt{3}} \Rightarrow 7\sqrt{3} = h - 7 \\ \Rightarrow 7\sqrt{3} + 7 &= h \Rightarrow h = 7(\sqrt{3} + 1) \text{ m.}\end{aligned}$$

Hence, height of cable tower is $7(\sqrt{3} + 1)$ m.

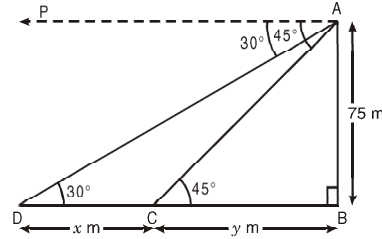
Q.13. As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45° . If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

Sol. Let AB be light house of height 75 m, angles of depression of two ships are 30° and 45° i.e. $\angle PAD = 30^\circ$ and $\angle PAC = 45^\circ$.
 $\angle ADC = \angle PAD = 30^\circ$ [alternate interior angles]
 $\angle ACB = \angle PAC = 45^\circ$ [alternate interior angles]

Let $CD = x$ m. and $BC = y$ m.

In right $\triangle ABC$, we have

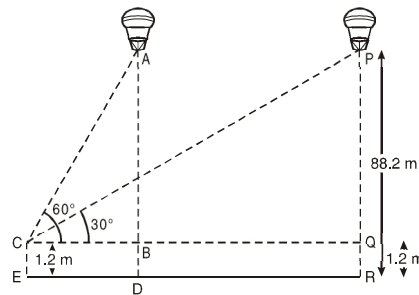
$$\begin{aligned}\tan 45^\circ &= \frac{AB}{BC} \\ \Rightarrow 1 &= \frac{75}{y} \Rightarrow y = 75 \text{ m} \quad \dots(1)\end{aligned}$$



and in right $\triangle ABD$, we have $\tan 30^\circ = \frac{AB}{BD}$

$$\begin{aligned}\Rightarrow \frac{1}{\sqrt{3}} &= \frac{75}{CD + BC} \\ \Rightarrow \frac{1}{\sqrt{3}} &= \frac{75}{x + y} \Rightarrow x + y = 75\sqrt{3} \\ \Rightarrow x + 75 &= 75\sqrt{3} \text{ [Using (1)]} \\ \Rightarrow x &= 75\sqrt{3} - 75 \Rightarrow x = 75(\sqrt{3} - 1) \\ \text{Hence, distance between the two ships is } &75(\sqrt{3} - 1) \text{ m.}\end{aligned}$$

Q.14. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60° . After some time, the angle of elevation reduces to 30° (see figure). Find the distance travelled by the balloon during the interval.



Sol. Let A be position of balloon when its angle of elevation from eye of the girl is 60° and P be the position of balloon when angle of elevation is 30° . Then

$\angle PCQ = 30^\circ$, $\angle ACB = 60^\circ$
 and $PR = 88.2$ m
 $PQ = PR - QR = 88.2 - 1.2 = 87$ m
 In right $\triangle PQC$, we have

$$\tan 30^\circ = \frac{PQ}{QC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{87}{QC}$$

$$\Rightarrow QC = 87\sqrt{3} \quad \dots(1)$$

and in right $\triangle ABC$, we have $\tan 60^\circ = \frac{AB}{BC}$

$$\Rightarrow \sqrt{3} = \frac{87}{BC} \quad [\because AB = PQ = 87\text{m}]$$

$$\Rightarrow BC = \frac{87}{\sqrt{3}} \Rightarrow BC = \frac{87\sqrt{3}}{3} \quad \dots(2)$$

Now distance travelled by balloon

$$= AP = BQ = QC - BC = 87\sqrt{3} - \frac{87\sqrt{3}}{3}$$

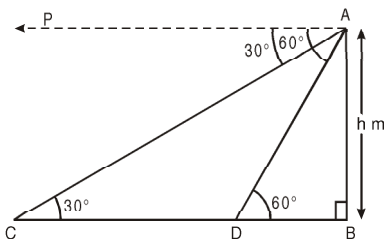
[From (1) and (2)]

$$= 87\sqrt{3} \left(1 - \frac{1}{3}\right) = 87\sqrt{3} \times \frac{2}{3} = 58\sqrt{3}$$

Hence, distance travelled by balloon is $58\sqrt{3}$ m.

Q.15. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30° , which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60° . Find the time taken by the car to reach the foot of the tower from this point.

Sol. Let AB be vertical tower of height h m. Let the speed of the car be v m/sec. At the point C angle of depression of car be 30° i.e. $\angle PAC = 30^\circ$ and it reaches to D six seconds later. Then angle of depression of car is 60° i.e. $\angle PAD = 60^\circ$.



$\angle ACD = \angle PAC = 30^\circ$
 [alternate interior angles]

$\angle ADB = \angle PAD = 60^\circ$
 [alternate interior angles]

Distance travelled by car in 6 sec = $6v$ metres

So, $CD = 6v$ metres

[$\because$ Distance = speed $\times$ time]

Let car takes t seconds to reach the point B from the point D

$\therefore$ Distance travelled by car in t sec = vt metres

In right $\triangle ABD$, we have

$$\tan 60^\circ = \frac{AB}{BD} \Rightarrow \sqrt{3} = \frac{h}{vt}$$

$$\Rightarrow h = \sqrt{3}vt \quad \dots(1)$$

and in right $\triangle ABC$, we have

$$\tan 30^\circ = \frac{AB}{BC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{CD + BD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{6v + vt} \Rightarrow \sqrt{3}h = 6v + vt$$

$$\Rightarrow \sqrt{3} \times \sqrt{3}vt = 6v + vt \quad \text{[Using (1)]}$$

$$\Rightarrow 3vt = 6v + vt \Rightarrow 2vt = 6v$$

$$\Rightarrow t = \frac{6v}{2v} = 3 \text{ sec.}$$

Hence, the car takes 3 sec. to reach the foot of tower.

10

Circles

Textbook Exercises 10.1

Q.1. How many tangents can a circle have?

Sol. A circle can have infinitely many tangents.

Q.2. Fill in the blanks :

- (i) A tangent to a circle intersects it in point(s).
- (ii) A line intersecting a circle in two points is called a
- (iii) A circle can have parallel tangents at the most.

(iv) The common point of a tangent to a circle and the circle is called

Ans. (i) One, (ii) Secant, (iii) Two, (iv) Point of contact.

Q.3. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that $OQ = 12$ cm. Length PQ is:

(A) 12 cm (B) 13 cm (C) 8.5 cm (D) $\sqrt{119}$ cm

Sol. Let O be the centre of the circle.

Let P be a point on circle such that $OP = 5$ cm.

Now, PQ is a tangent at P and OP is a radius through P.

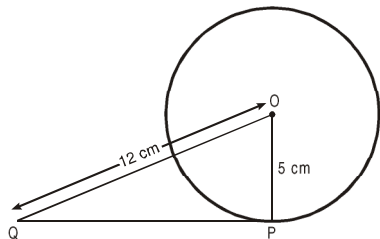
Since, the tangent at a point on a circle is perpendicular to the radius through the point of contact.

$$\therefore OP \perp PQ.$$

In right $\triangle OPQ$, we have

$$OQ^2 = OP^2 + PQ^2$$

[By Pythagoras theorem]



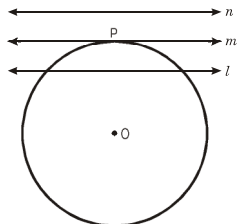
$$\Rightarrow (12)^2 = (5)^2 + PQ^2 \Rightarrow PQ^2 = 144 - 25$$

$$\Rightarrow PQ^2 = 119 \Rightarrow PQ = \sqrt{119} \text{ cm}$$

Hence, Length of PQ = $\sqrt{119}$ cm.

So, correct option is (D).

Q.4. Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.



Sol. In figure line m is parallel to line n and also tangent to the given circle at the point P. Another line l is parallel to the given line n and also a secant to the circle.

Textbook Exercises 10.2

In question 1 to 3, choose the correct option and give justification :

Q.1. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is:

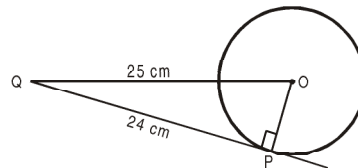
- (a) 7 cm (b) 12 cm (c) 15 cm (d) 24.5 cm

Sol. $OP \perp PQ$

In right $\triangle OPQ$, we have

$$OQ^2 = QP^2 + OP^2$$

[By Pythagoras theorem]



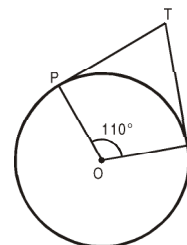
$$\Rightarrow 25^2 = 24^2 + OP^2 \Rightarrow OP^2 = 25^2 - 24^2$$

$$\Rightarrow OP^2 = (25 + 24)(25 - 24) \Rightarrow OP^2 = 49$$

$$\Rightarrow OP = \sqrt{49} = 7$$

$\therefore$ Radius = 7 cm So, correct option is (a).

Q.2. In the given figure, if TP and TQ are two tangents to a circle with centre O, so that $\angle POQ = 110^\circ$, then $\angle PTQ$ is equal to :



- (a) 60° (b) 70° (c) 80° (d) 90°

Sol. $\angle OPT = \angle OQT = 90^\circ$

In a quadrilateral POQT, we have

$$\angle OPT + \angle OQT + \angle POQ + \angle PTQ = 360^\circ$$

$$\Rightarrow 90^\circ + 90^\circ + 110^\circ + \angle PTQ = 360^\circ$$

$$\Rightarrow 290^\circ + \angle PTQ = 360^\circ \Rightarrow \angle PTQ = 360^\circ - 290^\circ$$

$$\Rightarrow \angle PTQ = 70^\circ$$

The correct option is (b).

Q.3. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80° , then $\angle POA$ is equal to :

- (a) 50° (b) 60° (c) 70° (d) 80°

Sol. $OA \perp AP$ and $OB \perp PB$

In $\triangle OAP$ and $\triangle OBP$, we have

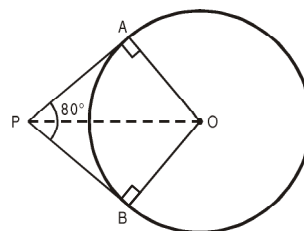
$$AO = OB \quad (\text{radii of the same circle})$$

$$\angle OAP = \angle OBP \quad (\text{each is } 90^\circ)$$

$$PO = OP \quad (\text{common})$$

$$\triangle OAP \cong \triangle OBP \quad (\text{By R.H.S. congruence})$$

$$\angle APO = \angle BPO \quad (\text{By CPCT})$$



$$\angle APO = \frac{80}{2} = 40^\circ$$

In $\triangle APO$,

$$\angle APO + \angle OAP + \angle POA = 180^\circ$$

$$\Rightarrow 40^\circ + 90^\circ + \angle POA = 180^\circ$$

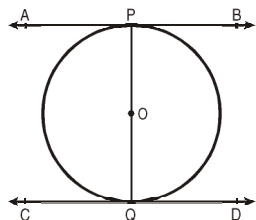
$$\Rightarrow 130^\circ + \angle POA = 180^\circ$$

$$\Rightarrow \angle POA = 180^\circ - 130^\circ = 50^\circ$$

Hence, correct option is (a).

Q.4. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Sol. Given : AB, CD are two tangents at the ends of a diameter PQ of a circle with centre O.



To Prove : AB $\parallel$ CD

Proof : OP $\perp$ AB [By theorem 10.1]

OQ $\perp$ CD [By theorem 10.1]

$$\angle APQ = 90^\circ \text{ and } \angle CQP = 90^\circ$$

$$\angle APQ + \angle CQP = 90^\circ + 90^\circ = 180^\circ$$

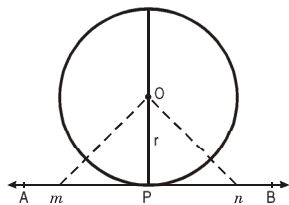
But these are co-interior angles.

$\therefore$ AB $\parallel$ CD **Hence Proved**

Q.5. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

Given : A circle with centre O and a tangent AB at point P.

To Prove : OP passes through centre.



Proof : AB is a tangent at point P and OP is the radius of circle.

If we have some points m, n on tangent AB. Then we find that OP is the shortest line from O in comparison of Om and On.

Therefore, OP $\perp$ AB.

Hence, perpendicular OP drawn to tangent

line at P passes through the centre (O) of the circle.

Hence Proved

Q.6. The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.

Sol. OB $\perp$ AB

In right $\triangle ABO$, we have $AO^2 = AB^2 + OB^2$

[By Pythagoras theorem]

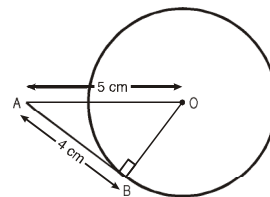
$$\Rightarrow 5^2 = 4^2 + OB^2$$

$$\Rightarrow OB^2 = 5^2 - 4^2$$

$$\Rightarrow OB^2 = 25 - 16$$

$$\Rightarrow OB^2 = 9$$

$$\Rightarrow OB = \sqrt{9} = 3$$



Hence, radius of the circle is 3 cm.

Q.7. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Sol. Let O be the centre of two concentric circles of radii 5 cm and 3 cm.

Let AB be a chord of the larger circle touching the smaller circle at P. Then

$$OP = 3 \text{ cm and } AO = 5 \text{ cm}$$

$$OP \perp AB \text{ and } AP = PB$$

In right $\triangle OPA$, we have

$$AO^2 = AP^2 + OP^2$$

$$\Rightarrow 5^2 = AP^2 + 3^2$$

$$\Rightarrow AP^2 = 5^2 - 3^2$$

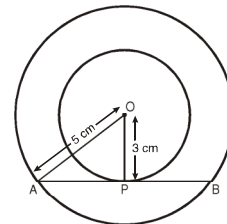
$$\Rightarrow AP^2 = (5 + 3)(5 - 3)$$

$$\Rightarrow AP^2 = 8 \times 2 = 16$$

$$\Rightarrow AP = \sqrt{16} = 4$$

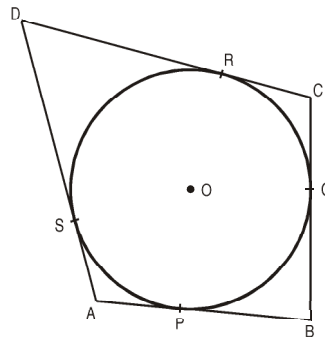
$$AB = 2 \times AP = 2 \times 4 = 8$$

Hence, length of chord is 8 cm.



Q.8. A quadrilateral ABCD is drawn to circumscribe a circle (see figure below). Prove that:

$$AB + CD = AD + BC$$



Given : A quadrilateral ABCD is drawn to circumscribe a circle.

To Prove : $AB + CD = AD + BC$

Proof : By theorem 10.2

We know that tangents drawn from an exterior point to a circle are equal in length.

$$AP = AS \dots(1) \text{ [Tangents from point A]}$$

$$PB = BQ \dots(2) \text{ [Tangents from point B]}$$

$$CR = QC \dots(3) \text{ [Tangents from Point C]}$$

$$DR = DS \dots(4) \text{ [Tangents from point D]}$$

Adding (1), (2), (3) and (4), we get

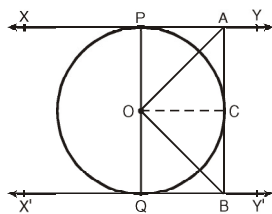
$$AP + PB + CR + DR$$

$$= AS + BQ + QC + DS$$

$$\Rightarrow AB + CD = AS + DS + BQ + QC$$

$$\Rightarrow AB + CD = AD + BC \text{ Hence Proved}$$

Q.9. In the given figure XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B. Prove that $\angle AOB = 90^\circ$.



[CBSE 2019]

Sol. Given : Tangent $XY \parallel$ Tangents $X'Y'$ and another Tangent AB with point of contact C intersect XY at A and X'Y' at B.

To Prove : $\angle AOB = 90^\circ$

Construction : Join OC

Proof : In $\triangle APO$ and $\triangle ACO$

$$AP = AC$$

$$\angle APO = \angle ACO = 90^\circ$$

$$AO = AO \quad (\text{Common})$$

$$\triangle APO \cong \triangle ACO$$

(By RHS congruence)

$$\angle PAO = \angle CAO \quad (\text{By CPCT})$$

$$\Rightarrow 2 \angle CAO = \angle PAC \dots(1)$$

$$\text{Similarly, } 2 \angle CBO = \angle QBC \dots(2)$$

$$\text{Now, } \angle PAC + \angle QBC = 180^\circ$$

[co-interior $\angle$ s of same side]

$$2 \angle CAO + 2 \angle CBO = 180^\circ$$

[Using (1) and (2)]

$$\Rightarrow \angle CAO + \angle CBO = \frac{180^\circ}{2} = 90^\circ$$

$$\Rightarrow \angle BAO + \angle ABO = 90^\circ$$

In $\triangle AOB$, we have

$$\angle BAO + \angle AOB + \angle ABO = 180^\circ$$

$$\Rightarrow \angle AOB + 90^\circ = 180^\circ \quad [\text{Using (3)}]$$

$$\Rightarrow \angle AOB = 180^\circ - 90^\circ = 90^\circ.$$

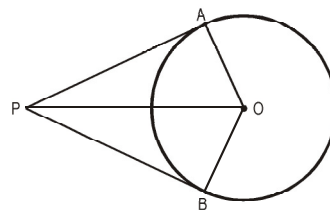
Hence Proved

Q.10. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.

Sol. Given : PA and PB be two tangents drawn from an external point P to a circle with centre O. Also, the line segments OA and OB are drawn.

To Prove : $\angle APB + \angle AOB = 180^\circ$

Proof : $PA \perp OA$ and $PB \perp OB$



$$\text{i.e. } \angle PAO = 90^\circ \text{ and } \angle PBO = 90^\circ$$

$$\angle PAO + \angle PBO = 180^\circ \dots(1)$$

We know that sum of $\angle$ s of a quadrilateral is 360° .

$$\angle APB + \angle AOB + \angle PAO + \angle PBO = 360^\circ$$

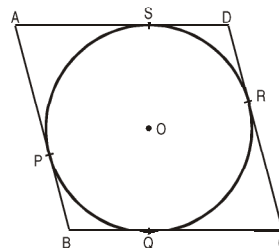
$$\Rightarrow \angle APB + \angle AOB + 180^\circ = 360^\circ$$

$$\Rightarrow \angle APB + \angle AOB = 360^\circ - 180^\circ$$

$$\Rightarrow \angle APB + \angle AOB = 180^\circ.$$

Hence Proved

Q.11. Prove that the parallelogram circumscribing a circle is a rhombus.



Sol. Given : A parallelogram ABCD circumscribes a circle with centre O.

To Prove : $AB = BC = CD = AD$

Proof : We know that lengths of tangents drawn from an exterior point to a circle are equal.

BQ = BP ...(1) [Tangents from Point B]

QC = RC ...(2) [Tangents from Point C]

SD = RD ...(3) [Tangents from point D]

AS = AP ...(4) [Tangents from point A]

Adding (1), (2), (3) and (4), we get

$$\begin{aligned} \text{BQ} + \text{QC} + \text{SD} + \text{AS} \\ = \text{BP} + \text{RC} + \text{RD} + \text{AP} \end{aligned}$$

$$\Rightarrow \text{BC} + \text{AD} = \text{BP} + \text{AP} + \text{RC} + \text{RD}$$

$$\Rightarrow \text{BC} + \text{AD} = \text{AB} + \text{CD}$$

$$\Rightarrow 2\text{BC} = 2\text{AB}$$

[$\because$ Opposite sides of ||gm are equal]

$$\Rightarrow \text{BC} = \text{AB}$$

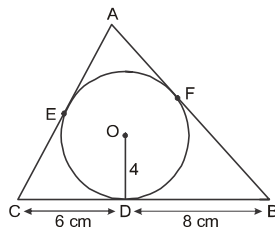
Since adjacent sides of a ||gm are equal

$$\therefore \text{AB} = \text{BC} = \text{CD} = \text{AD}.$$

Therefore, ABCD is a rhombus.

Hence Proved

Q.12. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see figure). Find the sides AB and AC.



Sol. Let tangents AB, BC and AC touch the circle at points E, D and F respectively. Join OE, OF, OA, OC and OB.

We know that lengths of tangents drawn from an external point to the circle are equal.

$$\text{CF} = \text{CD} = 6 \text{ cm}$$

$$\text{BE} = \text{BD} = 8 \text{ cm}$$

$$\text{AF} = \text{AE}$$

Let $\text{AF} = \text{AE} = x \text{ cm}$ $\text{AC} = (6 + x) \text{ cm}$
and $\text{AB} = (8 + x) \text{ cm}$

$$\therefore s = \frac{a+b+c}{2} \quad [\text{where } a, b \text{ and } c \text{ are lengths of sides of } \triangle ABC]$$

$$\Rightarrow s = \frac{14 + 6 + x + 8 + x}{2} = \frac{28 + 2x}{2}$$

$$\Rightarrow s = 14 + x$$

$$\begin{aligned} \text{Area of } \triangle ABC &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{(14+x)[14+x-14][14+x-6-x][14+x-8-x]} \\ &= \sqrt{(14+x)(x)(8)(6)} \\ &= \sqrt{48x(14+x)} \text{ cm}^2 \quad \dots(1) \end{aligned}$$

Now, $\text{OD} \perp \text{BC}$, $\text{OE} \perp \text{AB}$, $\text{OF} \perp \text{AC}$

$$\begin{aligned} \text{Area of } \triangle ABC &= \text{Area of } \triangle BOC \\ &\quad + \text{Area of } \triangle AOB \\ &\quad + \text{Area of } \triangle AOC \\ &= \frac{1}{2}(14) \times 4 + \frac{1}{2}(8+x) \times 4 + \frac{1}{2}(6+x) \times 4 \\ &= 28 + 16 + 2x + 12 + 2x \\ &= (56 + 4x) \text{ cm}^2 \quad \dots(2) \end{aligned}$$

From (1) and (2), we get

$$\sqrt{48x(14+x)} = 56 + 4x$$

Squaring both sides, we get

$$48x(14+x) = (56 + 4x)^2$$

$$\Rightarrow 48x(14+x) = 16(14+x)^2$$

$$\Rightarrow 3x(14+x) = (14+x)^2$$

$$3x = \frac{(14+x)^2}{(14+x)}$$

$$\Rightarrow 3x = 14 + x \Rightarrow 3x - x = 14$$

$$\Rightarrow 2x = 14 \Rightarrow x = \frac{14}{2} = 7$$

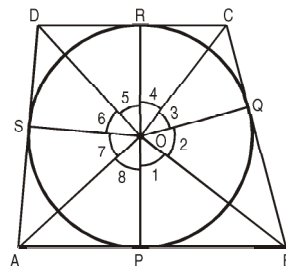
Length of $\text{AB} = 8 + x = 8 + 7 = 15 \text{ cm}$

Length of $\text{AC} = 6 + x = 6 + 7 = 13 \text{ cm}$

Hence, lengths of AB and AC are 15 cm and 13 cm.

Q.13. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle. **[CBSE 2019]**

Sol. Given : A quadrilateral ABCD circumscribes a circle with centre O.



To Prove : $\angle AOB + \angle COD = 180^\circ$

and $\angle AOD + \angle BOC = 180^\circ$

Construction : Join OP, OQ, OR and OS.

Proof : In $\triangle OPB$ and $\triangle OQB$,

$$\angle OPB = \angle OQB \quad (\text{each is } 90^\circ)$$

$$PB = QB$$

$$OB = OB \quad (\text{Common})$$

$$\triangle OPB \cong \triangle OQB \quad (\text{By RHS congruence})$$

$$\angle 1 = \angle 2 \quad (\text{By CPCT})$$

$$\text{Similarly, } \angle 3 = \angle 4$$

$$\angle 5 = \angle 6$$

$$\angle 7 = \angle 8$$

$\therefore$ Sum of $\angle$ s of a quadrilateral is 360° .

$$\therefore \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6$$

$$+ \angle 7 + \angle 8 = 360^\circ$$

$$\Rightarrow 2(\angle 2 + \angle 3) + 2(\angle 6 + \angle 7) = 360^\circ$$

$$[\because \angle 1 = \angle 2, \angle 4 = \angle 3, \angle 5 = \angle 6$$

$$\text{and } \angle 8 = \angle 7]$$

$$\text{and similarly, } 2(\angle 1 + \angle 8)$$

$$+ 2(\angle 4 + \angle 5) = 360^\circ$$

$$\Rightarrow (\angle 2 + \angle 3) + (\angle 6 + \angle 7) = 180^\circ$$

$$\text{and } (\angle 1 + \angle 8) + (\angle 4 + \angle 5) = 180^\circ$$

$$\Rightarrow \angle BOC + \angle AOD = 180^\circ$$

$$\text{and } \angle AOB + \angle COD = 180^\circ.$$

Hence Proved

11

Areas Related to Circles

Textbook Exercises 11.1

Unless stated otherwise, use $\pi = \frac{22}{7}$.

Q.1. Find the area of sector of a circle with radius 6 cm if angle of the sector is 60° .

Sol. We have,

$$\text{Radius} = 6 \text{ cm and } \theta = 60^\circ$$

$$\text{Area of the sector} = \frac{\pi r^2 \theta}{360^\circ}$$

$$= \frac{22}{7} \times \frac{6^2 \times 60^\circ}{360^\circ} = \frac{22}{7} \times \frac{6 \times 6}{6} = \frac{132}{7}$$

$$\text{Hence, area of sector} = \frac{132}{7} \text{ cm}^2$$

Q.2. Find the area of a quadrant of a circle whose circumference is 22 cm.

Sol. Let r be the radius of the circle whose circumference is 22 cm.

$$\therefore 2\pi r = 22 \Rightarrow r = \frac{22}{2\pi}$$

$$\Rightarrow r = \frac{22}{2 \times \frac{22}{7}} \Rightarrow r = \frac{22 \times 7}{2 \times 22}$$

$$\Rightarrow r = \frac{7}{2} \text{ cm}$$

$$\theta = 90^\circ$$

$$[\because \text{sector is quadrant } \therefore \theta = \frac{360}{4} = 90^\circ]$$

$$\text{Therefore, area of the quadrant} = \frac{\pi r^2 \theta}{360^\circ}$$

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{90^\circ}{360^\circ}$$

$$= \frac{22 \times 7}{4 \times 4} = \frac{77}{8} \text{ cm}^2$$

$$\text{Hence, area of the quadrant} = \frac{77}{8} \text{ cm}^2$$

Q.3. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

Sol. Angle describe by the minute hand in 60 minutes = 360°

Angle describe by the minute hand in 5 minutes

$$= \frac{360^\circ}{60} \times 5 = 30^\circ$$

$$\text{Now, } \theta = 30^\circ \text{ and radius} = 14 \text{ cm}$$

Area swept by the minute hand in 5 minutes

= Area of sector AOB

$$= \frac{\pi r^2 \theta}{360^\circ} = \frac{22}{7} \times \frac{14^2 \times 30^\circ}{360^\circ}$$

$$= \frac{22 \times 14 \times 14}{7 \times 12} = \frac{22 \times 2 \times 14}{12}$$

$$= \frac{11 \times 14}{3} = \frac{154}{3} \text{ cm}^2.$$

Hence, area swept by the minute hand

$$= \frac{154}{3} \text{ cm}^2.$$

Q.4. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding : (i) minor segment (ii) major sector. (use $\pi = 3.14$)

Sol. We have,

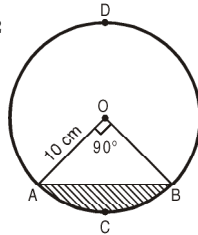
Radius = 10 cm and $\theta = 90^\circ$

Area of the minor segment ACB

$$\begin{aligned}
 &= \frac{\pi r^2 \theta}{360} - \text{Area of } \triangle AOB \\
 &= \frac{3.14 \times 10^2 \times 90^\circ}{360^\circ} - \frac{1}{2} \times 10 \times 10 \\
 &= \frac{3.14 \times 100}{4} - 50 \\
 &= 78.5 - 50 = 28.5 \text{ cm}^2
 \end{aligned}$$

Area of major sector ADB

$$\begin{aligned}
 &= \frac{\pi r^2 (360^\circ - 90^\circ)}{360^\circ} \\
 &= \frac{3.14 \times 10^2 \times 270^\circ}{360^\circ} \\
 &= 314 \times \frac{3}{4} = 235.5 \text{ cm}^2.
 \end{aligned}$$



Hence, area of minor segment = 28.5 cm^2 and area of major sector = 235.5 cm^2 .

Q.5. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre.

Find : (i) The length of the arc

(ii) Area of the sector formed by the arc

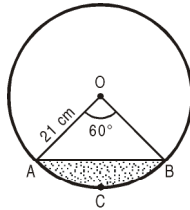
(iii) Area of the segment formed by the corresponding chord.

Sol. We have, Radius (r) = 21 cm

Sector angle (θ) = 60°

(i) Length of the arc = $\frac{\theta}{360^\circ} \times 2\pi r$

$$\begin{aligned}
 &= \frac{60^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 21 \\
 &= \frac{1}{6} \times 2 \times \frac{22}{7} \times 21 \\
 &= \frac{1}{3} \times 22 \times 3 \\
 &= 22 \text{ cm.}
 \end{aligned}$$



(ii) Area of the sector formed by the arc

$$\begin{aligned}
 &= \frac{\pi r^2 \theta}{360^\circ} = \frac{22}{7} \times \frac{21^2 \times 60^\circ}{360^\circ} \\
 &= \frac{22}{7} \times \frac{21 \times 21 \times 60^\circ}{360^\circ} \\
 &= \frac{22 \times 3 \times 21 \times 1}{6} \\
 &= 11 \times 21 \\
 &= 231 \text{ cm}^2.
 \end{aligned}$$

(iii) In $\triangle OAB$, OA = OB (equal radii)

$$\Rightarrow \angle OAB = \angle OBA$$

Let $\angle OAB = \angle OBA = x$

$$\therefore x + x + \angle AOB = 180^\circ$$

$$\Rightarrow 2x + 60^\circ = 180^\circ$$

$$\Rightarrow 2x = 180^\circ - 60^\circ = 120^\circ$$

$$\Rightarrow x = \frac{120^\circ}{2} = 60^\circ.$$

Therefore, $\triangle OAB$ is an equilateral triangle of side 21 cm

$$\begin{aligned}
 \therefore \text{Area of } \triangle OAB &= \frac{\sqrt{3}}{4} (\text{side})^2 \\
 &= \frac{\sqrt{3}}{4} \times 21^2 = \frac{441\sqrt{3}}{4} \text{ cm}^2
 \end{aligned}$$

Area of the segment formed by the corresponding chord

= Area of segment ACB.

= Area of the sector OACBO

- Area of $\triangle OAB$

$$\begin{aligned}
 &= \frac{\pi r^2 \theta}{360^\circ} - \frac{441\sqrt{3}}{4} \\
 &= \frac{22}{7} \times \frac{21^2 \times 60^\circ}{360^\circ} - \frac{441\sqrt{3}}{4} \\
 &= 11 \times 21 - \frac{441\sqrt{3}}{4} \\
 &= \left(231 - \frac{441\sqrt{3}}{4} \right) \text{ cm}^2.
 \end{aligned}$$

Hence, length of the arc = 22 cm, area of the sector = 231 cm^2 and area of the

segment (ACB) = $\left(231 - \frac{441\sqrt{3}}{4} \right) \text{ cm}^2$.

Q.6. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle.

[use $\pi = 3.14$ and $\sqrt{3} = 1.73$]

Sol. We have,

Radius (r) = 15 cm

Sector angle (θ) = 60°

In $\triangle AOB$

OA = OB (equal radii of the circle)

$$\Rightarrow \angle OAB = \angle OBA$$

$$\therefore \angle OAB + \angle OBA + \angle AOB = 180^\circ$$

$$\Rightarrow 2\angle OAB + 60^\circ = 180^\circ$$

$$\Rightarrow 2\angle OAB = 180^\circ - 60^\circ = 120^\circ$$

$$\Rightarrow \angle OAB = \frac{120^\circ}{2} = 60^\circ$$

Therefore, $\triangle AOB$ is an equilateral triangle.

Its side = 15 cm

$$\text{Area of } \triangle AOB = \frac{\sqrt{3}}{4} \times 15^2 = \frac{225\sqrt{3}}{4} \text{ cm}^2$$

Area of minor segment (ACB)

$$\begin{aligned}
 &= \text{Area of sector} - \text{area of } \triangle AOB \\
 &= \frac{\pi r^2 \theta}{360^\circ} - \frac{225\sqrt{3}}{4} \\
 &= \frac{3.14 \times 15^2 \times 60^\circ}{360^\circ} - \frac{225\sqrt{3}}{4} \\
 &= \frac{3.14 \times 15 \times 15}{6} - \frac{225 \times 1.73}{4} \\
 &= 117.75 - 97.31 \\
 &= 20.44 \text{ cm}^2.
 \end{aligned}$$

Area of major segment

$$\begin{aligned}
 &= \pi r^2 - \text{Area of minor segment} \\
 &= 3.14 \times 15^2 - 20.44 \\
 &= 706.5 - 20.44 = 686.06 \text{ cm}^2
 \end{aligned}$$

Hence, area of minor segment and major segment are 20.44 cm^2 and 686.06 cm^2 respectively.

Q.7. A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle.

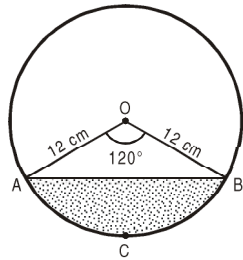
[use $\pi = 3.14$ and $\sqrt{3} = 1.73$]

Sol. We have, Radius (r) = 12 cm

$$\angle AOB = 120^\circ$$

Area of segment (ACB)

$$\begin{aligned}
 &= \frac{\pi r^2 \theta}{360^\circ} - r^2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} \\
 &= \frac{3.14 \times 12^2 \times 120}{360} - 12^2 \sin \frac{120^\circ}{2} \cos \frac{120^\circ}{2} \\
 &= 3.14 \times 12 \times 4 - 144 \sin 60^\circ \cos 60^\circ
 \end{aligned}$$



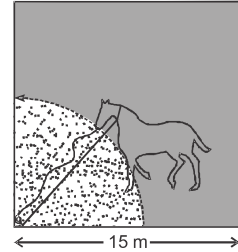
$$\begin{aligned}
 &= 3.14 \times 48 - 144 \times \frac{\sqrt{3}}{2} \times \frac{1}{2} \\
 &= 3.14 \times 48 - 36\sqrt{3} \\
 &= 150.72 - 36 \times 1.73 = 150.72 - 62.28 \\
 &= 88.44 \text{ cm}^2.
 \end{aligned}$$

Hence, area of segment (ACB) = 88.44 cm^2 .

Q.8. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope (see figure). Find

(i) The area of that part of the field in which the horse can graze.

(ii) The increase in the grazing area if the rope were 10 m long instead of 5 m. (use $\pi = 3.14$)



Sol. (i) We have,

Side of the square = 15 m

The area of the field in which the horse can graze represented in the figure by APR, a quadrant of the circle with radius 5 m.

The area which horse can graze

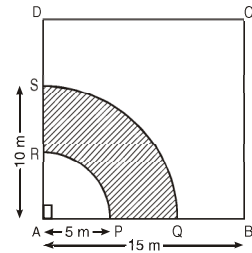
$$\begin{aligned}
 &= \frac{\pi r^2 \theta}{360^\circ} = \frac{3.14 \times 5^2 \times 90^\circ}{360^\circ} \\
 &= \frac{3.14 \times 25}{4} = 19.63 \text{ cm}^2.
 \end{aligned}$$

(ii) Increase in area when the rope is 10 m long

= Area of quadrant ASQ

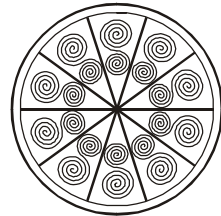
– Area of quadrant APR

$$\begin{aligned}
 &= \frac{\pi r^2 \theta}{360^\circ} - 19.63 \\
 &= \frac{3.14 \times 10^2 \times 90^\circ}{360^\circ} - 19.63 \\
 &= \frac{314}{4} - 19.63 \\
 &= 78.5 - 19.63 \\
 &= 58.87 \text{ cm}^2.
 \end{aligned}$$



Hence, area which horse can graze = 19.63 cm^2 and increase in area = 58.87 cm^2 .

Q.9. A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in figure. Find:



(i) The total length of the silver wire required.

(ii) The area of each sector of the brooch.

Sol. (i) We have,

$$\text{Radius of the circle} = \frac{35}{2} \text{ mm.}$$

$$\text{Circumference of the circle} = 2\pi r$$

$$= 2 \times \frac{22}{7} \times \frac{35}{2} = 110 \text{ mm}$$

The total length of wire required
 = circumference of the circle
 + length of 5 diameters
 = $110 + 5 \times 35 = 110 + 175 = 285$ mm.

(ii) Area of the circle = πr^2
 = $\frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} = \frac{11 \times 5 \times 35}{2}$
 = 962.5 mm^2
 Since circle is divided into 10 equal sectors.

Therefore, area of each sector

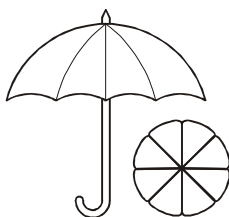
$$= \frac{\text{Area of the circle}}{10}$$

$$= \frac{962.5}{10} = 96.25 \text{ mm}^2.$$

Hence, (i) total length of wire required
 = 285 mm.

(ii) Area of each sector = 96.25 mm^2

Q.10. An umbrella has 8 ribs which are equally spaced (see figure). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.



Sol. We have, Radius of circle = 45 cm

Area of the circle = πr^2

$$= \pi \times 45^2 = 2025 \pi$$

$$= \frac{2025 \times 22}{7} = 6364.29 \text{ cm}^2$$

Since the ribs divide the circle into 8 sectors of equal area

Area between the two consecutive ribs = Area of each sector of the circle

$$= \frac{\text{Area of the circle}}{8} = \frac{6364.29}{8}$$

$$= 795.54 \text{ cm}^2.$$

Hence, area between the two consecutive ribs of the umbrella = 795.54 cm^2

Q.11. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115° . Find the total area cleaned at each sweep of the blades. [CBSE 2019]

Sol. We have, One blade of a wiper sweeps a sector area of circle of a radius 25 cm.

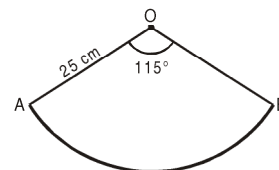
The sector angle (θ) = 115°

Area covered by one blade = $\frac{\pi r^2 \theta}{360^\circ}$

$$= \frac{22}{7} \times \frac{25^2 \times 115^\circ}{360^\circ}$$

$$= \frac{22 \times 625 \times 23}{7 \times 72}$$

$$= \frac{11 \times 625 \times 23}{7 \times 36}$$



The area covered by two blades

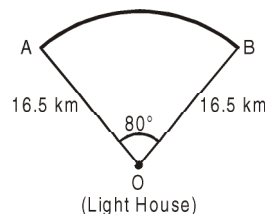
$$= \frac{2 \times 11 \times 625 \times 23}{7 \times 36}$$

$$= \frac{158125}{126} \text{ cm}^2.$$

Hence, total area cleaned at each sweep of the

blades = $\frac{158125}{126} \text{ cm}^2$

Q.12. To warn ships for underwater rocks, a light house spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. find the area of the sea over which the ships are warned. (use $\pi = 3.14$)



Sol. We have, Radius = 16.5 km

Sector angle (θ) = 80°

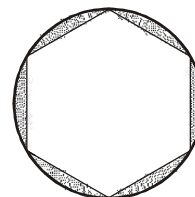
Area of the sea over which the ships are warned

$$= \frac{\pi r^2 \theta}{360} = \frac{3.14 \times 16.5 \times 16.5 \times 80}{360}$$

$$= \frac{3.14 \times 272.25 \times 2}{9} = 189.97 \text{ km}^2.$$

Hence, area of the sea over which the ships are warned = 189.97 km^2 .

Q.13. A round table cover has six equal designs as shown in figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of ₹ 0.35 per cm^2 . (Use $\sqrt{3} = 1.7$)



[CBSE 2011]

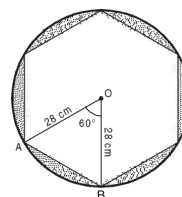
Sol. We have,

$$r = 28 \text{ cm}$$

$$\text{and } \theta = \frac{360^\circ}{6} = 60^\circ$$

In the figure $OA = OB$

$$\angle OAB = \angle OBA$$



$$\therefore \angle OAB + \angle OBA + \angle AOB = 180^\circ$$

$$\Rightarrow 2 \angle OAB + 60^\circ = 180^\circ$$

$$\Rightarrow 2 \angle OAB = 180^\circ - 60^\circ = 120^\circ$$

$$\Rightarrow \angle OAB = \frac{120}{2} = 60^\circ$$

$\therefore \triangle AOB$ is an equilateral triangle having side 28 cm.

Area of shaded designed portion
= Area of the sector OAB
– Area of $\triangle OAB$

$$\begin{aligned} &= \frac{\pi r^2 \theta}{360^\circ} - \frac{\sqrt{3}}{4} \times 28^2 \\ &= \frac{22}{7} \times \frac{28^2 \times 60^\circ}{360^\circ} - \frac{\sqrt{3}}{4} \times 28 \times 28 \\ &= \frac{22 \times 4 \times 28 \times 1}{6} - \sqrt{3} \times 7 \times 28 \end{aligned}$$

$$\begin{aligned} &= \frac{44 \times 28}{3} - 1.7 \times 196 \\ &= 410.67 - 333.2 = 77.47 \text{ cm}^2. \end{aligned}$$

The total area of six shaded designed portion
= $6 \times 77.47 = 464.82 \text{ cm}^2$.

Cost of making designs at the rate of 0.35 ₹/cm² = $464.82 \times 0.35 = ₹ 162.68$

Hence, cost of the making designs = ₹ 162.68

Q.14. Tick the correct answer in the following:
Area of a sector of angle p (in degrees) of a circle with radius R is

- (A) $\frac{p}{180^\circ} \times 2\pi R$ (B) $\frac{p}{180^\circ} \times \pi R^2$
(C) $\frac{p}{360^\circ} \times 2\pi R$ (D) $\frac{p}{720^\circ} \times 2\pi R^2$

Ans. (D)

12

Surface Areas and Volumes

Textbook Exercises 12.1

Unless stated otherwise, take $\pi = \frac{22}{7}$.

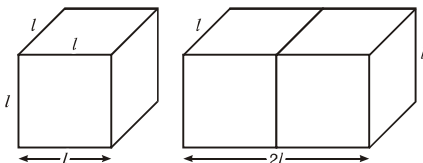
Q.1. 2 cubes each of volume 64 cm³ are joined end to end. Find the surface area of the resulting cuboid.

Sol. Let a be length of an edge of the cube,
Volume of cube = 64 cm³ (given)

$$\Rightarrow a^3 = 64$$

$$\Rightarrow a = \sqrt[3]{64}$$

$$\Rightarrow a = 4 \text{ cm}$$



Now, dimensions of the resulting cuboid made by joining two cubes are 8 cm $\times$ 4 cm $\times$ 4 cm.

$\therefore$ Length of cuboid = 8 cm

Breadth of cuboid = 4 cm

Height of cuboid = 4 cm

Surface area of the cuboid
= $2 (lb + bh + hl)$

$$\begin{aligned} &= 2 (8 \times 4 + 4 \times 4 + 4 \times 8) \\ &= 2 (32 + 16 + 32) = 2 \times 80 \\ &= 160 \text{ cm}^2 \end{aligned}$$

Hence, surface area of the cuboid = 160 cm²

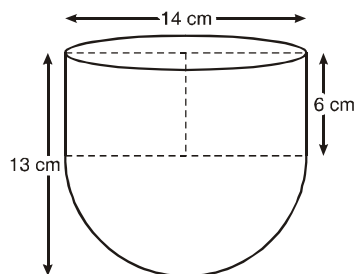
Q.2. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and total height of the vessel is 13 cm. Find the inner surface area of the vessel.

Sol. We have,

Diameter of the hemispherical portion
= 14 cm

Radius of the hemispherical portion

$$= \frac{14}{2} = 7 \text{ cm}$$



Total height of the vessel = 13 cm

Height of the cylindrical portion

$$= 13 - 7 = 6 \text{ cm}$$

Total inner surface area of the vessel

$$\begin{aligned} &= \text{C.S.A. of hemispherical portion} \\ &\quad + \text{C.S.A. of cylindrical portion} \\ &= 2\pi r^2 + 2\pi rh = 2\pi r(r + h) \\ &= 2 \times \frac{22}{7} \times 7(7 + 6) \\ &= 44 \times 13 = 572 \text{ cm}^2 \end{aligned}$$

Hence, total inner surface area of the vessel = 572 cm^2

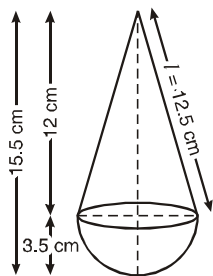
Q.3. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

Sol. We have, Radius (r) = 3.5 cm

Total height of the toy = 15.5 cm

height of the conical part

$$(h) = 15.5 - 3.5 = 12 \text{ cm}$$



Slant height of the conical part

$$\begin{aligned} (l) &= \sqrt{h^2 + r^2} = \sqrt{12^2 + (3.5)^2} \\ &= \sqrt{(12)^2 + (3.5)^2} \\ &= \sqrt{144 + 12.25} \\ &= \sqrt{156.25} = 12.5 \text{ cm} \end{aligned}$$

Total surface area of the toy

$$\begin{aligned} &= 2\pi r^2 + \pi rl = \pi r(2r + l) \\ &= \frac{22}{7} \times 3.5(2 \times 3.5 + 12.5) \\ &= 11 \times 19.5 = 214.5 \text{ cm}^2. \end{aligned}$$

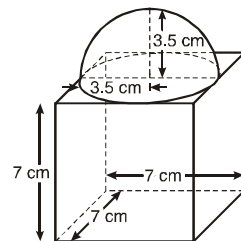
Hence, total surface area of the toy = 214.5 cm^2 .

Q.4. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.

Sol. We have,

Edge of cubical block = 7 cm

Greatest diameter of the hemisphere = edge of cube = 7 cm



$$\therefore \text{Radius of hemisphere} = \frac{7}{2} = 3.5 \text{ cm}$$

Surface area of the solid = The surface area of the cube + C.S.A. of the hemisphere – base area of the hemisphere

$$\begin{aligned} &= 6 \times (\text{edge})^2 + 2\pi r^2 - \pi r^2 \\ &= 6 \times (7)^2 + \pi r^2 \\ &= 6 \times 49 + \frac{22}{7} \times (3.5)^2 \\ &= 294 + 38.5 = 332.5 \text{ cm}^2 \end{aligned}$$

Hence, surface area of the solid = 332.5 cm^2

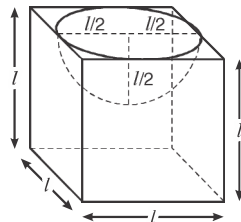
Q.5. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

Sol. We have,

Diameter of the hemisphere = l

Edge of the cube = diameter of the hemisphere = l

$$\text{radius of the hemisphere} = \frac{l}{2}$$



Surface Area of the remaining solid

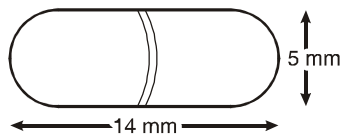
$$\begin{aligned} &= \text{surface area of the cube} \\ &\quad + \text{Inner C.S.A. of the hemisphere} \\ &\quad - \text{Area of the top of the hemisphere} \end{aligned}$$

$$\begin{aligned}
 &= 6l^2 + 2\pi r \times \left(\frac{l}{2}\right)^2 - \pi \left(\frac{l}{2}\right)^2 \\
 &= 6l^2 + \pi \times \left(\frac{l}{2}\right)^2 = 6l^2 + \frac{\pi l^2}{4} \\
 &= \frac{1}{4}(24l^2 + \pi l^2) = \frac{l^2}{4}(24 + \pi)
 \end{aligned}$$

Hence, surface area of the remaining solid

$$= \frac{1}{4}l^2(24 + \pi)$$

Q.6. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends (see in figure).



The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.

Sol. We have,

Radius of the cylindrical part and hemispherical

$$\text{part } (r) = \frac{5}{2} = 2.5 \text{ mm}$$

Length of the cylindrical part

$$\begin{aligned}
 (h) &= 14 - (2.5 + 2.5) \\
 &= 14 - 5 = 9 \text{ mm}
 \end{aligned}$$

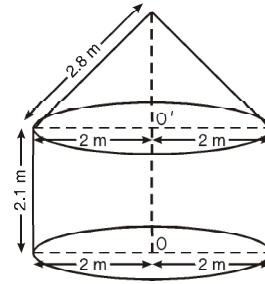
Surface area of the capsule

$$\begin{aligned}
 &= \text{C.S.A. of the cylindrical part} \\
 &\quad + \text{C.S.A. of two hemispherical parts}
 \end{aligned}$$

$$\begin{aligned}
 &= 2\pi rh + 2 \times 2\pi r^2 \\
 &= 2\pi r (h + 2r) \\
 &= 2 \times \frac{22}{7} \times 2.5 (9 + 2 \times 2.5) \\
 &= \frac{22}{7} \times 5 \times 14 = 220 \text{ mm}^2.
 \end{aligned}$$

$$\begin{aligned}
 \text{Hence, surface area of the capsule} \\
 &= 220 \text{ mm}^2
 \end{aligned}$$

Q.7. A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of ₹ 500 per m^2 . (Note that the base of the tent will not be covered with canvas.)



Sol. We have,

$$\text{Radius of conical portion radius } (r) = \frac{4}{2} = 2 \text{ m}$$

$$\text{Slant height } (l) = 2.8 \text{ m}$$

For cylindrical portion

$$\text{Radius } (r) = 2 \text{ m}$$

$$\text{Height } (h) = 2.1 \text{ m}$$

Area of canvas used for making the tent

$$\begin{aligned}
 &= \text{Surface area of tent} \\
 &= \text{C.S.A. of conical part} \\
 &\quad + \text{C.S.A. of cylindrical part} \\
 &= \pi rl + 2\pi rh = \pi r(l + 2h) \\
 &= \frac{22}{7} \times 2 (2.8 + 2 \times 2.1) \\
 &= \frac{44}{7} \times 7 = 44 \text{ m}^2.
 \end{aligned}$$

Cost of canvas at the rate of ₹ 500/ m^2

$$= 44 \times 500 = ₹ 22000$$

Hence, area of the canvas = 44 m^2

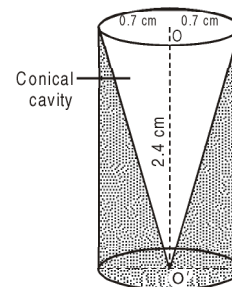
and cost of canvas = ₹ 22000.

Q.8. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2 .

Sol. We have, Height of the cylinder = 2.4 cm

Diameter of the cylinder = 1.4 cm

$$\therefore \text{Radius of the cylinder} = \frac{1.4}{2} = 0.7 \text{ cm}$$



$$\begin{aligned}\text{Slant height of the cone } (l) &= \sqrt{h^2 + r^2} \\ &= \sqrt{(2.4)^2 + (0.7)^2} = 2.5 \text{ cm}\end{aligned}$$

$$\begin{aligned}\text{Total surface area of the remaining solid} &= \text{C.S.A. of the cylinder} + \text{Area of base} \\ &\quad \text{of the cylinder} + \text{C.S.A. of the cone} \\ &= 2\pi rh + \pi r^2 + \pi rl = \pi r (2h + r + l) \\ &= \frac{22}{7} \times 0.7 (2 \times 2.4 + 0.7 + 2.5) \\ &= 2.2 (4.8 + 3.2) \\ &= 2.2 \times 8 \\ &= 17.6 \text{ cm}^2 = 18 \text{ cm}^2. \text{ (appx)}\end{aligned}$$

Hence, the total surface area of the remaining solid to the nearest $\text{cm}^2 = 18 \text{ cm}^2$.

Q.9. A wooden article was made by scooping out hemisphere from each end of a solid cylinder, as shown in figure. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the total surface area of the article.



Sol. We have,

Height of the cylinder (h) = 10 cm

Radius of cylinder and hemisphere

$$(r) = 3.5 \text{ cm}$$

$$\begin{aligned}\text{Total surface area of the article} &= \text{C.S.A of the cylinder} + 2 \\ &\quad (\text{surface area of a hemisphere}) \\ &= 2\pi rh + 2 \times 2\pi r^2 \\ &= 2\pi r (h + 2r) \\ &= 2 \times \frac{22}{7} \times 3.5 (10 + 2 \times 3.5) \\ &= 22 \times 17 = 374 \text{ cm}^2.\end{aligned}$$

Hence, total surface area of the article
= 374 cm^2

Textbook Exercises 12.2

Unless stated otherwise, take $\pi = \frac{22}{7}$

Q.1. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and height of the cone is equal to its radius. Find the volume of the solid in terms of π .

Sol. We have,

Radius of the hemisphere

$$(r) = 1 \text{ cm}$$

Radius of the cone

$$(r) = 1 \text{ cm}$$

Height of the cone

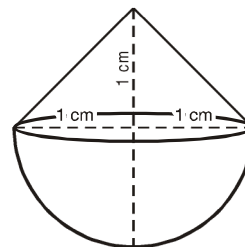
$$(h) = 1 \text{ cm}$$

Volume of the solid

$$\begin{aligned}&= \text{Volume of the hemisphere} \\ &\quad + \text{Volume of the cone}\end{aligned}$$

$$= \frac{2}{3} \pi r^3 + \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \pi r^2 (2r + h)$$



$$= \frac{1}{3} \pi \times 1^2 (2 \times 1 + 1)$$

$$= \frac{1}{3} \pi \times 1 \times 3 = \pi \text{ cm}^3.$$

Hence, volume of the solid

$$= \pi \text{ cm}^3.$$

Q.2. Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same)

Sol. We have,

Radius of two conical parts (r)

$$= \frac{3}{2} = 1.5 \text{ cm}$$

Radius of the cylindrical part (r)

$$= 1.5 \text{ cm}$$

Height of the conical part (h) = 2 cm

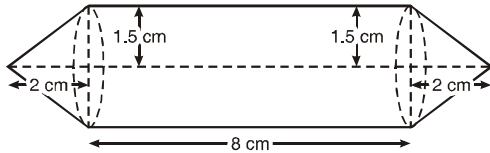
Height of the cylindrical part (H)

$$= 12 - (2 + 2) = 8 \text{ cm}.$$

Volume of the air contained in the model
 = Volume of the cylindrical part +
 Volumes of two conical parts

$$= \pi r^2 H + \frac{2 \times \pi r^2 h}{3}$$

$$= \pi r^2 \left[H + \frac{2 \times h}{3} \right]$$



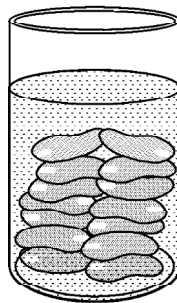
$$= \pi \times 1.5^2 \left[8 + \frac{2 \times 2}{3} \right] = \pi \times 2.25 \times \frac{28}{3}$$

$$= \frac{22}{7} \times 2.25 \times \frac{28}{3}$$

$$= 66 \text{ cm}^3.$$

Hence, volume of the air contained in the model = 66 cm^3

Q.3. A *gulab jamun*, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 *gulab jamuns*, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm. (see in figure.)



Sol. We have,

Radius of a cylindrical part (r) = 1.4 cm

Radius of hemispherical part (r) = 1.4 cm

Length of the cylindrical part of a *gulab jamun* (h)

$$= [5 - (1.4 + 1.4)]$$

$$= 5 - 2.8 = 2.2 \text{ cm}$$

Volume of *gulab jamun*

$$= \text{Volume of the cylindrical part}$$

$$+ \text{Volume of two hemispherical part}$$

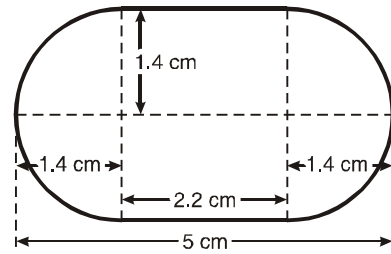
$$= \pi r^2 h + 2 \times \frac{2}{3} \pi r^3 = \pi r^2 \left(h + \frac{4}{3} r \right)$$

$$= \frac{22}{7} \times (1.4)^2 \left(2.2 + \frac{4}{3} \times 1.4 \right)$$

$$= 6.16 \times 4.0666 = 25.05 \text{ cm}^3$$

Volume of 45 *gulab jamuns*

$$= 25.05 \times 45 = 1127.25 \text{ cm}^3$$



Volume of the syrup

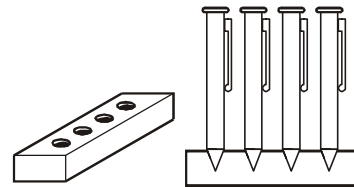
$$= 30\% \text{ of } 1127.25$$

$$= \frac{30}{100} \times 1127.25 = 338.18 \text{ cm}^3$$

Hence, volume of the syrup

$$= 338 \text{ cm}^3 \text{ (approx)}$$

Q.4. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand. (see in figure.)



Sol. We have,

Dimensions of the cuboidal part are 15 cm $\times$ 10 cm $\times$ 3.5 cm

Volume of the cuboid

$$= 15 \times 10 \times 3.5 = 525 \text{ cm}^3$$

Radius of each conical depression

$$(r) = 0.5 \text{ cm}$$

Depth (height) of each conical depression

$$(h) = 1.4 \text{ cm}$$

Volume of wood taken out to make four cavities

$$= 4 \times \text{volume of conical depression}$$

$$= 4 \times \frac{1}{3} \pi r^2 h$$

$$= 4 \times \frac{1}{3} \times \frac{22}{7} \times 0.5^2 \times 1.4$$

$$= \frac{88 \times 0.25 \times 0.2}{3}$$

$$= 1.47 \text{ cm}^3 \text{ (approx)}$$

Volume of the wood in the entire stand

$$= 525 - 1.47$$

$$= 523.53 \text{ cm}^3$$

Hence, volume of the wood in the entire stand
 $= 523.53 \text{ cm}^3$

Q.5. A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one fourth of the water flows out. Find the number of lead shots dropped in the vessel.

Sol. We have,

Radius of the base of the cone (r) = 5 cm

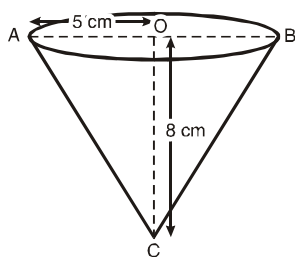
Height of the cone (h) = 8 cm

Volume of water of cone

$$\begin{aligned} &= \frac{1}{3} \pi r^2 h = \frac{1}{3} \times \pi \times 5^2 \times 8 \\ &= \frac{200\pi}{3} \text{ cm}^3 \end{aligned}$$

Volume of the water that flows out of the cone

$$= \frac{1}{4} \times \frac{200\pi}{3} = \frac{50\pi}{3} \text{ cm}^3$$



Radius of spherical lead

$$(r_1) = 0.5 \text{ cm}$$

Volume of spherical lead

$$\begin{aligned} &= \frac{4}{3} \pi r_1^3 = \frac{4}{3} \pi \times (0.5)^3 \\ &= \frac{\pi \times 0.5}{3} = \frac{\pi}{6} \text{ cm}^3 \end{aligned}$$

Let n spherical lead shots are dropped into the vessel. So that $\frac{1}{4}$ th of the water contained in the vessel flows out.

$\therefore$ Volume of n spherical lead shots

= Volume of water that flows out

$$\Rightarrow n \times \frac{\pi}{6} = \frac{50\pi}{3}$$

$$\Rightarrow n = \frac{50\pi}{3} \times \frac{6}{\pi} \Rightarrow n = 100$$

Hence, number of lead shots dropped in the vessel = 100.

Q.6. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8 g mass. (Use $\pi = 3.14$) **[CBSE 2019]**

Sol. We have,

First cylindrical part has radius 12 cm and height 220 cm

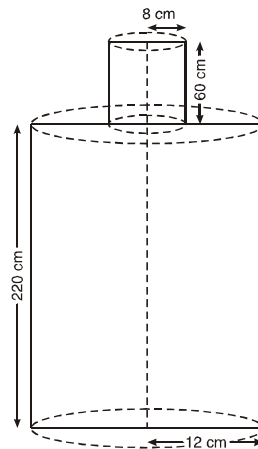
$$\begin{aligned} \text{Its volume} &= \pi \times 12^2 \times 220 \\ &= 31680\pi \text{ cm}^3 \end{aligned}$$

Second cylindrical part has radius 8 cm and height 60 cm

$$\text{Its volume} = \pi \times 8^2 \times 60 = 3840\pi \text{ cm}^3$$

Total volume of iron pole

$$\begin{aligned} &= 31680\pi + 3840\pi \\ &= 35520\pi = 35520 \times 3.14 \\ &= 111532.8 \text{ cm}^3 \end{aligned}$$



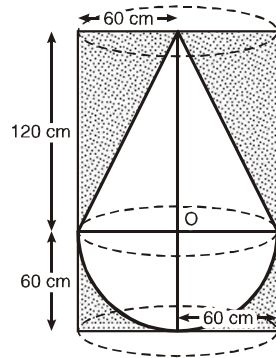
Total mass of the iron pole

$$\begin{aligned} &= 111532.8 \times 8 \text{ gm} \\ &= \frac{111532.8 \times 8}{1000} \text{ kg} \\ &= 892.2624 \text{ kg} = 892.26 \text{ kg} \end{aligned}$$

Hence, weight of iron pole

$$= 892.26 \text{ kg}$$

Q.7. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.



Sol. We have,

Radius of the cylinder

$$(r) = 60 \text{ cm}$$

Height of the cylinder

$$(H) = 180 \text{ cm}$$

Volume of water that the cylinder contain

$$\begin{aligned} &= \pi r^2 H \\ &= \pi \times 60^2 \times 180 \\ &= 648000\pi \text{ cm}^3 \end{aligned}$$

Radius of the conical part

$$(r) = 60 \text{ cm}$$

Radius of the hemispherical part

$$(r) = 60 \text{ cm}$$

Height of the conical part

$$(h) = 120 \text{ cm}$$

Volume of the solid = Volume of conical part
+ Volume of hemispherical part

$$\begin{aligned} &= \frac{1}{3} \pi r^2 h + \frac{2}{3} \pi r^3 \\ &= \frac{1}{3} \pi r^2 (h + 2r) \\ &= \frac{1}{3} \pi \times 60^2 (120 + 2 \times 60) \\ &= \frac{1}{3} \pi \times 3600 \times 240 \\ &= 288000\pi \text{ cm}^3 \end{aligned}$$

Volume of water left in the cylinder

$$\begin{aligned} &= \text{Volume of water that cylinder contains} - \text{Volume of solid} \\ &= 648000\pi - 288000\pi \\ &= 360000\pi \\ &= 360000 \times \frac{22}{7} \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} &= \frac{360000 \times 22}{7 \times 100 \times 100 \times 100} \text{ m}^3 \\ &\left[\because 1 \text{ cm}^3 = \frac{1}{100} \times \frac{1}{100} \times \frac{1}{100} \text{ m}^3 \right] \end{aligned}$$

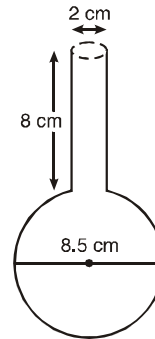
$$= \frac{22 \times 36}{700} \text{ m}^3 = 1.1314 \text{ m}^3$$

$$= 1.131 \text{ m}^3 \text{ (approx)}$$

Hence, Volume of water out in the cylinder

$$= 1.131 \text{ m}^3 \text{ (approx)}$$

Q.8. A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter ; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm^3 . Check whether she is correct, taking the above as the inside measurements and $\pi = 3.14$.



Sol. We have,

Radius of cylindrical neck

$$(r) = \frac{2}{2} = 1 \text{ cm}$$

Height of cylindrical neck

$$(h) = 8 \text{ cm}$$

Radius of the spherical part

$$(R) = \frac{8.5}{2} \text{ cm}$$

Amount of water in the vessel

= Volume of the cylindrical neck
+ Volume of the spherical part

$$\begin{aligned} &= \pi r^2 h + \frac{4}{3} \pi R^3 = \pi \left(r^2 h + \frac{4}{3} R^3 \right) \\ &= 3.14 \left[1^2 \times 8 + \frac{4}{3} \times \left(\frac{8.5}{2} \right)^3 \right] \\ &= 3.14 (8 + 102.354) \\ &= 3.14 \times 110.354 = 346.51 \text{ cm}^3 \end{aligned}$$

Hence, volume find by the child is not correct.

Correct answer is 346.51 cm^3

Textbook Exercises 13.1

Q.1. A survey was conducted by a group of students as a part of their environment awareness programme, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house.

Number of plants	0 - 2	2 - 4	4 - 6	6 - 8	8 - 10	10 - 12	12 - 14
Number of houses	1	2	1	5	6	2	3

Which method did you use for finding the mean, and why ?

Sol.

Number of plants	Number of houses (f_i)	Class mark (x_i)	$f_i x_i$
0 - 2	1	1	1
2 - 4	2	3	6
4 - 6	1	5	5
6 - 8	5	7	35
8 - 10	6	9	54
10 - 12	2	11	22
12 - 14	3	13	39
Total	$\Sigma f_i = 20$		$\Sigma f_i x_i = 162$

From the table, we have $\Sigma f_i = 20$, $\Sigma f_i x_i = 162$

$$\therefore \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} \Rightarrow \bar{x} = \frac{162}{20} = 8.1.$$

Hence, mean number of plants per house = 8.1 plants.

We used direct method to find the mean because numerical values of f_i 's and x_i 's were small.

Q.2. Consider the following distribution of daily wages of 50 workers of a factory.

Daily wages in (₹)	100 - 120	120 - 140	140 - 160	160 - 180	180 - 200
Number of workers	12	14	8	6	10

Find the mean daily wages of the workers of the factory by using an appropriate method.

Sol. Here, $h = 20$

Daily wages in (₹)	Number of workers (f_i)	Class mark (x_i)	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
100 - 120	12	110	-2	-24
120 - 140	14	130	-1	-14
140 - 160	8	150 = a	0	0
160 - 180	6	170	1	6
180 - 200	10	190	2	20
Total	$\Sigma f_i = 50$			$\Sigma f_i u_i = -12$

From the table, we have

$$\Sigma f_i = 50, \Sigma f_i u_i = -12, a = 150, h = 20$$

$$\begin{aligned} \therefore \bar{x} &= a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i} \right) \times h \\ \Rightarrow \bar{x} &= 150 + \left(-\frac{12}{50} \right) \times 20 \\ \Rightarrow \bar{x} &= 150 - 4.80 \\ \Rightarrow \bar{x} &= 145.20. \end{aligned}$$

Hence, the mean daily wages of the workers = ₹ 145.20.

Q.3. The following distribution shows the daily pocket allowance of children of a locality. The mean pocket allowance is ₹ 18. Find the missing frequency f .

Daily pocket allowance (in ₹)	11-13	13-15	15-17	17-19	19-21	21-23	23-25
Number of children	7	6	9	13	f	5	4

Sol. Here, $h = 2$

Daily pocket allowance (in ₹)	Number of children (f_i)	Class mark (x_i)	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
11 - 13	7	12	-3	-21
13 - 15	6	14	-2	-12
15 - 17	9	16	-1	-9
17 - 19	13	18 = a	0	0
19 - 21	f	20	1	f
21 - 23	5	22	2	10
23 - 25	4	24	3	12
Total	$\Sigma f_i = 44 + f$			$\Sigma f_i u_i = -20 + f$

From the table, we have

$$a = 18, h = 2, \Sigma f_i = 44 + f, \Sigma f_i u_i = -20 + f \text{ and mean } (\bar{x}) = ₹ 18$$

$$\begin{aligned} \therefore \bar{x} &= a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i} \right) \times h \\ \Rightarrow 18 &= 18 + \left(\frac{-20 + f}{44 + f} \right) \times 2 \\ \Rightarrow 0 &= \frac{-40 + 2f}{44 + f} \\ \Rightarrow 0 &= -40 + 2f \\ \Rightarrow 2f &= 40 \\ \Rightarrow f &= \frac{40}{2} = 20 \end{aligned}$$

Hence, the missing frequency (f) = 20.

Q.4. Thirty women were examined in a hospital by a doctor and the number of heart beats per minute were recorded and summarised as follows. Find the mean heart beats per minute for these women, choosing a suitable method.

Number of heart beats per minute	65 - 68	68 - 71	71 - 74	74 - 77	77 - 80	80 - 83	83 - 86
Number of women	2	4	3	8	7	4	2

Sol. here, $h = 3$

Number of heart beats per minute	Number of women (f_i)	Class mark (x_i)	$u_i = \frac{x_i - 75.5}{h}$	$f_i u_i$
65 - 68	2	66.5	-3	-6
68 - 71	4	69.5	-2	-8
71 - 74	3	72.5	-1	-3
74 - 77	8	75.5 = a	0	0
77 - 80	7	78.5	1	7
80 - 83	4	81.5	2	8
83 - 86	2	84.5	3	6
Total	$\Sigma f_i = 30$			$\Sigma f_i u_i = 4$

From the table, we have $a = 75.5$, $h = 3$, $\Sigma f_i = 30$, $\Sigma f_i u_i = 4$

$$\therefore \bar{x} = a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i} \right) \times h \Rightarrow \bar{x} = 75.5 + \left(\frac{4}{30} \right) \times 3$$

$$\Rightarrow \bar{x} = 75.5 + 0.4 \Rightarrow \bar{x} = 75.9$$

Hence, mean heart beats per minute = 75.9.

Q.5. In a retail market, fruit vendors were selling mangoes kept in packing boxes. These boxes contained varying number of mangoes. The following was the distribution of mangoes according to the number of boxes.

Number of mangoes	50 - 52	53 - 55	56 - 58	59 - 61	62 - 64
Number of boxes	15	110	135	115	25

Find the mean number of mangoes kept in a packing box. Which method of finding the mean did you choose?

Sol. The given series is an inclusive series. It is not necessary to change it in an exclusive series because the class interval includes both 50 and 52. So we take $h = 3$ (not 2).

Number of Mangoes	Numbers of boxes (f_i)	Class mark (x_i)	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
50 - 52	15	51	-2	-30
53 - 55	110	54	-1	-110
56 - 58	135	57 = a	0	0
59 - 61	115	60	1	115
62 - 64	25	63	2	50
Total	$\Sigma f_i = 400$			$\Sigma f_i u_i = 25$

From the table, we have,

$$\Sigma f_i = 400, \Sigma f_i u_i = 25, a = 57, h = 3$$

$$\therefore \bar{x} = a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i} \right) \times h \Rightarrow \bar{x} = 57 + \left(\frac{25}{400} \right) \times 3$$

$$\Rightarrow \bar{x} = 57 + 0.19 \Rightarrow \bar{x} = 57.19$$

Hence, the mean number of mangoes = 57.19 .

We used step-deviation method.

Q.6. The table below shows the daily expenditure on food of 25 households in a locality.

Daily expenditure (in ₹)	100 - 150	150 - 200	200 - 250	250 - 300	300 - 350
Number of households	4	5	12	2	2

Find the mean daily expenditure on food by a suitable method.

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Sol. Here $h = 50$

Daily expenditure in (₹)	Number of households (f_i)	Class mark (x_i)	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
100 - 150	4	125	-2	-8
150 - 200	5	175	-1	-5
200 - 250	12	225 = a	0	0
250 - 300	2	275	1	2
300 - 350	2	325	2	4
Total	$\Sigma f_i = 25$			$\Sigma f_i u_i = -7$

From the table, we have

$$\Sigma f_i = 25, \Sigma f_i u_i = -7, a = 225, h = 50$$

$$\therefore \bar{x} = a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i} \right) \times h \Rightarrow \bar{x} = 225 + \left(-\frac{7}{25} \right) \times 50$$

$$\Rightarrow \bar{x} = 225 - 14 \Rightarrow \bar{x} = 211$$

Hence, the mean daily expenditure on food = ₹ 211.

Q.7. To find out the concentration of SO_2 in the air (in parts per million, i.e., ppm), the data was collected for 30 localities in a certain city and is presented below:

Concentration of SO_2 (in ppm)	0.00-0.04	0.04-0.08	0.08-0.12	0.12-0.16	0.16-0.20	0.20-0.24
Frequency	4	9	9	2	4	2

Find the mean concentration of SO_2 in the air.

Sol.

Concentration of SO_2 (in ppm)	Frequency (f_i)	Class mark (x_i)	$f_i x_i$
0.00 - 0.04	4	0.02	0.08
0.04 - 0.08	9	0.06	0.54
0.08 - 0.12	9	0.10	0.90
0.12 - 0.16	2	0.14	0.28
0.16 - 0.20	4	0.18	0.72
0.20 - 0.24	2	0.22	0.44
Total	$\Sigma f_i = 30$		$\Sigma f_i x_i = 2.96$

From the table, we have $\Sigma f_i = 30, \Sigma f_i x_i = 2.96$

$$\therefore \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{2.96}{30} = 0.099$$

Hence, the mean concentration of SO_2 in the air = 0.099 ppm.

Q.8. A class teacher has the following absentee record of 40 students of a class for the whole term. Find the mean number of days a student was absent.

Number of days	0 - 6	6 - 10	10 - 14	14 - 20	20 - 28	28 - 38	38 - 40
Number of students	11	10	7	4	4	3	1

Sol.	Number of days	Number of students (f_i)	Class mark (x_i)	$d_i = x_i - a$	$f_i d_i$
	0 - 6	11	3	-14	-154
	6 - 10	10	8	-9	-90
	10 - 14	7	12	-5	-35
	14 - 20	4	17 = a	0	0
	20 - 28	4	24	7	28
	28 - 38	3	33	16	48
	38 - 40	1	39	22	22
	Total	$\Sigma f_i = 40$			$\Sigma f_i d_i = -181$

From the table, we have

$$\Sigma f_i = 40, \Sigma f_i d_i = -181, a = 17$$

$$\therefore \bar{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i} \Rightarrow \bar{x} = 17 + \left(-\frac{181}{40}\right)$$

$$\Rightarrow \bar{x} = 17 - 4.52 = 12.48$$

Hence, the mean number of days a student was absent = 12.48 days.

Q.9. The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate.

Literacy rate (in %)	45 - 55	55 - 65	65 - 75	75 - 85	85 - 95
Number of cities	3	10	11	8	3

Sol.	Literacy rate (in %)	Number of cities (f_i)	Class mark (x_i)	$d_i = x_i - a$	$f_i d_i$
	45 - 55	3	50	-20	-60
	55 - 65	10	60	-10	-100
	65 - 75	11	70 = a	0	0
	75 - 85	8	80	10	80
	85 - 95	3	90	20	60
	Total	$\Sigma f_i = 35$			$\Sigma f_i d_i = -20$

From the table, we have

$$\Sigma f_i = 35, \Sigma f_i d_i = -20, a = 70$$

$$\therefore \bar{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i} \Rightarrow \bar{x} = 70 + \left(-\frac{20}{35}\right) \Rightarrow \bar{x} = 70 - 0.57 = 69.43\%$$

Hence, the mean literacy rate = 69.43%.

Textbook Exercises 13.2

Q.1. The following table shows the ages of the patients admitted in a hospital during a year:

Age (in years)	5 - 15	15 - 25	25 - 35	35 - 45	45 - 55	55 - 65
Number of patients	6	11	21	23	14	5

Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.

Sol. The class 35 - 45 has maximum frequency. So, it is the modal class.

$\therefore l = 35, f_1 = 23, f_0 = 21, f_2 = 14$ and $h = 10$

$$\begin{aligned}
 \text{Mode} &= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\
 &= 35 + \left(\frac{23 - 21}{2 \times 23 - 21 - 14} \right) \times 10 \\
 &= 35 + \left(\frac{2}{46 - 21 - 14} \right) \times 10 = 35 + \frac{20}{11} \\
 &= 35 + 1.8 = 36.8 \text{ years}
 \end{aligned}$$

Now let us find the mean of the data. Here $h = 10$

Age (in years)	Number of patients (f_i)	Class mark x_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
5 - 15	6	10	-3	-18
15 - 25	11	20	-2	-22
25 - 35	21	30	-1	-21
35 - 45	23	40 = a	0	0
45 - 55	14	50	1	14
55 - 65	5	60	2	10
Total	$\Sigma f_i = 80$			$\Sigma f_i u_i = -37$

From the table, we have $\Sigma f_i = 80$, $\Sigma f_i u_i = -37$, $a = 40$, and $h = 10$

$$\begin{aligned}
 \text{Mean} &= a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i} \right) \times h \\
 &= 40 + \frac{-37}{80} \times 10 = 40 - 4.63 = 35.37 \text{ years.}
 \end{aligned}$$

Hence, mode = 36.8 years and mean = 35.37 years. Maximum number of patients admitted in the hospital are of the age 36.8 years (approx). While on an average the age of a patient admitted in the hospital is 35.37 years.

Q.2. The following data gives the information on the observed lifetimes (in hours) of 225 electrical components:

Lifetimes (in hours)	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100	100 - 120
Frequency	10	35	52	61	38	29

Determine the modal lifetimes of the components.

Sol. The class 60-80 has maximum frequency. So, it is the modal class.

$$\therefore l = 60, f_1 = 61, f_0 = 52, f_2 = 38 \text{ and } h = 20$$

$$\begin{aligned} \text{Mode} &= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\ &= 60 + \left(\frac{61 - 52}{2 \times 61 - 52 - 38} \right) \times 20 \\ &= 60 + \left(\frac{9}{122 - 52 - 38} \right) \times 20 \\ &= 60 + \frac{180}{32} = 60 + 5.625 \\ &= 65.625 \text{ hours.} \end{aligned}$$

Hence, the modal lifetimes of the electrical components = 65.625 years

Q.3. The following data gives the distribution of total monthly household expenditure of 200 families of a village. Find the modal monthly expenditure of the families. Also, find the mean monthly expenditure :

Expenditure (in ₹)	Number of families	Expenditure (in ₹)	Number of families
1000 - 1500	24	3000 - 3500	30
1500 - 2000	40	3500 - 4000	22
2000 - 2500	33	4000 - 4500	16
2500 - 3000	28	4500 - 5000	7

Sol. The class 1500 - 2000 has maximum frequency. So, it is the modal class.

$$\therefore l = 1500, f_1 = 40, f_0 = 24, f_2 = 33 \text{ and } h = 500$$

$$\begin{aligned} \text{Mode} &= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\ &= 1500 + \left(\frac{40 - 24}{2 \times 40 - 24 - 33} \right) \times 500 \\ &= 1500 + \left(\frac{16}{80 - 24 - 33} \right) \times 500 \\ &= 1500 + \frac{8000}{23} = 1500 + 347.83 = ₹ 1847.83 \end{aligned}$$

Now, let us find the mean. Here $h = 500$.

Expenditure (in ₹)	Number of families (f_i)	Class mark (x_i)	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
1000 - 1500	24	1250	-4	-96
1500 - 2000	40	1750	-3	-120
2000 - 2500	33	2250	-2	-66
2500 - 3000	28	2750	-1	-28
3000 - 3500	30	3250 = a	0	0
3500 - 4000	22	3750	1	22
4000 - 4500	16	4250	2	32
4500 - 5000	7	4750	3	21
Total	$\Sigma f_i = 200$			$\Sigma f_i u_i = -235$

From the table, we have

$$\Sigma f_i = 200, \Sigma f_i u_i = -235, a = 3250 \text{ and } h = 500$$

$$\begin{aligned} \text{Mean} &= a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i} \right) \times h = 3250 + \left(\frac{-235}{200} \right) \times 500 \\ &= 3250 - 587.5 = ₹ 2662.5 \end{aligned}$$

Hence, modal monthly expenditure = ₹ 1847.83

Mean monthly expenditure = ₹ 2662.5.

Q.4. The following distribution gives the state-wise teacher-student ratio in the higher secondary schools of India. Find the mode and mean of this data. Interpret the two measures.

Number of students per teacher	Number of states / U.T.
15 - 20	3
20 - 25	8
25 - 30	9
30 - 35	10
35 - 40	3
40 - 45	0
45 - 50	0
50 - 55	2

Sol. The class 30 - 35 has maximum frequency. So, it is the modal class.

$$\therefore l = 30, f_1 = 10, f_0 = 9, f_2 = 3, \text{ and } h = 5$$

$$\begin{aligned} \text{Mode} &= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\ &= 30 + \left(\frac{10 - 9}{2 \times 10 - 9 - 3} \right) \times 5 \\ &= 30 + \left(\frac{1}{8} \right) \times 5 \\ &= 30 + \frac{5}{8} = 30 + 0.625 = 30.6 \end{aligned}$$

Now, let us find the mean.

Number of students per teacher	Number of states /U.T. (f_i)	Class mark (x_i)	$d_i = x_i - a$	$f_i d_i$
15 - 20	3	17.5	-15	-45
20 - 25	8	22.5	-10	-80
25 - 30	9	27.5	-5	-45
30 - 35	10	32.5 = a	0	0
35 - 40	3	37.5	5	15
40 - 45	0	42.5	10	0
45 - 50	0	47.5	15	0
50 - 55	2	52.5	20	40
Total	$\Sigma f_i = 35$			$\Sigma f_i d_i = -115$

From the table, we have

$$\Sigma f_i = 35, \Sigma f_i d_i = -115, \text{ and } a = 32.5$$

$$\text{Mean} = a + \frac{\Sigma f_i d_i}{\Sigma f_i} = 32.5 + \left(\frac{-115}{35} \right) = 32.5 - 3.3 = 29.2$$

Hence, mode = 30.6 and mean = 29.2. We conclude that most of the states/U.T. have a student-teacher ratio of 30.6 and on an average, this ratio is 29.2.

Q.5. The given distribution shows the number of runs scored by some top batsmen of the world in one-day international cricket matches.

Runs scored	Number of batsmen	Runs scored	Number of batsmen
3000 - 4000	4	7000 - 8000	6
4000 - 5000	18	8000 - 9000	3
5000 - 6000	9	9000 - 10000	1
6000 - 7000	7	10000 - 11000	1

Find the mode of the data.

Sol. The class 4000 - 5000 has maximum frequency. So, it is the modal class.

$\therefore l = 4000, f_1 = 18, f_0 = 4, f_2 = 9$ and $h = 1000$.

$$\begin{aligned}
 \text{Mode} &= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\
 &= 4000 + \left(\frac{18 - 4}{2 \times 18 - 4 - 9} \right) \times 1000 \\
 &= 4000 + \left(\frac{14}{36 - 4 - 9} \right) \times 1000 \\
 &= 4000 + \left(\frac{14000}{23} \right) \\
 &= 4000 + 608.7
 \end{aligned}$$

Hence,

mode = 4608.7 runs

Q.6. A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below. Find the mode of the data.

Number of cars	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	7	14	13	12	20	11	15	8

Sol. The class 40 - 50 has maximum frequency. So, it is the modal class.

$\therefore l = 40, f_1 = 20, f_0 = 12, f_2 = 11$ and $h = 10$

$$\begin{aligned}
 \text{Mode} &= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\
 &= 40 + \left(\frac{20 - 12}{2 \times 20 - 12 - 11} \right) \times 10 \\
 &= 40 + \left(\frac{8}{40 - 12 - 11} \right) \times 10 = 40 + \frac{80}{17} = 40 + 4.7 = 44.7.
 \end{aligned}$$

Hence,

mode = 44.7 cars.

Textbook Exercises 13.3

Q.1. The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the median, mean and mode of the data and compare them.

Monthly consumption (in units)	Number of consumers
65 - 85	4
85 - 105	5
105 - 125	13
125 - 145	20
145 - 165	14
165 - 185	8
185 - 205	4

Sol. For calculating the median we prepare cumulative frequency distribution table as given below :

Monthly consumption (in units)	Number of consumers (frequency)	Cumulative frequency (cf)
65 - 85	4	4
85 - 105	5	9
105 - 125	13	22
125 - 145	20	42
145 - 165	14	56
165 - 185	8	64
185 - 205	4	68
Total	$n = \sum f_i = 68$	

$$n = 68 \Rightarrow \frac{n}{2} = \frac{68}{2} = 34$$

But 34 comes under the cumulative frequency 42 and the class interval against the cumulative frequency 42 is 125-145. So, it is the median class.

$\therefore l = 125, cf = 22, f = 20$ and $h = 20$

$$\therefore \text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h = 125 + \left(\frac{34 - 22}{20} \right) \times 20 = 125 + 12 = 137.$$

For calculating mean, we prepare the table as given below :

Monthly consumption (in ₹)	Number of consumers (f_i)	Class mark (x_i)	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
65 - 85	4	75	-3	-12
85 - 105	5	95	-2	-10
105 - 125	13	115	-1	-13
125 - 145	20	$135 = a$	0	0
145 - 165	14	155	1	14
165 - 185	8	175	2	16
185 - 205	4	195	3	12
Total	$\sum f_i = 68$			$\sum f_i u_i = 7$

From the table, we have

$$\Sigma f_i = 68, \Sigma f_i u_i = 7, a = 135, h = 20$$

$$\therefore \text{Mean} = a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i} \right) \times h = 135 + \frac{7}{68} \times 20 = 135 + \frac{140}{68} = 135 + 2.05 = 137.05.$$

For calculating mode, the class 125 - 145 has maximum frequency. So, it is the modal class.

$$\therefore l = 125, f_0 = 13, f_1 = 20, f_2 = 14, \text{ and } h = 20$$

$$\begin{aligned} \text{Mode} &= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h = 125 + \left(\frac{20 - 13}{2 \times 20 - 13 - 14} \right) \times 20 \\ &= 125 + \frac{7 \times 20}{13} = 125 + \frac{140}{13} = 125 + 10.76 = 135.76. \end{aligned}$$

Hence, median, mean and mode are 137, 137.05 and 135.76 respectively. The three measures are approximately same in this case.

Q.2. If the median of the distribution given below is 28.5, find the values of x and y .

Class Interval	Frequency
0 - 10	5
10 - 20	x
20 - 30	20
30 - 40	15
40 - 50	y
50 - 60	5
Total	60

Sol. For calculating the median, we prepare the cumulative frequency distribution table as given below:

Class Interval	Frequency	Cumulative Frequency
0-10	5	5
10-20	x	$5 + x$
20-30	20	$25 + x$
30-40	15	$40 + x$
40-50	y	$40 + x + y$
50-60	5	$45 + x + y$
Total	60	

$$n = 60$$

$$\Rightarrow 45 + x + y = 60$$

$$\Rightarrow x + y = 60 - 45$$

$$\Rightarrow x + y = 15 \quad \dots(1)$$

Median is 28.5, which lies in the class 20 - 30.

So, it is the median class.

$$\therefore l = 20, f = 20, cf = 5 + x \text{ and } h = 10.$$

$$\therefore \text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

$$\Rightarrow 28.5 = 20 + \frac{\left[\frac{60}{2} - (5 + x) \right]}{20} \times 10$$

$$\Rightarrow 28.5 = 20 + \frac{(30 - 5 - x)}{20} \times 10 \Rightarrow 28.5 = 20 + \frac{(25 - x)}{2}$$

$$\Rightarrow 28.5 - 20 = \frac{25 - x}{2} \Rightarrow 8.5 = \frac{25 - x}{2}$$

$$\Rightarrow 17 = 25 - x \Rightarrow x = 25 - 17 \Rightarrow x = 8$$

Putting the value of x in equation (1), we get

$$8 + y = 15 \Rightarrow y = 15 - 8 = 7.$$

Hence,

$$x = 8, y = 7.$$

Q.3. A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are given only to persons having age 18 years onwards but less than 60 year.

Age (in years)	No. of policy holders	Age (in years)	No. of policy holders
Below 20	2	Below 45	89
Below 25	6	Below 50	92
Below 30	24	Below 55	98
Below 35	45	Below 60	100
Below 40	78		

Sol. Let us prepare the table depicting class intervals with their respective frequencies and cumulative frequencies from the given data as below:

Age (in years)	Number of policy holders (frequency)	Cumulative Frequency
Below 20	2 = 2	2
20 - 25	6 - 2 = 4	6
25 - 30	24 - 6 = 18	24
30 - 35	45 - 24 = 21	45
35 - 40	78 - 45 = 33	78
40 - 45	89 - 78 = 11	89
45 - 50	92 - 89 = 3	92
50 - 55	98 - 92 = 6	98
55 - 60	100 - 98 = 2	100
Total	$n = \sum f_i = 100$	

$$n = 100 \Rightarrow \frac{n}{2} = \frac{100}{2} = 50$$

But 50 comes under the cumulative frequency 78 and the class interval against the cumulative frequency 78 is 35 - 40. So, it is the median class.

$\therefore l = 35, cf = 45, f = 33$ and $h = 5$

$$\begin{aligned}\therefore \text{Median} &= l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h \\ &= 35 + \left(\frac{50 - 45}{33} \right) \times 5 = 35 + \frac{25}{33} = 35 + 0.76 = 35.76\end{aligned}$$

Hence, median age = 35.76 years.

Q.4. The lengths of 40 leaves of a plant are measured correct to the nearest millimetre, and data obtained is represented in the following table :

Length (in mm)	Number of leaves
118 - 126	3
127 - 135	5
136 - 144	9
145 - 153	12
154 - 162	5
163 - 171	4
172 - 180	2

Find the median length of the leaves.

Sol. The series is in inclusive form. We convert it into exclusive form and prepare the cumulative frequency table as given below :

Length (in mm)	Number of leaves (frequency)	Cumulative Frequency
117.5 - 126.5	3	3
126.5 - 135.5	5	8
135.5 - 144.5	9	17
144.5 - 153.5	12	29
153.5 - 162.5	5	34
162.5 - 171.5	4	38
171.5 - 180.5	2	40
Total	$n = \sum f_i = 40$	

$$n = 40 \Rightarrow \frac{n}{2} = 20$$

But 20 comes under the cumulative frequency 29 and the class interval against the cumulative frequency 29 is 144.5 - 153.5. So, it is the median class.

$\therefore l = 144.5, cf = 17, f = 12$, and $h = 9$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h = 144.5 + \left(\frac{20 - 17}{12} \right) \times 9 = 144.5 + \frac{9}{4} = 144.5 + 2.25 = 146.75.$$

Hence, median length of leaves = 146.75 mm.

Q.5. The following table gives the distribution of the life time of 400 neon lamps :

Life time (in hours)	Number of lamps
1500 - 2000	14
2000 - 2500	56
2500 - 3000	60
3000 - 3500	86
3500 - 4000	74
4000 - 4500	62
4500 - 5000	48

Find the median life time of a lamp.

Sol. We prepare the cumulative frequency distribution table as given below :

Life time (in hours)	Number of lamps (Frequency)	Cumulative (Frequency)
1500 - 2000	14	14
2000 - 2500	56	70
2500 - 3000	60	130
3000 - 3500	86	216
3500 - 4000	74	290
4000 - 4500	62	352
4500 - 5000	48	400
Total	$n = \sum f_i = 400$	

$$n = 400 \Rightarrow \frac{n}{2} = \frac{400}{2} = 200$$

But 200 comes under the cumulative frequency 216 and the class interval against the cumulative frequency 216 is 3000 – 3500. So, it is the median class.

$\therefore l = 3000, cf = 130, f = 86, \text{ and } h = 500$

$$\begin{aligned} \therefore \text{Median} &= l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h = 3000 + \left(\frac{200 - 130}{86} \right) \times 500 \\ &= 3000 + \frac{35000}{86} = 3000 + 406.98 = 3406.98 \end{aligned}$$

Hence, median life time of a lamp = 3406.98 hours.

Q.6. 100 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in the English alphabets in the surnames was obtained as follows :

Number of letters	1-4	4-7	7-10	10-13	13-16	16-19
Number of surnames	6	30	40	16	4	4

Determine the median number of letters in the surnames. Find the mean number of letters in the surnames? Also, find the modal size of the surnames. **[CBSE 2008]**

Sol. For calculating the median, we prepare the cumulative frequency distribution table as given below:

Number of Letters	Number of Surnames (frequency)	Cumulative Frequency
1 - 4	6	6
4 - 7	30	36
7 - 10	40	76
10 - 13	16	92
13 - 16	4	96
16 - 19	4	100
Total	$n = 100$	

$$n = 100 \Rightarrow \frac{n}{2} = \frac{100}{2} = 50$$

But 50 comes under the cumulative frequency 76. The class interval against cumulative frequency 76 is 7 - 10. So, it is the median class.

$$\therefore l = 7, cf = 36, f = 40 \text{ and } h = 3$$

$$\begin{aligned} \therefore \text{Median} &= l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h \\ &= 7 + \left(\frac{50 - 36}{40} \right) \times 3 = 7 + \frac{14 \times 3}{40} \\ &= 7 + \frac{42}{40} = 7 + 1.05 = 8.05 \end{aligned}$$

For calculating the mean we prepare table as given below :

Number of Letters	Number of Surnames (frequency)	Mid value (x_i)	$d_i = x_i - a$	$f_i d_i$
1 - 4	6	2.5	-6	-36
4 - 7	30	5.5	-3	-90
7 - 10	40	8.5 = a	0	0
10 - 13	16	11.5	3	48
13 - 16	4	14.5	6	24
16 - 19	4	17.5	9	36
	$\Sigma f_i = 100$			$\Sigma f_i d_i = -18$

From the table, we have

$$\Sigma f_i = 100, \Sigma f_i d_i = -18, a = 8.5$$

$$\therefore \text{Mean} = a + \left(\frac{\Sigma f_i d_i}{\Sigma f_i} \right) = 8.5 + \left(\frac{-18}{100} \right) = 8.5 - \frac{18}{100} = 8.5 - 0.18 = 8.32.$$

For calculating mode, the class 7 - 10 has maximum frequency. So, it is the modal class.

$$\therefore l = 7, f_1 = 40, f_0 = 30, f_2 = 16 \text{ and } h = 3$$

$$\begin{aligned} \text{Mode} &= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h = 7 + \left(\frac{40 - 30}{2 \times 40 - 30 - 16} \right) \times 3 \\ &= 7 + \frac{10 \times 3}{34} = 7 + \frac{30}{34} = 7 + 0.88 = 7.88 \end{aligned}$$

Hence, median = 8.05, Mean = 8.32 and modal size = 7.88.

Q.7. The distribution below gives the weights of 30 students of a class. Find the median weight of the students.

Weight (in kg)	40 - 45	45 - 50	50 - 55	55 - 60	60 - 65	65 - 70	70 - 75
Number of students	2	3	8	6	6	3	2

Sol. We prepare the cumulative frequency distribution table as given below.

Weight (in kg)	Number of Students (frequency)	Cumulative (frequency)
40 - 45	2	2
45 - 50	3	5
50 - 55	8	13
55 - 60	6	19
60 - 65	6	25
65 - 70	3	28
70 - 75	2	30
	$n = 30$	

$$n = 30 \Rightarrow \frac{n}{2} = \frac{30}{2} = 15$$

But 15 comes under the cumulative frequency 19 and the class interval against the cumulative frequency 19 is 55-60. So, it is the median class.

$\therefore l = 55, cf = 13, f = 6$ and $h = 5$

$$\begin{aligned} \text{Median} &= l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h = 55 + \left(\frac{15 - 13}{6} \right) \times 5 \\ &= 55 + \frac{10}{6} = 55 + 1.67 = 56.67 \end{aligned}$$

Hence, median weight of the students = 56.67 kg.

Textbook Exercises 14.1

Q.1. Complete the following statements:

- (i) Probability of an event E + Probability of the event 'not E' =
- (ii) The probability of an event that cannot happen is Such an event is called
- (iii) The probability of an event that is certain to happen is Such an event is called
- (iv) The sum of the probabilities of all the elementary events of an experiment is
- (v) The probability of an event is greater than or equal to and less than or equal to

Sol. (i) 1 (ii) 0, impossible event

(iii) 1, sure or certain event

(iv) 1 (v) 0, 1

Q.2. Which of the following experiments have equally likely outcomes ? Explain.

- (i) A driver attempts to start a car. The car starts or does not start.
- (ii) A player attempts to shoot a basket ball. She/he shoots or misses the shot.
- (iii) A trial is made to answer a true-false question. The answer is right or wrong.
- (iv) A baby is born. It is a boy or a girl.

Sol. (i) The outcome is not equally, likely because when there is some defect the car does not start.

(ii) The outcome in this situation is not equally likely because the outcome depends on many factors such quality of the gun, training of the player etc.

(iii) The outcome in this situation are equally likely because out come in this trial the answer is right or wrong *i.e.*, one out of the two and both have equal chances to happen.

(iv) The outcome in this situation are equally likely because a baby is born can be either a boy or girl.

Q.3. Why is tossing a coin considered to be fair way of deciding which team should get the ball at the beginning of a football game ?

Sol. When we toss a coin, the outcomes head and tail are equally likely. So the result of an individual coin toss is completely unpredictable.

Q.4. Which of the following cannot be the probability of an event ?

- (a) $\frac{2}{3}$ (b) -1.5 (c) 15% (d) 0.7

Sol. The value of probability of an event cannot be negative or greater than 1. So, the correct option is (B).

Q.5. If $P(E) = 0.05$, what is the probability of "not E" ?

Sol. We have,

$$P(E) = 0.05$$

$$P(\text{not } E) = 1 - P(E)$$

$$= 1 - 0.05 = 0.95.$$

Hence, $P(\text{not } E) = 0.95$.

Q.6. A bag contains lemon flavoured candies only. Malini takes out one candy without looking into the bag. What is the probability that she takes out

- (i) an orange flavoured candy ?
- (ii) a lemon flavoured candy ?

Sol. (i) Let E denote the event to get an orange flavoured candy.

There is no outcome favourable to the event E because all the candies are lemon flavoured.

So, $P(E) = 0$.

(ii) Let F denote the event to get a lemon flavoured candy.

All the outcomes favourable to event F because all the candies in the bag lemon flavoured.

So $P(F) = 1$.

Hence, (i) $P(E) = 0$ (ii) $P(F) = 1$.

Q.7. It is given that in a group of 3 students, the probability of 2 students not having the same birthday is 0.992. What is the probability that the 2 students have the same birthday?

Sol. Let E denote the event that two students out of three have same birthday.

$$P(\text{not } E) = 0.992 \text{ (given)}$$

$$\begin{aligned} \text{Then } P(E) &= 1 - P(\text{not } E) \\ &= 1 - 0.992 \\ &= 0.008. \end{aligned}$$

$$\text{Hence, } P(E) = 0.008.$$

Q.8. A bag contains 3 red balls and 5 black balls. A ball is drawn at random from the bag. What is the probability that the ball drawn is (i) red ? (ii) not red ?

$$\begin{aligned} \text{Sol. Number of all possible out comes} \\ &= 3 + 5 = 8 \end{aligned}$$

$$\begin{aligned} \text{(i) Number of red balls} &= 3 \\ \text{Number of favourable outcomes} &= 3 \\ \therefore P(\text{red ball}) &= \frac{3}{8}. \end{aligned}$$

$$\begin{aligned} \text{(ii) Number of not red balls} \\ &= \text{black balls} = 5 \\ \text{Number of favourable outcomes} &= 5 \\ P(\text{not red ball}) &= \frac{5}{8}. \end{aligned}$$

$$\text{Hence, (i) } P(\text{red ball}) = \frac{3}{8}, \text{ (ii) } P(\text{not red ball}) = \frac{5}{8}.$$

Q.9. A box contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out of the box at random. What is the probability that the marble taken out will be

(i) red ? (ii) white ? (iii) not green ?

$$\begin{aligned} \text{Sol. Total number of possible out comes} \\ &= 5 + 8 + 4 = 17 \end{aligned}$$

$$\begin{aligned} \text{(i) Number of red marbles} &= 5 \\ \text{Number of favourable outcomes} &= 5 \\ P(\text{a red marble}) &= \frac{5}{17}. \end{aligned}$$

$$\begin{aligned} \text{(ii) Number of white marbles} &= 8 \\ \text{Number of favourable outcomes} &= 8 \\ P(\text{a white marble}) &= \frac{8}{17}. \end{aligned}$$

$$\begin{aligned} \text{(iii) Number of marbles which are not green} \\ &= 5 + 8 = 13 \\ \text{Number of favourable outcomes} &= 13 \\ \therefore P(\text{a marble not green}) &= \frac{13}{17}. \end{aligned}$$

$$\text{Hence, (i) } P(\text{a red marble}) = \frac{5}{17}$$

$$\text{(ii) } P(\text{a white marble}) = \frac{8}{17}$$

$$\text{(iii) } P(\text{a marble not green}) = \frac{13}{17}.$$

Q.10. A piggy bank contains hundred 50 P coins, fifty ₹ 1 coins, twenty ₹ 2 coins and ten ₹ 5 coins. If it is equally likely that one of the coins will fall out when the bank is turned upside down, what is the probability that the coin (i) will be a 50 P coin ? (ii) will not be a ₹ 5 coin ?

$$\begin{aligned} \text{Sol. Total number of coins} \\ &= 100 + 50 + 20 + 10 = 180 \end{aligned}$$

$$\text{Total number of possible outcomes} = 180$$

$$\begin{aligned} \text{(i) Number of 50 paise coins} &= 100 \\ \text{Number of favourable outcomes} &= 100 \end{aligned}$$

$$\therefore P(50 \text{ paise coins}) = \frac{100}{180} = \frac{5}{9}.$$

$$\begin{aligned} \text{(ii) Number of coins other than ₹ 5 coins} \\ &= 100 + 50 + 20 = 170 \end{aligned}$$

$$\text{Number of favourable outcomes} = 170$$

$$P(\text{will not be a ₹ 5 coins}) = \frac{170}{180} = \frac{17}{18}.$$

$$\begin{aligned} \text{Hence, (i) } P(50 \text{ P coins}) &= \frac{5}{9} \text{ (ii) } P(\text{will not be a ₹ 5 coins}) = \frac{17}{18}. \end{aligned}$$

Q.11. Gopi buys a fish from a shop for his aquarium. The shopkeeper takes out one fish at random from a tank containing 5 male fish and 8 female fish (see in figure).



What is the probability that the fish taken out is a male fish ?

$$\begin{aligned} \text{Sol. Total fish in the aquarium} \\ &= 5 + 8 = 13 \end{aligned}$$

$$\text{Total number of possible outcomes} = 13$$

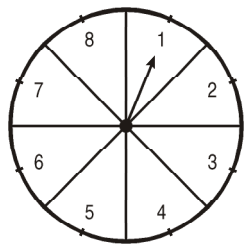
$$\text{Number of male fish} = 5$$

$$\text{Number of favourable outcomes} = 5$$

$$P(\text{male fish}) = \frac{5}{13}.$$

$$\text{Hence, } P(\text{male fish}) = \frac{5}{13}.$$

Q.12. A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8, (see in figure), and these are equally likely outcomes. What is the probability that it will point at



- (i) 8 ?
- (ii) an odd number ?
- (iii) a number greater than 2 ?
- (iv) a number less than 9 ?

Sol. Since arrow can come to rest at any one of the numbers 1, 2, 3, 4, 5, 6, 7, 8.

Total number of possible outcomes = 8

- (i) Number of favourable outcomes = 1

$$\therefore P(\text{Arrow point at 8}) = \frac{1}{8}.$$

- (ii) Odd numbers are 1, 3, 5, 7

Number of favourable outcomes = 4

$$\therefore P(\text{Arrow points at an odd number}) = \frac{4}{8} = \frac{1}{2}.$$

- (iii) The numbers greater than 2 are 3, 4, 5, 6, 7, 8

Number of favourable outcomes = 6

$$\therefore P(\text{Arrow points a number greater than 2}) = \frac{6}{8} = \frac{3}{4}.$$

- (iv) The numbers less than 9 are 1, 2, 3, 4, 5, 6, 7, 8.

Number of favourable outcomes = 8.

$$\therefore P(\text{Arrow points at a number less than 9}) = \frac{8}{8} = 1.$$

$$\text{Hence, (i) } P(\text{Arrow point at 8}) = \frac{1}{8}$$

$$\text{(ii) } P(\text{Arrow points at an odd number}) = \frac{1}{2}$$

$$\text{(iii) } P(\text{Arrow points at a number greater than 2}) = \frac{3}{4} \quad \text{(iv) } P(\text{Arrow points at a number less than 9}) = 1.$$

Q.13. A die is thrown once. Find the probability of getting

- (i) a Prime number
- (ii) a number lying between 2 and 6
- (iii) an odd number.

Sol. In a single throw of a die, all possible outcomes are 1, 2, 3, 4, 5, 6.

Total number of possible outcomes = 6

- (i) Let E_1 be event of getting a prime number. Prime numbers are 2, 3, 5
Number of favourable outcomes = 3

$$\therefore P(\text{getting a prime number})$$

$$= P(E_1) = \frac{3}{6} = \frac{1}{2}.$$

- (ii) Let E_2 be event of getting a number lying between 2 and 6

The numbers lying between 2 and 6 are 3, 4, 5

Number of favourable outcomes = 3

$$P(E_2) = \frac{3}{6} = \frac{1}{2}.$$

- (iii) Let E_3 be event of getting an odd number
The odd numbers are 1, 3, 5

Number of favourable outcomes = 3

$$P(E_3) = \frac{3}{6} = \frac{1}{2}.$$

$$\text{Hence, (i) } P(E_1) = \frac{1}{2} \quad \text{(ii) } P(E_2) = \frac{1}{2}$$

$$\text{(iii) } P(E_3) = \frac{1}{2}.$$

Q.14. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting of (i) a king of red colour (ii) a face card (iii) a red face card. (iv) the jack of hearts (v) a spade (vi) the queen of diamonds.

Sol. Total number of all possible outcomes = 52

- (i) Number of king of red colour = 2

Number of favourable outcomes = 2

$P(\text{getting a king of red colour})$

$$= \frac{2}{52} = \frac{1}{26}.$$

- (ii) We know that kings, queens and jacks are the face cards
 Number of face cards = 12
 Number of favourable outcomes = 12
 $P(\text{getting a face card}) = \frac{12}{52} = \frac{3}{13}$.
- (iii) Number of red face cards = 6
 Number of favourable outcomes = 6
 $P(\text{getting a red face card}) = \frac{6}{52} = \frac{3}{26}$.
- (iv) Number of jack of hearts = 1
 Number of favourable outcomes = 1
 $P(\text{getting the jack of hearts}) = \frac{1}{52}$.
- (v) Number of spades = 13
 Number of favourable outcomes = 13
 $P(\text{getting a spade}) = \frac{13}{52} = \frac{1}{4}$.
- (vi) Number of queen of diamonds = 1
 Number of favourable outcome = 1
 $P(\text{getting the queen of diamonds}) = \frac{1}{52}$.

Hence, (i) $P(\text{getting a king of red colour})$

$$= \frac{1}{26} \quad \text{(ii) } P(\text{getting a face card}) = \frac{3}{13}$$

$$\text{(iii) } P(\text{getting a red face card}) = \frac{3}{26}$$

$$\text{(iv) } P(\text{getting the jack of hearts}) = \frac{1}{52} \quad \text{(v)}$$

$$P(\text{getting a spade}) = \frac{1}{4} \quad \text{(vi) } P(\text{getting the queen of diamonds}) = \frac{1}{52}.$$

Q.15. Five cards—the ten, jack, queen, king and ace of diamonds, are well- shuffled with their face downwards. One card is then picked up at random.

- (i) What is the probability that the card is the queen ?
 (ii) If the queen is drawn and put aside, what is the probability that the second card picked up is (a) an ace ? (b) a queen?

Sol. (i) The total number of cards = 5
 Total number of possible outcomes = 5
 Out of these 5 cards only one is queen
 Number of favourable outcomes = 1

$$\therefore P(\text{a queen}) = \frac{1}{5}.$$

- (ii) After the queen has been drawn and put a side total number of cards = 5 – 1 = 4

Number of possible outcomes = 4

These four cards include one ace and no queen

$$P(\text{an ace}) = \frac{1}{4}, P(\text{a queen}) = 0.$$

$$\text{Hence, (i) } P(\text{a queen}) = \frac{1}{5} \quad \text{(ii) (a) } P(\text{an ace}) = \frac{1}{4} \quad \text{(b) } P(\text{a queen}) = 0.$$

Q.16. 12 defective pens are accidentally mixed with 132 good ones. It is not possible to just look at a pen and tell whether or not it is defective. One pen is taken out at random from this lot. Determine the probability that the pen taken out is a good one.

Sol. Number of total pens = 12 + 132 = 144

Total number of possible outcomes = 144

Number of good pens = 132

Number of favourable outcomes = 132

$$P(\text{good pens}) = \frac{132}{144} = \frac{11}{12}.$$

$$\text{Hence, } P(\text{good pens}) = \frac{11}{12}.$$

Q.17. (i) A lot of 20 bulbs contain 4 defective ones. One bulb is drawn at random from the lot. What is the probability that this bulb is defective?

(ii) Suppose the bulb drawn in (i) is not defective and is not replaced. Now one bulb is drawn at random from the rest. What is the probability that this bulb is not defective ?

Sol. (i) Total number of bulbs = 20

Total number of possible outcomes = 20

Number of bulbs defective in this lot = 4

Number of favourable outcomes = 4

$$P(\text{a defective bulb}) = \frac{4}{20} = \frac{1}{5}$$

(ii) According to question, the bulb drawn is not defective and it is not replaced.

Total number of bulbs contains the lot = 19

Total number of possible outcomes = 19

Number of good bulbs are

$$= 19 - 4 = 15$$

Number of favourable outcomes = 15

$$\therefore P(\text{a non defective bulb}) = \frac{15}{19}$$

Hence, (i) $P(\text{a defective bulb}) = \frac{1}{5}$ (ii) $P(\text{a}$

non defective bulb) = $\frac{15}{19}$

Q.18. A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears.

(i) a two-digit number

(ii) a perfect square number

(iii) a number divisible by 5.

Sol. Total number of possible outcomes = 90

(i) Two digit numbers from 1 to 90 are 10, 11, 12, 13,..... 90

Total two digit numbers = 81

Number of favourable outcomes = 81

$\therefore P(\text{a disc bearing a two digit number})$

$$= \frac{81}{90} = \frac{9}{10}$$

(ii) Perfect square numbers from 1 to 90 are 1, 4, 9, 16, 25, 36, 49, 64, 81.

Total perfect square numbers = 9

Number of favourable outcomes = 9

$\therefore P(\text{a disc bearing a perfect square$

number) = $\frac{9}{90} = \frac{1}{10}$

(iii) The numbers divisible by 5 from 1 to 90 are 5, 10, 15, 20, 90.

Total numbers divisible by 5 from 1 to 90 = 18

Number of favourable outcomes = 18

$P(\text{a disc bearing a number divisible by 5})$

$$= \frac{18}{90} = \frac{1}{5}$$

Hence, (i) $P(\text{a disc bearing a two digit number})$

= $\frac{9}{10}$ (ii) $P(\text{disc bearing a perfect square$

number) = $\frac{1}{10}$ (iii) $P(\text{a disc bearing a number$

divisible by 5) = $\frac{1}{5}$.

Q.19. A child has a die whose six faces shows the letters as given below :

A B C D E A

The die is thrown once. What is the probability of getting (i) A ? (ii) D ?

Sol. In throwing the die any one of the six faces may come upward.

Total number of possible outcomes = 6

(i) Since there are two faces with letter A

$\therefore$ Number of favourable outcomes = 2

$$\therefore P(\text{getting A}) = \frac{2}{6} = \frac{1}{3}$$

(ii) There is one face with letter D

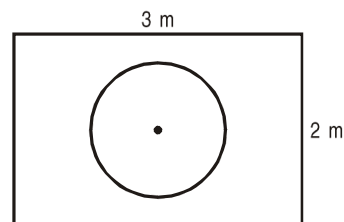
Number of favourable outcome = 1

$$P(\text{getting D}) = \frac{1}{6}$$

Hence, (i) $P(\text{getting A}) = \frac{1}{3}$

(ii) $P(\text{getting D}) = \frac{1}{6}$

Q.20. * Suppose you drop a die at random on the rectangular region shown in figure. What is the probability that it will land inside the circle with diameter 1 m ?



Sol. Total area of the rectangular region

$$= l \times b = 3 \times 2 = 6 \text{ m}^2$$

Area of the circular region

$$= \pi r^2$$

$$= \pi \times \left(\frac{1}{2}\right)^2 = \frac{\pi}{4} \text{ m}^2$$

$P(\text{the die lands inside the circle})$

$$= \frac{\text{Area of the circular region}}{\text{Total area of rectangular region}}$$

$$= \frac{\frac{\pi}{4}}{6} = \frac{\pi}{24}$$

Hence, $P(\text{the die lands inside the circle})$

$$= \frac{\pi}{24}$$

(* This Question is not from the point of Board Examination.)

Q.21. A lot consists of 144 ball pens of which 20 are defective and the others are good. Nuri

will buy a pen if it is good, but will not buy if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that

(i) She will buy it ?

(ii) She will not buy it ?

Sol. Total number of pens = 144

Total number of possible outcomes = 144

(i) Number of defective pens = 20

The number of good pens

$$= 144 - 20 = 124$$

Number of favourable outcomes

$$= 124$$

P (Nuri will buy good pen)

$$= \frac{124}{144} = \frac{31}{36}$$

(ii) P (Nuri will not buy a good pen)

$$= P$$

(selecting a defective pen)

$$= \frac{20}{144} = \frac{5}{36}$$

Hence, (i) P (Nuri will buy a good pen) = $\frac{31}{36}$,

(ii) P (Nuri will not buy a good pen) = $\frac{5}{36}$.

Q.22. Refer to example 25 of NCERT (i) complete the following table :

Event sum on 2 dice	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$						$\frac{5}{36}$				$\frac{1}{36}$

(ii) A student argues that 'there are 11 possible out comes 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12. Therefore, each of them has a probability $\frac{1}{11}$. Do you agree with this argument? Justify your answer.

Sol. Total number of possible outcomes of the toss of the pair of two dice = 36

(i) (1, 2), (2, 1), favour the event of the getting the sum 3

$$\therefore P(\text{getting the sum 3}) = \frac{2}{36}$$

Cases (1, 3), (2, 2), (3, 1) favour the event of getting sum 4

$$\therefore P(\text{getting the sum 4}) = \frac{3}{36}$$

Cases (1, 4), (2, 3), (3, 2), (4, 1) favour the event of getting the sum 5

$$\therefore P(\text{getting the sum 5}) = \frac{4}{36}$$

Cases (1, 5), (2, 4), (3, 3), (4, 2), (5, 1) favour the event of getting the sum 6

$$\therefore P(\text{getting the sum 6}) = \frac{5}{36}$$

Cases (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1) favour the event of getting the sum 7

$$\therefore P(\text{getting the sum 7}) = \frac{6}{36}$$

Cases (2, 6), (3, 5), (4, 4), (5, 3), (6, 2) favour the event of getting the sum 8

$$\therefore P(\text{getting the sum 8}) = \frac{5}{36}$$

Cases (3, 6), (4, 5), (5, 4), (6, 3) favour the event of getting the sum 9

$$\therefore P(\text{getting the sum 9}) = \frac{4}{36}$$

Cases (4, 6), (5, 5), (6, 4) favour the event of getting the sum 10

$$\therefore P(\text{getting the sum 10}) = \frac{3}{36}$$

Cases (5, 6), (6, 5), (6, 4) favour the event of getting the sum 11

$$\therefore P(\text{getting the sum 11}) = \frac{2}{36}$$

Event Sum on 2 dice	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

- (ii) No, as the eleven different outcomes are not equally likely.

Q.23. A game consists of tossing a one rupee coin 3 times and noting its outcome each time. Hanif wins if all the tosses give the same result *i.e.*, three heads or three tails, and loses otherwise. Calculate the probability that Hanif will lose the game.

Sol. When a coin tossed three times, the possible outcomes are (H, H, H), (H, H, T), (H, T, T), (H, T, H), (T, H, H), (T, H, T), (T, T, H) and (T, T, T)

Total number of possible outcomes = 8

Number of cases when all the tosses give the same results are two *i.e.*, (H, H, H) and (T, T, T)

It is given that Hanif wins when all the tosses give the same results,

Number of cases in which Hanif will lose the game = $8 - 2 = 6$

Number of favourable outcomes = 6

$\therefore P$ (Hanif will lose the game)

$$= \frac{6}{8} = \frac{3}{4}$$

Hence, P (Hanif will lose the game) = $\frac{3}{4}$

Q.24. A die is thrown twice. What is the probability that

- (i) 5 will not come up either time ?

- (ii) 5 will come up at least once ?

[Hint—Throwing a die twice and throwing two dice simultaneously are treated as the same experiment.]

Sol. A die is thrown twice

Total number of possible outcomes = 36

- (i) Out of these 36 outcomes, 5 comes at least once in the following cases :

(1, 5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5), (5, 1), (5, 2), (5, 3), (5, 4), (5, 6)

Total number of these cases = 11

$\therefore$ Number of cases when 5 will not come up either time = $36 - 11 = 25$

Number of favourable outcomes = 25

$\therefore P$ (5 will not come up either time)

$$= \frac{25}{36}$$

- (ii) Number of cases when 5 comes up at least once = 11

Number of favourable outcomes

$$= 11$$

$\therefore P$ (5 will come up at least once)

$$= \frac{11}{36}$$

Hence, (i) P (5 will not come up either time)

$$= \frac{25}{36}$$

- (ii) P (5 will come up at least once)

$$= \frac{11}{36}$$

Q.25. Which of the following arguments are correct and which are not correct? Give reasons for your answer.

- (i) If two coins are tossed simultaneously there are three possible outcomes —two heads, two tails or one of each. Therefore, for each of these outcomes, the probability is $1/3$.

- (ii) If a die is thrown, there are two possible outcomes — an odd number or an even number. Therefore, the probability of getting an odd number is $1/2$.

Sol. When two coins are tossed.

- (i) The possible outcomes are (H, H), (H, T), (T, H), (T, T)

Total number of possible outcomes = 4

$$P(H, H) = \frac{1}{4}$$

$$P(T, T) = \frac{1}{4}$$

$$P(H, T \text{ and } T, H) = \frac{2}{4} = \frac{1}{2}$$

Hence, argument is incorrect.

- (ii) When a die is thrown

The possible outcomes are 1, 2, 3, 4, 5, 6

Total number of possible outcomes = 6

We have odd numbers or even numbers

Odd numbers are 1, 3, 5

Number of favourable outcomes = 3

Probability (getting an odd number)

$$= \frac{3}{6} = \frac{1}{2}$$

Hence, argument is correct.

